# A REVERSE CONVERTER FOR THE FIVE COPRIME MODULI SET $\{2^n-1, 2^n, 2^n+1, 2^{n-1}-1, 2^{n+1}-1\}$

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## Abstract

Residue to binary conversion is presented for the five moduli set $\{2^n-1, 2^n, 2^n+1, 2^{n-1}-1, 2^{n+1}-1\}$ in this paper. A novel converter for the moduli set using modular adders, multipliers, and carry save adders is proposed using a cyclic jump method. The binary representation, hardware implementation and comparison with a state-of- the- art scheme put the proposed converter ahead. The moduli set is carefully selected to provide for larger dynamic range needed for digital signal processing.

**Keywords: Residue Number system; Moduli set; Dynamic Range; Cyclic Jump Technique**

## i. Introduction

Residue Number System (RNS) is an emerging area of research. This is because of its suitability for the implementation of high-speed digital signal processing devices and its inherent parallelism, modularity, fault tolerance and carry free propagation properties [7]. Arithmetic operations such as addition and multiplication are performed more easily and efficiently in RNS than conventional two's complement number systems [6].

The traditional moduli set $\{2^n+1,\ 2^n,\ 2^n-1\}$, has been one of the most popularly studied in RNS.

The moduli set $\{2^n-1, 2^n, 2^n+1, 2^{n-1}-1, 2^{n+1}-1\}$ which shares a common factor of 2 between the third, fourth and fifth moduli has been applied. This moduli set offers consecutiveness and allows for equal width adders and multipliers in hardware design. This gives it high study significance than the traditional moduli sets [5].

## ii. Fundamentals of Residue Number System(RNS)

RNS is presented using relatively prime moduli set $\{m_i\}_{i=1,k}$ such that, the greatest common divisor(gcd) of $(m_i, m_j) = 1$ for $i \neq j$, while $M = \prod_{i=1}^{k} m_i$, is the dynamic range. The residues of a decimal number $X$ is obtained as $x_i = |X|_{m_i}$.A decimal number X can therefore be represented in RNS as $X = (x_1, x_2, x_3, \dots, x_k)$, $0 \leq x_i \leq m_i$. This representation is unique for any integer $X \in [0, M-1]$. $|X|_{m_i}$is the modulo operation of $X$ with respect to $m_i$ [1],[4].

Two standard conversion techniques exist for performing reversion conversions in RNS. They are The Chinese Remainder Theorem and the Mixed Radix Conversion Method. However, other derived versions for performing backward conversion also exist.

## iii. Mixed Radix Conversion

The Mixed Radix Conversion (MRC) approach is an alternative method to the CRT for performing reverse conversion. This method does not involve the use of the large modulo-M computation as is required by the CRT. This method is used to perform `residue to binary conversion of $(x_1, x_2, x_3, x_4, x_5)$ based on the moduli set $\{m_1, m_2, \dots m_5\}$ as follows;

$$X = a_1 + a_2m_1 + a_3m_1m_2 + a_4m_1m_2m_3 + a_5m_1m_2m_3m_4 + a_nm_1m_2m_3 \dots m_{k-1} \qquad (2)$$

Where $a_{i,i=1,k}$ are the Mixed Radix Digits (MRDs) which can be computed below as shown in [2],[3],[8] ;

$$a_1 = x_1$$
$$a_2 = \left|(x_2 - a_1)|m_1^{-1}|_{m_2}\right|_{m_2}$$
$$a_3 = \left|((x_3 - a_1)|m_1^{-1}|_{m_3} - a_2)|m_2^{-1}|_{m_3}\right|_{m_3}$$
$$\vdots$$
$$a_k = \left|(((x_k - a_1)|m_1^{-1}|_{m_k} - a_2)|m_2^{-1}|_{m_k} - \dots - a_{k-1})|m_{k-1}^{-1}|_{m_k}\right|_{m_k}$$

## iv. The Cyclic Jump Technique

A cyclic jump approach to reverse conversion is presented in this paper. The technique uses the first residue as an initial position and then jumps to new locations until a final point is reached. The various jumps are then summed when all residues turn to zero, to arrive at the decimal number $X$. This technique is an MRC based approach.

