Inventory Model for Perishable Items with Deterioration and Time Dependent Demand Rate

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Abstract - *In this paper, an inventory management of the perishable items is needed in order to avoid the relevant losses due to their deterioration. This research work develops an inventory model for perishable items, constrained by both physical and freshness condition degradations. By working with perishable items that eventually deteriorates, this inventory model also takes into consideration the expiration date, a salvage value, and the cost of deterioration. In addition, the holding cost is modelled as a quadratic function of time. The proposed inventory model jointly determines the optimal price, the replenishment cycle time, and the order quantity, which together result in maximum total profit per unit of time. The inventory model has a wide application since it can be implemented in several fields such as food goods (milk, vegetables, and meat), organisms, and ornamental flowers, among others. Some numerical examples are presented to illustrate the use of the inventory model. The results show that increasing the value of the shelf-life results in an increment in price, inventory cycle time, quantity ordered, and profits that are generated for all price demand functions. Finally, a sensitivity analysis is performed, and several managerial insights are provided.*

Key Words: Inventory, Perishable Items, Deterioration, Salvage value, Optimal price, Replenishment cycle time.

1.INTRODUCTION

Deterioration of physical goods is one of the important factors in any inventory and production system. If the rate of deterioration is very low its effect can be ignored. But in many practical situations deterioration plays an important role. It is important to control and maintain the inventories of decaying items for the model corporation. Most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables- and foodstuffs suffer from depletion by direct spoilage while kept in store. Highly volatile liquids such as alcohol, gasolines etc. undergo physical depletion over time through the process of evaporation. Electronic goods, photographic film, grain, chemicals, pharmaceuticals etc. deteriorate through a gradual loss of potential or utility with the passage of time. Thus deterioration of physical goods in stock is very realistic feature. Deterioration rate of any item is either constant or time dependent. When deterioration is time dependent, time is accompanied by proportional loss in the value of the product. Realization of this factor motivated modelers to consider the deterioration factor as one of the modeling aspects.

Mashud et al. [1] determined the optimal replenishment policy of deteriorating goods for the classical newsboy inventory problem by considering multiple just-in-time deliveries. Additionally, Mashud et al. [2] derived an inventory model for deteriorating products that calculates the optimal vales for replenishment time, price, and green investment cost.

Given that the inherent perishability can occur immediately, Pal et al. [3] addressed a production-inventory model for deteriorating products when the production cost depends on both production order quantity and production rate. Bhunia A.K and M. Maiti, [4] were the first proponent for developing a deterministic inventory model for deteriorating items with finite rate of replenishment dependent on inventory level. Cheng TCE [5] discussed an economic order quantity model with demand-dependent unit production cost and imperfect production processes. In most of the inventory models, authors have taken constant rate of deterioration. In practice it can be observed that constant rate of deterioration occurs rarely. Most of the items deteriorate as fast as the time passes. It can be noticed that deterioration does not depends upon time only: it can be affected due to weather conditions, humidity, and storage conditions etc. Therefore it is much more realistic to consider deterioration rate as two or three parameter Weibull distribution function. Many researchers considered the constant demand rate in their inventory models, but the assumption of constant demand is not always applicable in real situations.

Tirkolaee et al. [8] noted that the inherent perishability widely occurs in food goods (e.g., milk, vegetables, and meat), organisms, and ornamental flowers. These authors also stated that the time window between preparation and sales of perishable items is very significant for producers and purchasers. Giri et al. [9] noted such demand pattern for fashionable products which initially increases exponentially with time for a period of time after that it becomes steady rather than increasing exponentially. The ramp-type demand demonstrates a time period classified demand pattern. In different time periods the demand is either constant or its rate of change is different.

According to Yavari et al. [13], one of the challenges when managing inventories is the inherent perishability of many items, which means their freshness and quality decrease over time, and these cannot be sold after their expiration date. Many researchers have modified inventory policies by considering the "time proportional partial backlogging rate".

The present section deals with a mathematical model of tie economic order quantity for finite time horizon. This inventory model has been developed for a single deteriorating item by considering the constant-deterioration rate. In this model, a new demand pattern has been introduced in which the demand follows quadratic pattern in the initial period of time and becomes linear later on. It is believed that such type of demand is quite realistic for newly launched product in the market. Shortages are allowed and the backlogging rate is dependent on the duration on waiting time and taken as exponential decreasing function of time. The effect due to change of deterioration parameter has been considered for different parameters numerically.

2. ASSUMPTIONS AND NOTATIONS

To develop inventory models for perishable items with variable demand and partial back logging. The following notations and assumption are used.

(i) I (t) be the inventory level at time t, $t \geq 0$

(ii) t_1 is the time at which shortage starts and T is the length of replenishment cycle $0 \leq t_1 \leq T$.

(iii) Replenishment rate is infinite and lead time is zero.

