

# RELIABILITY ANALYSIS OF CONCRETE BEAMS: A Complete Review

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**Abstract** - When we consider a structure or a building, there is a possibility of uncertainties or variability in material properties and loading. This review paper shows the studies of a complete review of literature on reinforced concrete beams which is based on structural reliability analysis and is solved by different probability-based design methods. The probability-based design approach has widespread acceptance and is a more precise and accurate way of treating uncertainties and variabilities in a structure. Using structural engineering, reliability in a structure is a performance or functionality that can be measured. The methods which are reviewed in this paper are FORM (First Order Reliability Method), Monte-Carlo Simulation, TCM (Three-Term Conjugate Map), Fault Tree Analysis, and Hasofer-Lind Method.

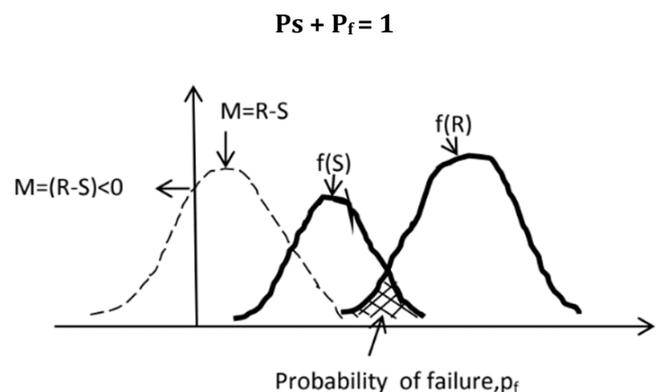
**Key Words:** First Order Reliability Method, Three-Term Conjugate Method, Fault Tree Analysis, Hasofer-Lind method, Structural Reliability.

## 1. INTRODUCTION

In a structure, there are many individual structural components, one of the important structural elements is a beam which carries the load from the walls and is made to cause bending, shear, and torsion and they are always dependent on the parameters such as depth, length, materials, and cross-sectional shapes. Analysis of reinforced concrete beams is designed by BS5950 (1990) and it was explained by FORM, at the serviceability and ultimate limit states. A random and stochastic approach is considered for design variables such as span, load ratio, and intensity of live loads. The serviceability limit state tells us that the appearance, cracking, vibration, exposure condition, and thermal resistance whereas the ULS tells us about the loading form in bending and shear. Steel beams transfer transverse loads by flexural action into a column in a structure. The function of several variables, most of them are random variables has a resistance in the structural member and the applied loads. Hence, the probabilistic approach enables the use of the design of structural safety to be treated more rationally.

Reliability methods are probability-based approaches that are used in assessing the performance and safety of structural components or systems. When the probability-based components or the system for a specific period meets

some specified demands under specified environmental conditions are defined as Reliability. Therefore, Reliability has the ability of the structural system to fulfill its design purpose for a specific time duration. The probability approaches in the reliability analysis take into account the uncertainties in the structural component variables or the system as a whole. Uncertainties are inherent in the component strength, random variables, and corresponding loading effects. Reliability may also be defined as the structure that can be applied without any damage for a certain amount of time under some probabilistic situations.



**Fig -1:** Graphical representation of Probability of failure ( $P_f$ ) and Margin of Safety ( $M$ )

Due to the presence of more structural components in the system, there is a possibility of failure. A failure mode in the system is considered by combining the structural elements that lead to the failure of the structural system. Identifying the failure modes in a structural system is important for the appropriate design and maintenance of the structural system. Reinforced concrete beams are subjected simultaneously to torsion, shear, and bending which is the case for a system reliability analysis is required when a whole structure or just a structural element has many modes.