**v. Jump Technique for the 5- Moduli Set $\{2^n - 1, 2^n, 2^n + 1, 2^{n-1} - 1, 2^{n+1} - 1\}$**

**General algorithm for a five moduli set**

1. The first jump is defined by the number $J_1$ which normally corresponds to the first residue in X i.e. $J_1 = r_1$.

The first location $L_1$ after the jump $J_1$ is defined by $L_1 = X - J_1$

Thus

$$L_1 = X - J_1 = \begin{bmatrix} |r_1 - J_1|_{m_1} = r_1' \\ |r_2 - J_1|_{m_2} = r_2' \\ |r_3 - J_1|_{m_3} = r_3' \\ |r_4 - J_1|_{m_4} = r_4' \\ |r_5 - J_1|_{m_5} = r_5' \end{bmatrix}$$

2. The second jump is defined by the number $J_2$ , such that:

$J_2 = m_1 K_2$ and $|r_2' - J_2|_{m2} = 0 \implies - m_1 K_2|_{m2} = 0$

$K_2 = \left|\frac{r_2'}{m_1}\right|_{m_2}$

The second location $L_2$ is given by $L_2 = r - J_1 - J_2$

Thus

$$L_2 = X - J_1 - J_2 = L_1 - J_2 = \begin{bmatrix} |r_1' - J_2|_{m_1} = r_1'' \\ |r_2' - J_2|_{m_2} = r_2'' \\ |r_3' - J_2|_{m_3} = r_3'' \\ |r_4' - J_2|_{m_4} = r_4'' \\ |r_5' - J_2|_{m_5} = r_5'' \end{bmatrix}$$

3. The third jump is defined by $J_3$ , such that

$J_3 = m_1 m_2 K_3$ and $|r_3'' - J_3|_{m_3} = 0 \implies |r_3'' - m_1 m_2 K_3|_{m_3} = 0$ $K_3 = \left|\frac{r_3''}{m_1 m_2}\right|_{m_3}$

The third location is given by $L_3 = r_1 - J_1 - J_2 - J_3$

Thus

$$L_3 = X - J_1 - J_2 - J_3 = L_2 - J_3 = \begin{bmatrix} |r_1'' - J_3|_{m_1} = r_1''' \\ |r_2'' - J_3|_{m_2} = r_2''' \\ |r_3'' - J_3|_{m_3} = r_3''' \\ |r_4'' - J_3|_{m_4} = r_4''' \\ |r_5'' - J_3|_{m_5} = r_5''' \end{bmatrix}$$

4. The fourth jump is defined by $J_4$ , such that:

$J_4 = m_1 m_2 m_3 K_4$ and $|r_4''' - J_4|_{m_4} = 0 \implies |r_4''' - m_1 m_2 m_3 K_4|_{m_4} = 0$ $K_4 = \left|\frac{r_4'''}{m_1 m_2 m_3}\right|_{m_4}$

The fourth location is given by $L_4 = r_1 - J_1 - J_2 - J_3 - J_4$

Thus

$$L_4 = X - J_1 - J_2 - J_3 - J_4 = L_3 - J_4 = \begin{bmatrix} |r_1''' - J_4|_{m_1} = r_1'''' \\ |r_2''' - J_4|_{m_2} = r_2'''' \\ |r_3''' - J_4|_{m_3} = r_3'''' \\ |r_4''' - J_4|_{m_4} = r_4'''' \\ |r_5''' - J_4|_{m_5} = r_5'''' \end{bmatrix}$$

5. The fifth jump is defined by $J_5$ such that :

$J_5 = m_1 m_2 m_3 m_4 K_5$ and $|r_4'''' - J_5|_{m_5} = 0$ $|r_4'''' - m_1 m_2 m_3 m_4 K_5|_{m_5} = 0$

$K_4 = \left|\frac{r_4''''}{m_1 m_2 m_3 m_4}\right|_{m_5}$

The fourth location is given by $L_5 = r_1 - J_1 - J_2 - J_3 - J_4 - J_5$

Thus $\implies$

$$L_4 = X - J_1 - J_2 - J_3 - J_4 - J_5 = L_4 - J_5 = \begin{bmatrix} |r_1'''' - J_5|_{m_1} = r_1''''' \\ |r_2'''' - J_5|_{m_2} = r_2''''' \\ |r_3'''' - J_5|_{m_3} = r_3''''' \\ |r_4'''' - J_5|_{m_4} = r_4''''' \\ |r_5'''' - J_5|_{m_5} = r_5''''' \end{bmatrix}$$