(iv) There is no repair OR replenishment of deteriorated units during the period.

(v) A single item is considered over the prescribed period T units of time.

(vi) S be the initial inventory level after fulfilling backorders.

(vii) In this model θ be the constant rate of defective units out of on hand inventory at any time, $\theta > 0$.

(viii) Demand rate D(t) is assumed to be a function of time such that

$D(t) = a + bt + c\{t - (t-\mu)H(t-\mu)\}t$ where $H(t-\mu)$ is the Heaviside's functions defined as follows.

$$H(t-\mu) = \begin{Bmatrix} 1 \text{ if } & t \geq \mu \\ 0 \text{ if } & t < \mu \end{Bmatrix}$$

and 'a' is the initial rate of demand, b is the rate with which the demand rate increase. The rate of change in demand rate itself increase at a rate 'c'. *a*, *b* and *c* are positive constant.

(ix) Unsatisfied demand is back logged at a rate exp $(-\delta x)$, where x is the time up to next replenishment. The backlogging parameters δ is a positive constant.

(x) C_1, C_2, C_3 and C_4 are the holding cost per unit time, unit purchase cost per unit, shortage cost per unit per unit time and the unit cost of lost sales respectively. C' is the inventory ordering cost per order.

3. FORMULATION AND SOLUTION OF THE MODEL

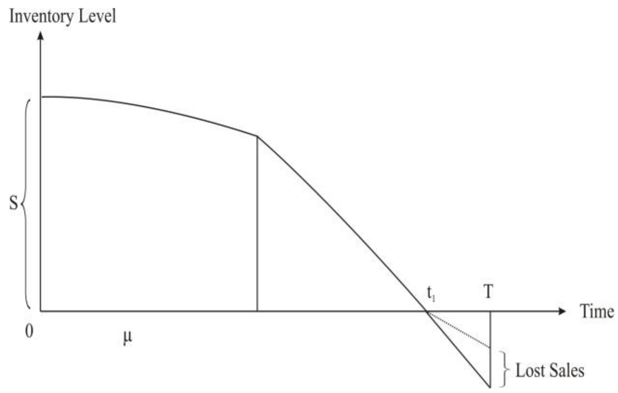

Figure 1

The depletion of inventory during the interval $(0, t_1)$ is due to joint effect of demand and deteriorating of items and the demand is partially backlogged in the interval (t_1, T). The depletion of inventory is given in the (figure 1).

The governing deferential equations of the proposed inventory system in the interval (0, T) are

$$I'(t) + \theta I(t) = -(a + bt + ct^2), 0 \leq t \leq \mu \quad ... (2.1)$$

$$I'(t) + \theta I(t) = -(a + kt), \quad \mu \leq t \leq t_1 \quad ... (2.2)$$

$$I'(t) = -(a + kt)e^{-\delta t}, \quad t_1 \leq t \leq T \quad ... (2.3)$$

where $k = b + c\mu$

with the conditions

$$I(0) = S \text{ and } I(t_1) = 0 \quad ... (2.4)$$

Solution of the equation (2.1)

$$\frac{dI(t)}{dt}+\theta I(t)=-\left(a+bt+ct^2\right)$$

$$I\cdot F\cdot = e^{\int \theta dt} = e^{\theta t}$$

Now

$$\int_0^t d\left\{e^{\theta t}I(t)\right\}dt=-a\int_0^t e^{\theta t}dt-\int_0^t t\cdot e^{\theta t}dt$$

$$-C\int_0^t t^2e^{\theta t}dte^{\theta t}I(t)-S$$

$$=-\frac{a}{\theta}\left[e^{\theta t}\right]_0^t-\frac{b}{\theta}\left[t\cdot e^{\theta t}\right]_0^t+\frac{b}{\theta^2}\left[e^{\theta t}\right]_0^t$$

$$-\frac{C}{\theta}\left[t^2e^{\theta t}\right]_0^t+\frac{C2}{\theta^2}\left[t\cdot e^{\theta t}\right]_0^t+\frac{b}{\theta}\left[t\cdot e^{\theta t}\right]_0^t$$

$$+\frac{b}{\theta^2}\left[e^{\theta t}\right]_0^t-\frac{C}{\theta}\left[t^2e^{\theta t}\right]_0^t+\frac{C2}{\theta^2}\left[t\cdot e^{\theta t}\right]_0^t$$

$$-\frac{2C}{\theta^3}\left[e^{\theta t}\right]_0^t$$

$$e^{\theta t}I(t)-S=-\frac{a}{\theta}\left[e^{\theta t}-1\right]-\frac{b}{\theta}\left[te^{\theta t}\right]$$

$$+\frac{b}{\theta^2}\left[e^{\theta t}-1\right]-\frac{C}{\theta}\left[t^2e^{\theta t}\right]+\frac{2C}{\theta^2}\left[te^{\theta t}\right]$$