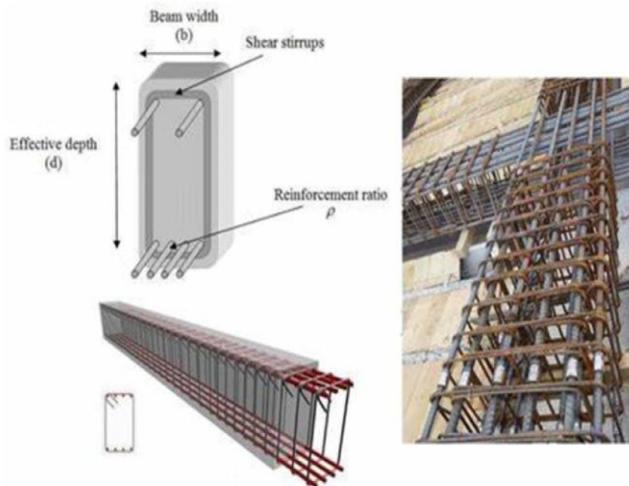


Fig -2: Typical representation of the Reinforced concrete beam (Source – Internet).

Structural Reliability is the modern and best way to deal with or handle uncertainties in engineering design. FORM, SORM, HL Method, Point Estimate method, Monte-Carlo Simulation, Three-Term Conjugate Map (TCM), MVFOSM, and Fault Tree Analysis are the different types of reliability approaches that can be used to find out the Safety Index ( $\beta$ ), and Probability of Failure ( $P_f$ ).

## 2. LITERATURE REVIEW

**2.1 Fault Tree Analysis:** A fault tree can be explained by a graphical logical flowchart that describes the inter-relationships of the basic structural components that are composing the system and leads to the occurrence of a predefined undesired top event in the tree. For structural engineering systems, the undesired top event is defined as structural system failure. The fault tree analysis identifies the failures (faults) that can tend to unintended work. Therefore, the fault tree shows the different modes (the sequence of basic component failures) that lead to the system failure.

The approximate and rigorous results are provided which might consider the structural building as a single system or a unit which will be emphasized because of the reliability which was utilized in the analysis instead of the structural reliability.

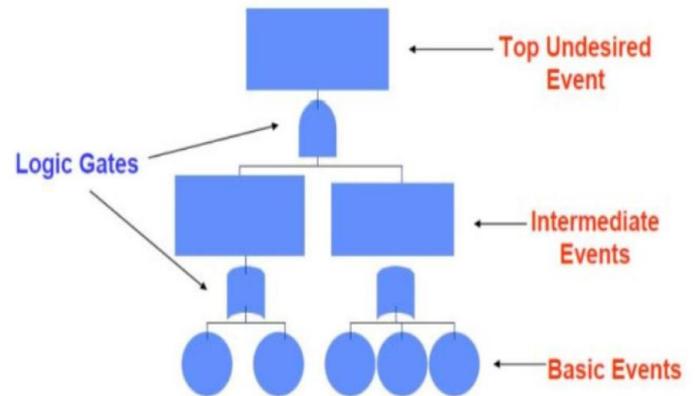


Fig -3: Fault Tree Diagram

**2.2 First Order Reliability Method (FORM):** The reliability of structural elements can be assessed and utilized by basic design approaches. For a perfect solution for the safety factor and probability of failure, the input is given as standard deviation values, mean values, load values, and resistance values in certain limit states.

It is vastly used in applications of estimation of structural reliability. Taylor expansion is involved in this procedure to find the equations in the limit state, which cannot be explained by the average values in the functions. Normally distributed variables are studied in this paper.

Using the Rosenblatt transformation, the method deals with the conversion which is excluding normal variables to the normal variables which give standard values and it is inclusive of zero average value that is the mean value, and the variance is always kept equal to one. The main agenda is to find the failure point which is defined as the distance from the origin of the reduced values and the minimum distance of the limit state surface of the reduced variables.