Where$(r_1''''', r_2''''', r_3''''', r_4''''', r_5''''') = (0, 0, 0, 0,0)$

Therefore, the decimal number is given by:

$X = J_1 + J_2 + J_3 + J_4 + J_5$

**vi. Numerical illustrations**

For example, given the moduli set $\{2^n - 1, 2^n, 2^n + 1, 2^{n-1} - 1, 2^{n+1} - 1\}$ and assuming we are given

a residue number $X = (2, 5, 5, 2, 2)$

Therefore we have, $m_1 = 2^n - 1$ , $m_2 = 2^n$ ,

$m_3 = 2^n + 1$ , $m_4 = 2^{n-1} - 1$ and $m_5 = 2^{n+1} - 1$

When n = 3 , $m_1 = 7$ , $m_2 = 8$ , $m_3 = 9$ , $m_4 = 3$, , $m_5 = 15$ , $r_1 = 2$, $r_2 = 5$, $r_3 = 5$ , $r_4 = 2$ and

$r_5 = 2$

1. The first jump is defined by the number $J_1$ which normally corresponds to the first residue in X.

Thus $J_1 = 2$

The first location is defined by: $(X - J_1)$

Therefore:

$$\Longrightarrow X - 2 = \begin{bmatrix} |2-2|_7 = 0 \\ |5-2|_8 = 3 \\ |5-2|_9 = 3 \\ |2-2|_3 = 0 \\ |2-2|_{15} = 0 \end{bmatrix}$$

2. The second jump is defined by the number $J_2$, such that:

$J_2 = 7K_2$ and $|3 - J_2|_8 = 0$

$|3 - 7K_2|_8 = 0 \Longrightarrow K_2 = \left|\frac{3}{7}\right|_8 = 5$

Thus $J_2 = 35$

The second location is defined by: $(X - J_1 - J_2)$

Therefore:

$$X - 2 - 35 = \begin{bmatrix} |2-35|_7 = 0 \\ |3-35|_8 = 0 \\ |3-35|_9 = 4 \\ |0-35|_3 = 1 \\ |0-35|_{15} = 10 \end{bmatrix}$$

3. The third jump is defined by the number $J_3$, such that:

$J_3 = 7 * 8 * K_3$ and $|4 - J_3|_9 = 0$

$|4 - 56K_3|_9 = 0 \Longrightarrow K_3 = \left|\frac{4}{56}\right|_9 = 2$

Thus $J_3 = 112$

The third location is defined by: $(X - J_1 - J_2 - J_3)$

Therefore:

$$X - 2 - 35 - 112 = \begin{bmatrix} |0-112|_7 = 0 \\ |0-112|_8 = 0 \\ |4-112|_9 = 0 \\ |1-112|_3 = 0 \\ |10-112|_{15} = 3 \end{bmatrix}$$

4. The fourth jump is defined by the number $J_4$, such that:

$J_4 = 7*8*9K_4$ and $|0 - J_4|_3 = 0$

$|0 - 504K_4|_3 = 0 \Longrightarrow K_4 = 1$

Thus $J_4 = 504$

The fourth location is defined by: $(X - J_1 - J_2 - J_3 - J_4)$

Therefore:

$$X - 2 - 35 - 112 - 504 = \begin{bmatrix} |0-504|_7 = 0 \\ |0-504|_8 = 0 \\ |0-504|_9 = 0 \\ |0-504|_3 = 0 \\ |3-504|_{15} = 9 \end{bmatrix}$$

5. The fifth jump is defined by the number $J_5$, such that:

$J_5 = 7*8*9*3K_5$ and $|9 - J_5|_{15} = 0$

$|9 - 1512K_5|_{15} = 0 \Longrightarrow K_5 = \left|\frac{9}{1512}\right|_{15} = 2$