$$-\frac{2C}{\theta^3}\left[e^{\theta t}-1\right]$$

$$=e^{\theta t}\left[-\frac{a}{\theta}-\frac{bt}{\theta}+\frac{b}{\theta^2}-\frac{ct^2}{\theta}+\frac{2ct}{\theta^2}-\frac{2c}{\theta^3}\right]$$

$$+\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^3}$$

$$=\frac{e^{\theta t}}{\theta^3}\left[-a\theta^2-b\theta^2t+b\theta-c\theta^2t^2+2c\theta t-2c\right]$$

$$+\frac{1}{\theta^3}\left[a\theta^2-b\theta+2c\right]$$

$$=-\frac{e^{\theta t}}{\theta^3}\left[a\theta^2+b\theta^2t-b\theta+c\theta^2t^2-2c\theta t-2c\right]$$

$$+\frac{1}{\theta^3}\left[a\theta^2-b\theta+2c\right]$$

$$I(t)=\left\{S+\frac{1}{\theta^3}\left(a\theta^2-b\theta+2c\right)\right\}e^{-\theta t}$$

$$-\frac{1}{\theta^3}\left\{a\theta^2+b\theta(\theta t-1)\right.$$

$$\left.+c\left(\theta^2t^2-2\theta t+2\right)\right\},\qquad 0\le t\le\mu \qquad ...(2.5)$$

Now the solution of the equation (2.2)

$$\frac{dI(t)}{dt}+\theta I(t)=-(a+kt)$$

$$\int_{t_1}^t d\left\{e^{\theta t}I(t)\right\}dt=-\int_{t_1}^t (a+kt)\cdot e^{\theta t}dt$$

$$e^{\theta t}I(t)-0=-\int_{t_1}^t ae^{\theta t}dt-k\int_{t_1}^t t\cdot e^{\theta t}dt$$

$$=-\frac{a}{\theta}\left[e^{\theta t}\right]_{t_1}^t-\frac{k}{\theta}\left[t\cdot e^{\theta t}\right]_{t_1}^t+\frac{k}{\theta^2}\left[e^{\theta t}\right]_{t_1}^t$$

$$=e^{\theta t}\left[-\frac{a}{\theta}-\frac{kt}{\theta}+\frac{k}{\theta^2}\right]+e^{\theta t_1}\left[\frac{a}{\theta}+\frac{t_1k}{\theta}-\frac{k}{\theta^2}\right]$$

$$e^{\theta t}I(t)=-\frac{e^{\theta t}}{\theta^2}(a\theta+k\theta t-k)$$

$$+\frac{e^{\theta t_1}}{\theta^2}(a\theta+k\theta t_1-k)$$

$$I(t)=\frac{1}{\theta^2}\left\{a\theta+k(\theta t_1-1)\right\}e^{\theta(t_1-t)}$$

$$-\frac{1}{\theta^2}\left\{a\theta+k(\theta t-1)\right\},\qquad (\mu\le t\le t_1) \qquad ...(2.6)$$

By the equations (2.5) and (2.6) we get initial inventory level after fulfilling backorders (S)

$$\frac{1}{\theta^2}\{a\theta+k(\theta t_1-1)\}e^{\theta(t_1-t)}-\frac{1}{\theta^2}\{a\theta+k(\theta t-1)\}$$

$$=\left\{S+\frac{1}{\theta}^3\left(a\theta^2-b\theta+2c\right)\right\}e^{-\theta t}$$

$$-\frac{1}{\theta^3}\left\{a\theta^2+b\theta(\theta t-1)+c\left(\theta^2t^2-2\theta t+2\right)\right\}$$

Multiplying on both sides by $e^{\theta t}$

$$S=-\frac{1}{\theta^3}\left(a\theta^2-b\theta+2c\right)$$

$$+\frac{e^{\theta t}}{\theta^3}\left\{a\theta^2+b\theta(\theta t-1)+c\left(\theta^2t^2-2\theta t+2\right)\right\}$$

$$+\frac{e^{\theta t_1}}{\theta^2}\{a\theta+k(\theta t_1-1)\}-\frac{1}{\theta^2}\{a\theta+k(\theta t-1)\}e^{\theta t}$$

$$=-\frac{1}{\theta^3}\left(a\theta^2-b\theta+2c\right)+\frac{1}{\theta^2}\{a\theta+k(\theta t_1-1)\}e^{\theta t_1}$$

$$+\frac{e^{\theta t}}{\theta^3}\{a\theta^2+b\theta(\theta t-1)$$

$$+c\left(\theta^2t^2-2\theta t+2\right)-a\theta^2-k\theta^2t+k\theta\}$$

$$=-\frac{1}{\theta^3}\left(a\theta^2-b\theta+2c\right)+\frac{1}{\theta^2}\{a\theta+k(\theta t_1-1)\}e^{\theta t_1}$$