One of the performance equations formulated for the reinforced concrete beam is,

$$Z = R - S \dots\dots (R - \text{Resistance and } S - \text{Effect of loads})$$

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

### Probability of Failure, $P_f = \Phi(-\beta)$

For a reinforced concrete beam, when all the parameters are considered involving the effect of loads, and hence the equation is enumerated as,

$$g_1(X) = 0.87f_y A_s \left( d - \frac{0.87f_y A_s}{0.9f_{cu} b} \right) - \frac{(1.4DL + 1.6LL)L^2}{8}$$

Where, area of tension steel,  $A_s$  (mm<sup>2</sup>)  
 Yield strength of steel,  $f_y$  (N/mm<sup>2</sup>)  
 Beam width,  $b$  (mm)  
 Length of the beam,  $L$  (mm)  
 Compressive strength of concrete,  $f_{cu}$  (N/mm<sup>2</sup>)

The reliability index and probability of success for various steel beams were determined using the probability-based reliability of the analysis. The developed model with different parameters like length, depth, and strength was a stochastic model. Different dimensions of concrete beams were constructed and built with varying dimensions like depth, length, and thickness. It proves that if the safety index increases it will increment along the depth and a slight decrease in the probability of failure, which automatically increases the probability of success. For every 2m increment in the length of the beam, there is a decrement in the safety factor. The  $P_f$  always remains constant when the safety index is directly proportional to the characteristic strength of beams.

In one of the studies, it was found that the performance equations for every failure mode such as shear failure mode, bending failure, and deflection mode.

- Bending failure mode:

$$g(x) = 0.156f_{cu}bd^2 - 0.125(1.4\alpha + 1.6)Q_kL^2$$

- Deflection failure mode:

$$g(x) = \frac{L}{250} - \frac{5(1.4\alpha + 1.6)Q_kL^4}{384EI}$$

- Shear failure mode:

$$g(x) = 0.8\sqrt{f_{cu}} - \frac{(1.4\alpha + 1.6)Q_kL}{2b \times d}$$

Where, Length of the beam,  $L$  (mm)  
 $\alpha$  is the DL-LL ratio  
 Compressive strength of the concrete,  $f_y$  (N/mm<sup>2</sup>)

FORM uses the combination of analytical and approximate values and it solves the problems at each stage. Firstly, the parameters have been defined as Normal, Log-normal, or Gumbel distribution which may or may not be independent of each other. The limit state which was the original limit state is now converted into the surface of the new limit state.

**2.3 Monte Carlo Simulation:** Some of the simulation methods is very helpful for finding a solution for the safety index in a structure. Simulation is the process of conducting experiments on a model instead of applying experiments directly to the system or the components. A model, physical or mathematical, is a representation of the system or the components to study its performance. Monte Carlo simulation is the most common simulation technique. MCS deals with the non-probabilistic methods to probabilistically assess output variables of interest. The uncertainties in the material properties, loads, geometry, and various models are taken into consideration for evaluating the probability of failure and safety index.

A particular reliability index is found using varying lengths, depth, and characteristic strength, and changes in the probability of failure were observed.

$$g_1 = A_s - \frac{200}{f_y} b_w d$$

$$g_2 = A_s - \frac{0.85\beta_1 f'_c}{f_y} \frac{87000}{87000 + f_y} b_w d$$

Where,  $d$  – effective depth of the beam  
 $B_w$  – width of the beam  
 $A_s$  – area of tension reinforcement  
 $f_c$  – compressive strength  
 $f_y$  – yield strength of steel

It was clear that, when a relatively low value of Monte Carlo simulation, it can be very accurate to predict the probability of success in the pre-defined limit state. The  $P_f$  values are lower than  $10^5$  for MCS and vary significantly in the random variables, degrees of freedom, and complications in the problem.

**2.4 Three-term Conjugate Map (TCM):** It is based on one of the important methods in the reliability approach which is the basic probability-based design approach. The finite step-length concept is used for a formulation that was extracted from a non-linear conjugate map and is called the Three-term Conjugate Finite Step-Length Method (TCFS). The sensitivity vector consists of three terms which are namely the first term will be the gradient vector, the conjugate gradient in the intermediate term, and finally the last is the previous gradient vector in the succeeding event. Identifying the multi-story building failure of reinforced concrete beams is one of the main applications of the TCM method which is plotted against many formulations in the structural reliability of FORM methods.