Thus $J_5 = 3024$

The fifth location is defined by: $(X - J_1 - J_2 - J_3 - J_4 - J_5)$

Therefore:

$$X - 2 - 35 - 112 - 504 - 3024 = \begin{bmatrix} |0-3024|_7 = 0 \\ |0-3024|_8 = 0 \\ |0-3024|_9 = 0 \\ |0-3024|_3 = 0 \\ |9-3024|_{15} = 0 \end{bmatrix}$$

Therefore the corresponding decimal number is: $2 + 35 + 112 + 504 + 3024 = 3677$

i.e. $(2, 5, 5, 2, 2) \longrightarrow 3677$

**vii. Binary Representation for the Moduli Set** $\{2^n - 1, 2^n, 2^n + 1, 2^{n-1} - 1, 2^{n+1} - 1\}$ Given the moduli set $\{2^n - 1, 2^n, 2^n + 1, 2^{n-1} - 1, 2^{n+1} - 1\}$, its binary representation is given as follows;

**Step 1:** First jump ( $J_1$), which is equal to first residue $r_1$. i.e $\boldsymbol{J_1 = r_1}$

First location $L_1$ is defind as $L_1 = r - J_1$

$r_1 = (r_{1,2n} \dots r_{1,0})$
$r_2 = (r_{2,2n-3} \dots r_{2,1}, r_{2,0})$
$r_3 = (r_{3,2n-2} \dots r_{3,1}, r_{3,0})$
$r_4 = (r_{4,2n-1} \dots r_{4,1}, r_{4,0})$
$r_5 = (r_{4,2n-1} \dots r_{5,1}, r_{5,0})$

$$Thus\ L_1 = r - J_1 = \begin{bmatrix} |r_1 - J_1|m_1 = r_1' \\ |r_2 - J_1|m_2 = r_2' \\ |r_3 - J_1|m_3 = r_3' \\ |r_4 - J_1|m_4 = r_4' \\ |r_5 - J_1|m_5 = r_5' \end{bmatrix}$$

$$= \begin{bmatrix} |(r_{1,2^n-1} \dots r_{1,0}) - (r_{1,2^n-1} \dots r_{1,0})|_{2^n-1} \\ |(r_{2,2^n} \dots r_{2,1}, r_{2,0}) - (r_{1,2^n-1} \dots r_{1,0})|_{2^n} \\ |(r_{3,2^n+1} \dots r_{3,1}, r_{3,0}) - (r_{1,2^n-1} \dots r_{1,0})|_{2^n+1} \\ |(r_{4,2^{n-1}-1} \dots r_{4,1}, r_{4,0}) - (r_{1,2^n-1} \dots r_{1,0})|_{2^{n-1}-1} \\ |(r_{5,2^{n+1}-1} \dots r_{5,1}, r_{4,0}) - (r_{1,2^n-1} \dots r_{1,0})|_{2^{n+1}-1} \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ |(r_{2,2^n} \dots r_{2,1}, r_{2,0}) + (\overline{r_{1,5}} \dots \overline{r_{1,0}})|_{2^n} \\ |(r_{3,2^n+1} \dots r_{3,1}, r_{3,0}) + (\overline{r_{1,5}} \dots \overline{r_{1,0}})|_{2^n+1} \\ |(r_{4,2^{n-1}-1} \dots r_{4,1}, r_{4,0}) + (\overline{r_{1,5}} \dots \overline{r_{1,0}})|_{2^{n-1}-1} \\ |(r_{5,2^{n+1}-1} \dots r_{5,1}, r_{5,0}) + (\overline{r_{1,5}} \dots \overline{r_{1,0}})|_{2^{n+1}-1} \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ r'_{2,2^n}, r'_{2,0} \\ r'_{3,2^n+1}, r'_{3,0} \\ r'_{4,2^{n-1}-1}, r'_{4,0} \\ r'_{5,2^{n+1}-1}, r'_{5,0} \end{bmatrix}$$

**Step 2:** Second jump ( $J_2$ ) is defined by:

$J_2 = m_1 k_2$ and $|r_2' - J_2|_{m_2} = 0$

$\Longrightarrow |r_2' - m_1 k_2|_{m_2} = 0 = k_2 = \left|\frac{r_2'}{m_1}\right|_{m_2}$

the expression (*) is continuously iterated by repeatedly adding the modulus $m_2$ until such a time that $|r_2' - m_1 k_2|_{m_2} = 0$