$$+\frac{e^{\theta t}}{\theta^3}\left[b\theta^2t-b\theta+c\theta^2t^2\right.$$

$$\left.-2c\theta t+2c-\theta^2tb-\theta^2tc\mu+b\theta+c\mu\theta\right]$$

Putting $t=\mu$

$$S=-\frac{1}{\theta^3}\left(a\theta^2-b\theta+2c\right)$$

$$+\frac{1}{\theta^2}\{a\theta+k(\theta t_1-1)\}e^{\theta t_1}+\frac{e^{\theta t}}{\theta^3}\left\{b\theta^2\mu-b\theta+c\theta^2t^2\right.$$

$$\left.-2c\theta\mu+2c-\theta^2\mu b-\theta^2\mu^2c+b\theta+c\mu\theta\right]$$

$$S=-\frac{1}{\theta^3}\left(a\theta^2-b\theta+2c\right)+\frac{1}{\theta^2}\{a\theta+k(\theta t_1-1)\}e^{\theta t_1}$$

$$+\frac{e^{\theta\mu}}{\theta^3}(2c-c\mu\theta)$$

$$S=-\frac{1}{\theta^3}\left(a\theta^2-b\theta+2c\right)+\frac{1}{\theta^2}\{a\theta+k(\theta t_1-1)\}e^{\theta t_1}$$

$$-\frac{c}{\theta^3}(-2+\mu\theta)e^{\theta\mu} \quad \text{... (2.7)}$$

Using (2.7) in (2.5), we get

$$I(t)=-\frac{1}{\theta^3}\left(a\theta^2-b\theta+2c\right)e^{-\theta t}$$

$$+\frac{1}{\theta^2}\{a\theta+k(\theta t_1-1)\}e^{\theta(t_1-1)}-\frac{c}{\theta^3}(\mu\theta-2)e^{\theta(\mu-t)}$$

$$+\frac{1}{\theta^3}\left(a\theta^2-b\theta+2c\right)e^{-\theta t}$$

$$-\frac{1}{\theta^3}\left\{a\theta^2+b\theta(\theta t-1)+c\left(\theta^2t^2-2\theta t+2\right)\right\}$$

$$I(t)=\frac{1}{\theta^2}\{a\theta+k(\theta t_1-1)\}e^{\theta(t_1-t)}$$

$$-\frac{c}{\theta^3}(\theta\mu-2)e^{\theta(\mu-t)}-\frac{1}{\theta^3}\{a\theta^2+b\theta(\theta t-1)$$

$$+c\left(\theta^2t^2-2\theta t+2\right)\},(0\le t\le\mu) \quad \text{... (2.8)}$$

Now the solution of the equation (2.4)

$$\frac{dI(t)}{dt}=-(a+kt)e^{-\delta t}$$

$$dI(t)=-(a+kt)e^{-\delta t}dt$$

$$\int_{t_1}^{t} d\{I(t)\}dt=-a\int_{t_1}^{t} e^{-\delta t}dt-k\int_{t_1}^{t} t\cdot e^{-\delta t}dt$$

$$I(t)-0=\frac{a}{\delta}\left[e^{-\delta t}\right]_{t_1}^{t}+\frac{k}{\delta}\left[t\cdot e^{-\delta t}\right]_{t_1}^{t}+\frac{k}{\delta^2}\left[e^{-\delta t}\right]_{t_1}^{t}$$

$$=\frac{a}{\delta}\left[e^{-\delta t}-e^{-\delta t_1}\right]+\frac{k}{\delta}\left[t\cdot e^{-\delta t}-t_1e^{-\delta t_1}\right]$$

$$+\frac{k}{\delta^2}\left[e^{-\delta t}-e^{-\delta t_1}\right]$$

$$=e^{-\delta t}\left[\frac{a}{\delta}+\frac{tk}{\delta}+\frac{k}{\delta^2}\right]+e^{-\delta t_1}\left[-\frac{a}{\delta}-\frac{t_1k}{\delta}-\frac{k}{\delta^2}\right]$$

$$I(t)=\frac{1}{\delta^2}\left[\{a\delta+k(\delta t+1)\}e^{-\delta t}\right.$$

$$\left.-\{a\delta+k(\delta t_1+1)\}e^{-\delta t_1}\right],\quad t_1\le t\le T \quad ...(2.9)$$