FORM method may or may not provide stable results without the variations in solution by non-linear formulations. Using the probability-based approaches, TCM approaches a structural analysis of reinforced concrete beam which is tested in both forms of corrosion variations namely pitting corrosion and uniform corrosion, in this case, a special brittle failure mode is included for the formulations of the performance equation.

The distance between the reinforcement and the compressive fiber of the extreme layer and the width of the section increases directly the value of the system reliability of the system and it also implies other parameters.

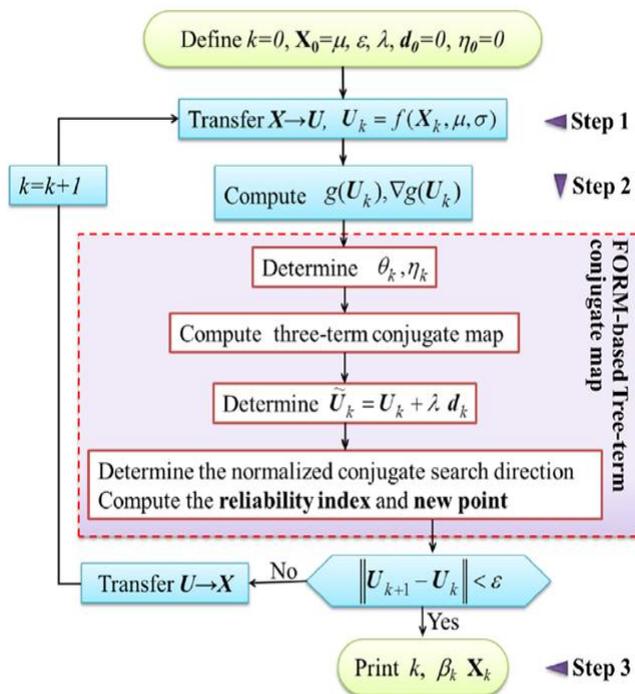


Fig -4: Flowchart for TCFS procedure.

**2.5 Hasofer-Lind Method:** The reliability index ( $\beta$ ) can be explained as the ratio of the average value of a particular event to the deviation of the same particular event. It can also be explained as the minimum length from the failure region to the origin of a design point. This can be explained by a common equation,

$$Z=R-S$$

$$\beta = \mu / \sigma$$

It also explains how the reliability index is defined by Mean Value First Order Second Moment (MVFOSM) method by the terms R and S which is the resistance and the effect of loads. We can tell that it is almost similar to MVFOSM if both the variables that are R and S are normally distributed.

### 3. RESULTS AND DISCUSSIONS

- Some uncertainties in the design are not consistent which are in a deterministic approach and have failed to put into consideration the measuring safety factors such as load factor and the factor of safety.
- The increment in the depth increases the safety index factor which automatically decreases the probability of failure. This is explained graphically in chart -1 by taking the values of the depth and the safety index.

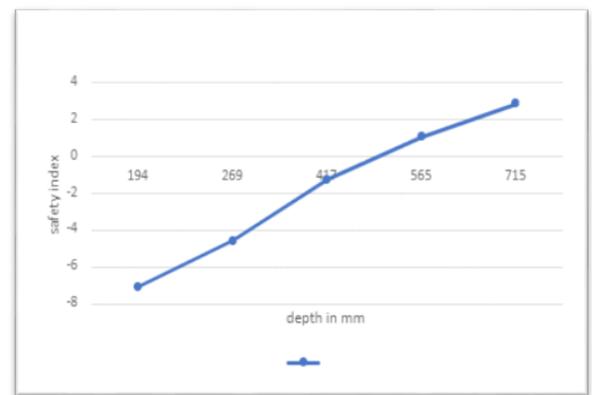


Chart -1: Graphical representation of Safety index vs Depth in mm.