$\boldsymbol{J_2 = m_1 k_2 = 2^2 k_2 = k_{2,2n-2}, \dots\ k_{2,0}.00}$

$= \boldsymbol{J_{2,2n}, \dots J_{2,0}}$

Location $L_2$ is given as: $L_2 = r - J_1 - J_2 = L_1 – J_2$

$$L_2 = r - J_1 - J_2 = L_1 – J_2 = \begin{bmatrix} |r_1 - J_2|m_1 = r_1'' \\ |r_2 - J_2|m_2 = r_2'' \\ |r_3 - J_2|m_3 = r_3'' \\ |r_4 - J_2|m_4 = r_4'' \\ |r_5 - J_2|m_5 = r_5'' \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ |(r_{2,2^n} \dots r_{2,1}, r_{2,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^n} \\ |(r_{3,2^n+1} \dots r_{3,1}, r_{3,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^n+1} \\ |(r_{4,2^{n-1}-1} \dots r_{4,1}, r_{4,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^{n-1}-1} \\ |(r_{5,2^{n+1}-1} \dots r_{5,1}, r_{4,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^{n+1}-1} \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ 00 \dots 00 \\ |(r_{3,2^n+1} \dots r_{3,1}, r_{3,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^n+1} \\ |(r_{4,2^{n-1}-1} \dots r_{4,1}, r_{4,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^{n-1}-1} \\ |(r_{5,2^{n+1}-1} \dots r_{5,1}, r_{4,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^{n+1}-1} \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ 00000 \\ r''_{3,2^n+1}, r''_{3,0} \\ r''_{4,2^{n-1}-1}, r''_{4,0} \\ r''_{5,2^{n+1}-1}, r''_{5,0} \end{bmatrix}$$

**Step 3**: Third jump is defined by $J_3$ and this is given by;

$J_3 = m_1 m_2 k_3$ and $|r_3'' - J_3|m_3 = 0 \ni |r_3'' - m_1 m_2 k_3|m_3 = 0$

$$K_3 = \left|\frac{r_3''}{m_1 m_2}\right| m_3$$

$\boldsymbol{J_3 = m_1 m_2 k_3 = 2^2 2n - 1. K_3}$

$= (2^n + 1). \boldsymbol{K_{3,2^n+1}, \dots\ K_{3,0}00}$

$= 2^n \boldsymbol{K_{3,2^n+1}, \dots\ K_{3,0} + K_{3,2^n+1}, \dots\ K_{3,0}}$

Third Location ($L_3$) after the jump $J_3$ is given by:

$$L_3 = r_1 – J_1 – J_2 – J_3 = L_2 – J_3 = \begin{bmatrix} |r_1 - J_3|m_1 = r_1''' \\ |r_2 - J_3|m_2 = r_2''' \\ |r_3 - J_3|m_3 = r_3''' \\ |r_4 - J_3|m_4 = r_4''' \\ |r_5 - J_3|m_5 = r_5''' \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ |(r_{2,2^n} \dots r_{2,1}, r_{2,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^n} \\ |(r_{3,2^n+1} \dots r_{3,1}, r_{3,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^n+1} \\ |(r_{4,2^{n-1}-1} \dots r_{4,1}, r_{4,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^{n-1}-1} \\ |(r_{5,2^{n+1}-1} \dots r_{5,1}, r_{4,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^{n+1}-1} \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ 00 \dots 00 \\ 00 \dots 00 \\ |(r_{4,2^{n-1}-1} \dots r_{4,1}, r_{4,0}) - (J_{2,2^n}, \dots J_{r_{4,1}, r_{4,0}})|_{2^{n-1}-1} \\ |(r_{5,2^{n+1}-1} \dots r_{5,1}, r_{5,0}) - (J_{2,2^n}, \dots J_{2,0})|_{2^{n+1}-1} \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ 00 \dots 00 \\ 00 \dots 00 \\ r'''_{4,2n-1}, r'''_{4,0} \\ r'''_{5,2n-1}, r'''_{5,0} \end{bmatrix}$$