Hence the inventory holding cost (C_H) during the period (0, T) becomes

$$C_H=C_1\left[\int_0^{\mu} I(t)dt+\int_{\mu}^{t_1} I(t)dt\right]$$

$$C_H=\frac{C_1}{\theta^2}\int_0^{\mu}\{a\theta+k(\theta t_1-1)\}e^{\theta(t_1-t)}dt$$

$$-\frac{C_1C}{\theta^3}\int_0^{\mu}(\theta\mu-2)e^{\theta(\mu-t)}dt$$

$$-\frac{C_1}{\theta^3}\int_0^{\mu}\left\{a\theta^2+b\theta(\theta t-1)+C\left(\theta^2t^2-2\theta t+2\right)\right\}dt$$

$$+\frac{C_1}{\theta^2}\int_{\mu}^{t_1}\{a\theta+k(\theta t_1-1)\}e^{\theta(t_1-t)}dt$$

$$-\frac{C_1}{\theta^2}\int_{\mu}^{t_1}\{a\theta+k(\theta t-1)\}dt$$

Solving all integration in the above equation separately now first integration term

$$I=\frac{C_1}{\theta^2}\int_0^{\mu}\{a\theta+k(\theta t_1-1)\}e^{\theta(t_1-t)}dt$$

$$=\frac{C_1}{\theta^2}\{a\theta+k(\theta t_1-1)\}e^{\theta t_1}\int_0^{\mu}e^{-\theta t}dt$$

$$=\frac{C_1}{\theta^2}\{a\theta+k(\theta t_1-1)\}\frac{e^{\theta t_1}}{\theta}\left[1-e^{-\theta\mu}\right]$$

$$I=\frac{C_1}{\theta^3}\{a\theta+k(\theta t_1-1)\}\left[e^{\theta t_1}-e^{\theta(t_1-\mu)}\right] \quad ... (p)$$

Now second integration term

$$II=\frac{C_1C}{\theta^3}\int_0^{\mu}(\theta\mu-2)e^{\theta(\mu-t)}dt$$

$$=\frac{C_1C}{\theta^3}\theta(\theta\mu-2)e^{\theta\mu}\int_0^{\mu}e^{-\theta t}dt$$

$$=\frac{C_1C}{\theta^3}(\theta\mu-2)\frac{e^{\theta\mu}}{\theta}\left[-e^{-\theta t}\right]_0^{\mu}$$

$$II=-\frac{C_1C}{\theta^4}(\theta\mu-2)\left(1-e^{-\theta\mu}\right) \quad ... (q)$$

Now third integration term

$$III=\frac{C_1}{\theta^3}\int_0^{\mu}\left\{a\theta^2+b\theta(\theta t-1)+C\left(\theta^2t^2-2\theta t+2\right)dt\right\}$$

$$=\frac{C_1}{\theta^3}\left[a\theta^2 t+b\theta^2\frac{t^2}{2}-b\theta t\right.$$

$$\left.+C\theta^2\frac{t^3}{3}-2\theta\cdot C\cdot\frac{t^2}{2}+2Ct\right]_0^{\mu}$$

$$=\frac{C_1}{\theta^3}\left[a\theta^2\mu+b\theta^2\frac{\mu^2}{2}-b\theta\mu+C\theta^2\frac{\mu^3}{3}\right.$$

$$-2\theta\cdot C\cdot\frac{\mu^2}{2}+2C\mu a\theta^2\mu+b\theta^2\frac{\mu^2}{2}-b\theta\mu$$

$$\left.+C\theta^2\frac{\mu^3}{3}-2\theta\cdot C\cdot\frac{\mu^2}{2}+2C\mu\right]$$

$$III=C_1\left\{\frac{a\mu}{\theta}+\frac{b\mu^2}{2\theta}-\frac{b\mu}{\theta^2}+\frac{C\mu^3}{3\theta}-\frac{\mu^2 C}{\theta^2}+\frac{2C\mu}{\theta^3}\right\} \quad \text{... (r)}$$

Now the fourth integration term

$$IV=C_1\left[\int_{\mu}^{t_1}\frac{1}{\theta^2}\left\{a\theta+k\left(\theta t_1-1\right)\right\}e^{\theta(t_1-t)}dt\right.$$

$$\left.-\frac{1}{\theta^2}\int_{\mu}^{t_1}\left\{a\theta+k\left(\theta t-1\right)\right\}dt\right]$$

$$=\frac{C_1}{\theta^2}\left\{a\theta+k\left(\theta t_1-1\right)\right\}e^{\theta t_1}\int_{\mu}^{t_1}e^{-\theta t}dt$$

$$-\frac{C_1}{\theta^2}\left(a\theta-k\right)\int_{\mu}^{t_1}dt-\frac{C_1 k}{\theta}\int_{\mu}^{t_1}dt$$

$$=\frac{C_1}{\theta^2}\left\{a\theta+k\left(\theta t_1-1\right)\right\}\frac{e^{\theta t_1}}{\theta}\left[e^{-\theta\mu}-e^{-\theta t_1}\right]$$

$$-\frac{C_1}{\theta^2}\left(a\theta-k\right)\left(t_1-\mu\right)-\frac{C_1 k}{2\theta}\left[t_1^2-\mu^2\right]$$

$$=\frac{C_1}{\theta^3}\left\{a\theta+k\left(\theta t_1-1\right)\right\}\left(e^{\theta(t_1-\mu)}-1\right)$$