- We can see from fig no.6 that the longer the length of the beam, it is unsafe for the reinforced concrete beam which also increases the tendency to fail. The length of the beam is increased by 2m each up to 10m to show that the safety index is inversely proportional to the length of the beam.

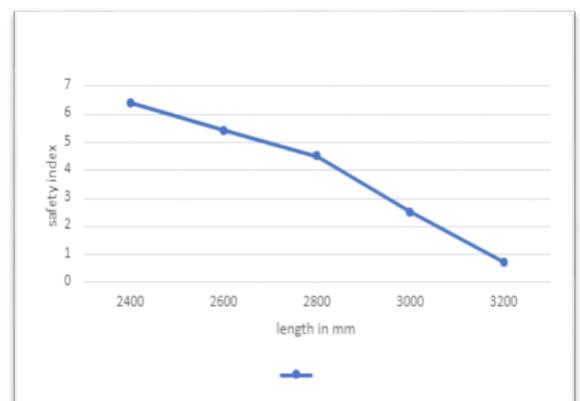
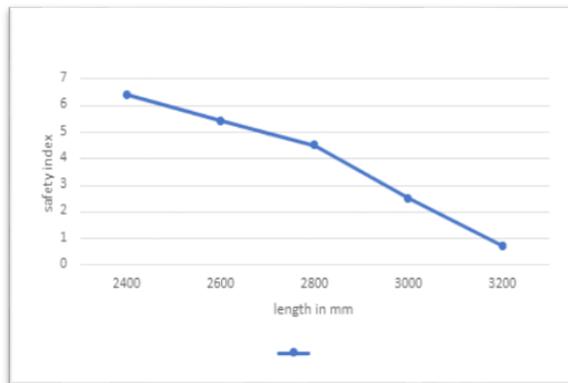


Chart -2: Safety index vs Length (mm)

- When the graph is plotted against the characteristic strength and the safety index, it shows that it varies linearly that is it is directly proportional to each

other and the probability of failure remains constant. This is shown in chart -3.



**Chart -3:** Safety index vs Characteristic strength (kN/m<sup>2</sup>)

#### 4. CONCLUSIONS

- The probability of failure ( $P_f$ ) remains constant when the safety index is directly proportional to characteristic strength.
- Probability of failure has a constant whole number of 1.0, when there is a decrement in the reliability index which automatically shows an increment in both the parameters namely characteristic strength and depth.
- The distance between the reinforcement and the compressive fiber of the extreme layer and the width of the section increases directly the value of the system reliability of the system and it also implies other parameters like compressive strength of concrete and specified strength of the beams.
- When the Hasofer-Lind method is used, the calculations were calculated for capacity, durability, and serviceability to find the reliability of the structural design in reinforced concrete beams.
- From the above review papers, it can be concluded that when we need to decrease the failure rate the cross-sectional area of the beam and steel should be increased.
- To produce more accurate results the system's reliability has its merits in predicting the life period of the building, but it can be costly and time-consuming.
- The Three-term Conjugate Finite Step-Length method shows very accurate results for non-linear structural components and is also more efficient

when compared to some of the probability-based design approaches.