**Step 4:** Fourth jump ($J_4$) is given by:

$J_4 = m_1 m_2\, m_3\, k_4$ and $|r_4''' - J_4|_{m_4} = 0$

$= |r_4''' - m_1 m_2 m_3 k_4|_{m_4} = 0\,, K_4 = \left|\frac{r_4'''}{m_1 m_2 m_3}\right|_{m_4}$

$K_4 = \left|\frac{r_4'''}{m_1 m_2 m_3}\right|_{m_4} = 0$

$\boldsymbol{J_4 = m_1 m_2\, m_3\, k_4}$

Fourth Location ($L_4$) after the jump $J_4$ is given by:

$L_4 = r - J_1 - J_2 - J_3 - J_4 = L_3 - J_4$

Also, $L_4 = r - J_1 - J_2 - J_3 - J_4 = L_3 - J_4$

$$= \begin{bmatrix} |r_1 - J_4|m_1 = r_1'''' \\ |r_2 - J_4|m_2 = r_2'''' \\ |r_3 - J_4|m_3 = r_3'''' \\ |r_4 - J_4|m_4 = r_4'''' \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ \left|\left(r_{5,2^{n+1}-1} \ldots r_{5,1}, r_{4,0}\right) - \left(J_{2,2^n}, \ldots J_{2,0}\right)\right|_{2^{n+1}-1} \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ r''''_{5,2n-1}, r''''_{5,0} \end{bmatrix}$$

**Step 5:** Firth jump ($J_5$) is given by:

$J_5 = m_1 m_2\, m_3\, m_4\, k_5$ and $|r_5'''' - J_5|_{m_5} = 0$

$= |r_5'''' - m_1 m_2 m_3 m_4\, k_5|_{m_5} = 0\,,$

$$K_5 = \left|\frac{r_5''''}{m_1 m_2 m_3 m_4}\right|_{m_5}$$

$K_5 = \left|\frac{r_4'''}{m_1 m_2 m_3 m_4}\right|_{m_5} = \left|\frac{0}{20.19.18}\right|_4 = |0|_4 = 0$

$\boldsymbol{J_5 = m_1 m_2\, m_3\, m_4\, k_5 = 20.19.18.0 = 0}$

Firth Location ($L_5$) after the jump $J_5$ is given by:

$L_5 = r - J_1 - J_2 - J_3 - J_4 - J_5 = L_4 - J_5$

And, $L_5 = r - J_1 - J_2 - J_3 - J_4 - J_5 = L_4 - J_5$

$$= \begin{bmatrix} |r_1 - J_5|m_1 = r_1''''' \\ |r_2 - J_5|m_2 = r_2''''' \\ |r_3 - J_5|m_3 = r_3''''' \\ |r_4 - J_5|m_4 = r_4''''' \\ |r_5 - J_5|m_5 = r_5''''' \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ \left|\left(r_{2,2^n} \ldots r_{2,1}, r_{2,0}\right) - \left(J_{2,2^n}, \ldots J_{2,0}\right)\right|_{2^n} \\ \left|\left(r_{3,2^n+1} \ldots r_{3,1}, r_{3,0}\right) - \left(J_{2,2^n}, \ldots J_{2,0}\right)\right|_{2^n+1} \\ \left|\left(r_{4,2^{n-1}-1} \ldots r_{4,1}, r_{4,0}\right) - \left(J_{2,2^n}, \ldots J_{2,0}\right)\right|_{2^{n-1}-1} \\ \left|\left(r_{5,2^{n+1}-1} \ldots r_{5,1}, r_{4,0}\right) - \left(J_{2,2^n}, \ldots J_{2,0}\right)\right|_{2^{n+1}-1} \end{bmatrix}$$

$$=$$

$$= \begin{bmatrix} 00000 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ \left|\left(r_{5,2^{n+1}-1} \ldots r_{5,1}, r_{5,0}\right) - \left(J_{2,2^{n+1}}, \ldots J_{r_{5,1}, r_{5,0}}\right)\right|_{2^{n+1}-1} \end{bmatrix}$$

$$= \begin{bmatrix} 00000 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ 00 \ldots 00 \end{bmatrix}$$

where $(r_1''''', r_2''''', r_3''''', r_4''''', r_5''''') = (0,0,0,0,0)$

The binary number is computed as follows;