$$-\frac{C_1}{\theta^2}\left(a\theta-k\right)\left(t_1-\mu\right)-\frac{C_1 k}{2\theta}\left(t_1^2-\mu^2\right)$$

$$=\frac{C_1}{\theta^3}\left\{a\theta+k\left(\theta t_1-1\right)\right\}\left(e^{\theta(t_1-\mu)}-1\right)$$

$$-\frac{C_1}{\theta^2}\left(a\theta-k\right)t_1+\frac{C_1}{\theta^2}\left(a\theta\mu-b\mu-c\mu^2\right)$$

$$-\frac{C_1 k}{2\theta}\left(t_1^2-\mu^2\right)$$

$$IV=\frac{C_1}{\theta^3}\left\{a\theta+k\left(\theta t_1-1\right)\right\}\left(e^{\theta(t_1-\mu)}-1\right)$$

$$-\frac{C_1}{\theta^2}\left(a\theta-k\right)t_1+\frac{ac_1\mu}{\theta}-\frac{C_1 b\mu}{\theta^2}$$

$$-\frac{C_1 C\mu^2}{\theta^2}-\frac{C_1 k}{2\theta}\left(t_1^2-\mu^2\right) \quad \text{... (s)}$$

After adding p, q, r and s, we get C_H as

$$C_H=\frac{C_1}{\theta^3}\left\{a\theta+k\left(\theta t_1-1\right)\right\}\left\{e^{\theta t_1}-e^{\theta(t_1-\mu)}\right\}$$

$$+\frac{C_1 C}{\theta^4}\left(\theta\mu-2\right)\left(1-e^{\theta\mu}\right)$$

$$-C_1\left(\frac{a\mu}{\theta}+\frac{b\mu^2}{2\theta}-\frac{b\mu}{\theta^2}+\frac{c\mu^3}{3\theta}-\frac{\mu^2 c}{\theta^2}+\frac{2c\mu}{\theta^3}\right)$$

$$+\frac{C_1}{\theta^3}\left\{a\theta+k\left(\theta t_1-1\right)\right\}\left(e^{\theta(t_1-\mu)}-1\right)$$

$$-\frac{C_1}{\theta^2}\left(a\theta-k\right)t_1+\frac{c_1 a\mu}{\theta}$$

$$-\frac{c_1 b\mu}{\theta^2}-\frac{c_1 c\mu^2}{\theta^2}-\frac{c_1 k}{2\theta}\left(t_1^2-\mu^2\right)$$

$$=\frac{C_1}{\theta^3}\{a\theta+k(\theta t_1-1)\}\left(e^{\theta t_1}-1\right)$$

$$+\frac{c_1 c}{\theta^4}(\theta\mu-2)\left(1-e^{\theta\mu}\right)-\frac{c_1 b\mu^2}{2\theta}-\frac{cc_1\mu^3}{3\theta}$$

$$-\frac{2c_1 c\mu}{\theta^3}-\frac{c_1}{\theta^2}(a\theta-k)t_1-\frac{c_1 k}{2\theta}\left(t_1^2-\mu^2\right)$$

$$=\frac{C_1}{\theta^3}\{a\theta+k(\theta t_1-1)\}\left(e^{\theta t_1}-1\right)$$

$$+\frac{c_1 c}{\theta^4}(\theta\mu-2)\left(1-e^{\theta\mu}\right)-\frac{c_1 b\mu^2}{2\theta}$$

$$-\frac{c_1}{3\theta^3}\left(c\mu^3\theta^2+6c\mu\right)-\frac{c_1}{\theta^2}(a\theta-k)t_1$$

$$-\frac{c_1 k}{2\theta}\left(t_1^2-\mu^2\right)=\frac{C_1}{\theta^3}\{a\theta+k(\theta t_1-1)\}\left(e^{\theta t_1}-1\right)$$

$$+\frac{c_1 c}{\theta^4}(\theta\mu-2)\left(1-e^{\theta\mu}\right)-\frac{c_1 b\mu^2}{2\theta}$$

$$-\frac{c_1 c\mu}{3\theta^3}\left(\theta^2\mu^2+6\right)-\frac{c_1}{\theta^2}(a\theta-k)t_1-\frac{c_1 k}{2\theta}\left(t_1^2-\mu^2\right)$$

$$C_H=C_1\left[\frac{c}{\theta^4}(\theta\mu-2)\left(1-e^{\theta\mu}\right)\right.$$

$$+\frac{1}{\theta^3}\{a\theta+k(\theta t_1-1)\}\left(e^{\theta t_1}-1\right)$$

$$-\frac{b\mu^2}{2\theta}-\frac{c\mu}{3\theta^3}\left(\theta^2\mu^2+6\right)$$

$$\left.-\frac{1}{\theta^2}(a\theta-k)t_1-\frac{k}{2\theta}\left(t_1^2-\mu^2\right)\right] \quad ...(2.10)$$