#### REFERENCES

- [1] Daniel Ndakuta Kolo, James Isiwu Aguwa, Theophilus Yisa Tsado, Mohammed Abdullahi, Abdulazeez Yusuf, Sikiru Folahan Oritola1 "Reliability Studies On Reinforced Concrete Beam Subjected To Bending Forces With Natural Stone As Coarse Aggregate" Asian Journal of Civil Engineering (2021).
- [2] O J Aladegboye, D A Opeyemi, O D Atoyebi1, S L Akingbonmire and E M Ibitogbe "Reliability Analysis Of Reinforced Concrete Beam Using Varying Properties" 2020 Earth and Environmental Science.
- [3] Marin Grubišić, Jelena Ivošević, and Ante Grubišić "Reliability Analysis of Reinforced Concrete Frame by Finite Element Method with Implicit Limit State Functions" 2019, www.mdpi.com/journal/buildings.
- [4] U. Hamza, A. Aboshio, M. T. Abdulwahab and A. Suleiman "Reliability Assessment of Reinforced Concrete Panelled-Beams" 2021.
- [5] A.U. Barambu, O.A.U. Uche, and M.T. Abdulwahab "Reliability-Based Code Calibration for Load and Safety Factors for the Design of a Simply Supported Steel Beam" SEP: Journal of Research Information in Civil Engineering, Vol.14, No.1, 2017.
- [6] Behrooz Keshtegar, Shun-Peng Zhu "Three-Term Conjugate Approach for Structural Reliability Analysis" journal homepage: [www.elsevier.com/locate/apm](http://www.elsevier.com/locate/apm).
- [7] Dimitri V. Val "Deterioration of Strength of RC Beams Due To Corrosion and Its Influence on Beam Reliability" 2007.
- [8] Mohamed El Amine Ben Seghier, Behrooz Keshtegar and Hussam Mahmoud "Time-Dependent Reliability Analysis Of Reinforced Concrete Beams Subjected To Uniform And Pitting Corrosion And Brittle Fracture".
- [9] Hasan A Kamal, Ibrahim A Assakkaf, Hasan J Karam and Naser O Al-Enzi "Reliability Assessment of Existing Reinforced Concrete Structures" 2013.
- [10] Z I Syed, M Kewalramani and E S Hejah "Structural Reliability Of Dapped End Beams With Different Reinforcement Layouts Under Dynamic Loading" IOP Conf. Series: Materials Science and Engineering 75 (2019) 012003, 2018.
- [11] S O A Olawale, O P Akintunde, M O Afolabi1 and O A Agbede "Comparison of Reliability-Based Design (Rbd) With Eurocode 2 for A Singly Reinforced Concrete

Beam” IOP Conf. Series: Materials Science and Engineering.

- [12] Paolo Castaldo, Diego Gino, Gabriele Bertagnoli, Giuseppe Mancini “Resistance Model Uncertainty in Non-Linear Finite Element Analyses Of Cyclically Loaded Reinforced Concrete Systems” Engineering Structures 211 (2020) 110496.
- [13] Structural Reliability and Durability Assessment of Reinforced Concrete Structures.
- [14] Abubakar Idris and Pius Edache “Reliability Analysis of Simply Supported Steel Beams” Australian Journal of Basic and Applied Sciences, 1(1): 20-29, 2007 ISSN 1991-8178.
- [15] Renjian Lu, Yuanhui Luo, Joel P. Conte “Reliability Evaluation of Reinforced Concrete Beams” Structural Safety 14 (1994) 277-298.
- [16] P.Hari Prasad, T.Sumathi Uma Maheswari “Reliability Analysis of the Sectional Beams Due To Distribution of Shearing Stress” Research Inventory: International Journal of Engineering and Science Vol.6, Issue 4 (April 2016), PP -83-90 Issn (e): 2278-4721, Issn (p):2319-648.
- [17] A. S. Nowak, Chan-Hee Park, J.R. Casas “Reliability Analysis of Prestressed Concrete Bridge Girders: Comparison of Eurocode, Spanish Norma Iap, And Aashto Irfd” 8th ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability.
- [18] Reliability-based analysis and design for civil engineering- Devaraj V and Ravindra R.
- [19] Ditlevsen, O. and Madsen, H. O. (2005). Structural Reliability Methods. John Wiley & Sons, Chichester.
- [20] Goutham DR and Manjunath K 2016 “Reliability Analysis of Grid Floor Slabs” International Research Journal of Engineering and Technology (IRJET), 3(6). E-ISSN: 2395-0056, www.irjet.net p-ISSN: 2395-0072.
- [21] Choi SK, Grandhi R and Canfield RA 2006 “Reliability-Based Structural Design” Springer Science and Business.