$\boldsymbol{X = r_{1,3} \ldots r_{1,0} + J_{2,2n}, \ldots J_{2,0} + J_{3,4n}, \ldots J_{3,0} + J_{4,2n,\ldots,}J_{4,0} + J_{5,2n,\ldots,}J_{5,0}}$

$$= \underbrace{\underbrace{\mathbf{00 \ldots 00}}_{(4n-3)bit}\overbrace{\boldsymbol{r_{1,3} \ldots r_{1,0}}}^{4\,bit} + \underbrace{\mathbf{00 \ldots 00}}_{2n\,bit}\overbrace{\boldsymbol{J_{2,2n}, \ldots J_{2,0}}}^{(2n+1)bit} + \#\; \underbrace{\mathbf{00 \ldots 00}}_{2n\,bit}\underbrace{\boldsymbol{J_{3,4n}, \ldots J_{3,0}}}_{(4n+1)\,bit} + \underbrace{\boldsymbol{J_{4,2n-1}, \ldots J_{4,0}}}_{(4n+1)\,bit} + \underbrace{\boldsymbol{J_{5,2n-1}, \ldots J_{5,0}}}_{(4n+1)\,bit}}_{(4n+1)bit}$$

## v. Proposed Hardware Implementation Scheme

The hardware implementation for the new 5- moduli set reverse converter is as shown in figure 1; In the implementation, the residues are passed through various adder channels to yield the respective new residues. The full conversion takes place in just 5 steps making the speed very good.

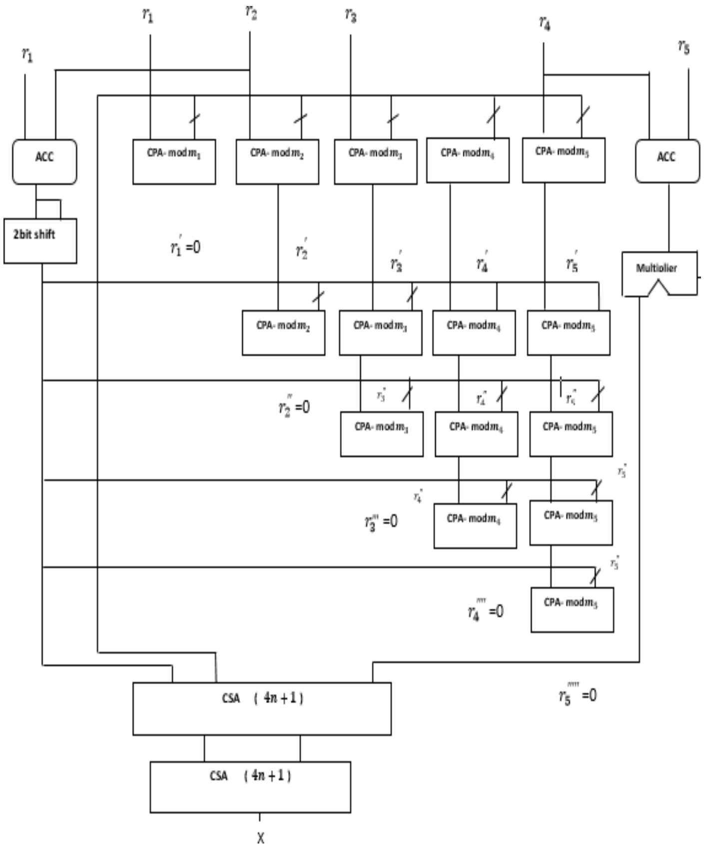


**Figure 1: Hardware Architecture for the proposed cyclic jump method**

## viii. Conclusion

The paper proposed a novel converter for the selected 5-moduli set $\{2^n - 1, 2^n, 2^n + 1, 2^{n-1} - 1, 2^{n+1} - 1\}$. The proposed converter is very fast and efficient in performing reverse conversions compared with a state-of-the-art converter presented in [7]. This is because, it uses fewer steps to generate the decimal value for any given conversion. The converter is best suited for applications requiring very large dynamic ranges such as DSP operations.

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