The cost due to deterioration of units (C_D) during the period (0, T)

$$C_D=C_2\left[\int_0^{\mu}\theta I(t)dt+\int_{\mu}^{t_1}\theta I(t)dt\right]$$

$$=C_2\theta\left[\int_0^{\mu}I(t)dt+\int_{\mu}^{t_1}I(t)dt\right]$$

$$C_D=C_2\theta\left[\frac{c}{\theta^4}(\theta\mu-2)\left(1-e^{\theta\mu}\right)\right.$$

$$+\frac{1}{\theta^3}\{a\theta+k(\theta t_1-1)\}\left(e^{\theta t_1}-1\right)$$

$$-\frac{b\mu^2}{2\theta}-\frac{c\mu}{3\theta^3}\left(\theta^2\mu^2+6\right)$$

$$\left.-\frac{1}{\theta^2}(a\theta-k)t_1-\frac{k}{2\theta}\left(t_1^2-\mu^2\right)\right] \quad ...(2.11)$$

The cost due to shortage of units (C_s) during the given period is given by

$$C_s=-C_3\int_{t_1}^{T}I(t)dt$$

$$C_s=-\frac{C_3}{\delta^2}\int_{t_1}^{T}\left[\{a\delta+k(\delta t+1)\}e^{-\delta t}\right.$$

$$\left.-\{a\delta+k(\delta t_1+1)\}e^{-\delta t_1}\right]dt+\frac{C_3}{\delta^2}\{a\delta+k(\delta t_1+1)\}$$

$$=-\frac{C_3}{\delta^2}a\delta\int_{t_1}^{T}e^{-\delta t}dt-\frac{C_3 k\delta}{\delta^2}\int_{t_1}^{T}t\cdot e^{-\delta t}dt-\frac{C_3 k}{\delta^2}\int_{t_1}^{T}e^{-\delta t}dt$$

$$+\frac{C_3}{\delta^2}\{a\delta+k(\delta t_1+1)\}\,e^{-\delta t_1}\cdot\left(\int_{t_1}^{T}dt\right)$$

$$=-\frac{C_3 a}{\delta}\int_{t_1}^{T}e^{-\delta t}dt-\frac{C_3 k}{\delta}\int_{t_1}^{T}t\cdot e^{-\delta t}dt-\frac{C_3 k}{\delta^2}\int_{t_1}^{T}e^{-\delta t}dt$$

$$+\frac{C_3}{\delta^2}\{a\delta+k(\delta t_1+1)\}\, e^{-\delta t_1}\cdot\left(\int_{t_1}^{T}dt\right)$$

$$=-C_3\left\{\frac{a}{\delta}+\frac{k}{\delta^2}\right\}\int_{t_1}^{T}e^{-\delta t}dt-\frac{C_3k}{\delta}\int_{t_1}^{T}t\cdot e^{-\delta t}dt$$

$$+\frac{C_3}{\delta^2}\{a\delta+k(\delta t_1+1)\}e^{-\delta t_1}(T-t_1)$$

$$=\frac{C_3}{\delta}\left\{\frac{a}{\delta}+\frac{k}{\delta^2}\right\}\left(e^{-\delta T}-e^{-\delta t_1}\right)$$

$$+\frac{C_3k}{\delta^2}\left(T\cdot e^{-\delta T}-t_1e^{-\delta t_1}\right)+\frac{C_3k}{\delta^3}\left(e^{-\delta T}-e^{-\delta t_1}\right)$$

$$+\frac{C_3}{\delta^2}\{a\delta+k(\delta t_1+1)\}\cdot e^{-\delta t_1}(T-t_1)$$

$$=\frac{e^{-\delta T}}{\delta^3}(C_3a\delta+2C_3k+C_3Tk\delta)$$

$$-\frac{e^{\delta t_1}}{\delta^3}(C_3a\delta+2C_3k+C_3t_1k\delta)$$

$$+\frac{C_3}{\delta^2}\{a\delta+k(\delta t_1+1)\}e^{-\delta t_1}(T-t_1)$$

$$C_s=\frac{C_3}{\delta^3}\Big[e^{-\delta T}\{a\delta+k(2+\delta T)\}$$

$$-e^{-\delta t_1}\{a\delta+k(2+t_1\delta)\}$$

$$+\{a\delta^2+\delta k(\delta t_1+1)\}e^{-\delta t_1}(T-t_1) \qquad ...(2.12)$$

The opportunity cost due to lost sales (C_0) is given by

$$C_0=C_4\int_{t_1}^{T}\left(1-e^{-\delta t}\right)(a+kt)dt$$

$$=C_4\int_{t_1}^{T}(a+kt)dt-C_4\int_{t_1}^{T}e^{-\delta t}(a+kt)dt$$

$$=C_4\int_{t_1}^{T}(a+kt)dt-C_4\int_{t_1}^{T}ae^{-\delta t}dt-C_4k\int_{t_1}^{T}t\cdot e^{-\delta t}dt$$

$$=C_4[at]_{t_1}^{T}+\frac{C_4k}{2}\left[t^2\right]_{t_1}^{T}+\frac{C_4a}{\delta}\left[e^{-\delta t}\right]_{t_1}^{T}$$

$$+\frac{C_4k}{\delta}\left[te^{-\delta t}\right]_{t_1}^{T}+\frac{C_4k}{\delta^2}\left[e^{-\delta t}\right]_{t_1}^{T}$$

$$=C_4a(T-t_1)+\frac{C_4k}{2}\left(T^2-t_1^2\right)$$

$$+e^{-\delta T}\left(\frac{C_4a}{\delta}+\frac{TC_4k}{\delta}+\frac{C_4k}{\delta^2}\right)$$

$$-e^{-\delta t_1}\left(\frac{C_4a}{\delta}+\frac{t_1C_4k}{\delta}+\frac{C_4k}{\delta^2}\right)$$

$$=C_4a(T-t_1)+\frac{C_4k}{2}\left(T^2-t_1^2\right)$$

$$+\frac{C_4e^{-\delta T}}{\delta^2}\{a\delta+k(1+\delta T)\}$$

$$-\frac{C_4e^{-\delta t_1}}{\delta^2}\{a\delta+k(1+t_1\delta)\}$$

$$C_0=C_4\Big[a(T-t_1)+\frac{k}{2}\left(T^2-t_1^2\right)$$

$$+\frac{e^{-\delta T}}{\delta^2}\{a\delta+k(1+T\delta)\}$$

$$-\frac{e^{-\delta t_1}}{\delta^2}\{a\delta+k(1+t_1\delta)\}\Big] \qquad ...(2.13)$$

The total average cost (k) of the inventory system permit time is given by

$$K=\frac{\left(C'+C_H+C_D+C_s+C_0\right)}{T}=\frac{R}{T}$$

where R is the total cost of ht inventory cycle

$$K=\frac{1}{T}\left[C'+\frac{C_1C}{\theta^4}(\theta\mu-2)\left(1-e^{\theta\mu}\right)\right.$$

$$+\frac{C_1}{\theta^3}\{a\theta+k(\theta t_1-1)\}\left(e^{\theta t_1}-1\right)-\frac{c_1b\mu^2}{2\theta}$$

$$-\frac{CC_1\mu}{3\theta^3}\left(\theta^2\mu^2+6\right)-\frac{C_1}{\theta^2}(a\theta-k)t_1$$

$$-\frac{C_1k}{2\theta}\left(t_1^2-\mu^2\right)+\frac{C_2C}{\theta^3}(\theta\mu-2)\left(1-e^{\theta\mu}\right)$$

$$+\frac{C_2}{\theta^2}\{a\theta+k(\theta t_1-1)\}\left(e^{\theta t_1}-1\right)-\frac{C_2b\mu^2}{2}$$

$$-\frac{CC_2\mu}{3\theta^2}\left(\theta^2\mu^2+6\right)-\frac{C_2}{\theta}(a\theta-k)t_1$$

$$-\frac{C_2k}{2}\left(t_1^2-\mu^2\right)+\frac{C_3}{\delta^3}\left\{e^{-\delta T}\{a\delta+k(2+\delta T)\}\right.$$

$$-e^{-\delta t_1}\{a\delta+k(2+t_1\delta)\}$$

$$\left.+\left\{a\delta^2+\delta k(\delta t_1+1)\right\}e^{-\delta t_1}(T-t_1)\right\}$$

$$+C_4\left\{a(T-t_1)+\frac{k}{2}\left(T^2-t_1^2\right)\right.$$

$$+\frac{1}{\delta^2}\{a\delta+k(\delta T+1)\}e^{-\delta T}$$

$$\left.\left.-\frac{e^{-\delta t_1}}{\delta^2}\{a\delta+k(\delta t_1+1)\}\right\}\right]$$

To minimize (k), the optimal values of t_1 & T can be obtained by solving $\frac{\partial k}{\partial t_1}=0$ and $\frac{\partial k}{\partial T}=0$ simultaneously.

Now

$$\frac{\partial k}{\partial t_1}=\frac{C_1(a\theta-k)e^{\theta t_1}}{\theta^2}+\frac{C_1kt_1e^{\theta t_1}}{\theta}$$

$$+\frac{C_1ke^{\theta t_1}}{\theta^2}-\frac{C_1k}{\theta^2}-\frac{C_1a}{\theta}+\frac{C_1k}{\theta^2}-\frac{C_1kt_1}{\theta}$$

$$+C_2ae^{\theta t_1}-\frac{C_2ke^{\theta t_1}}{\theta}+C_2kt_1e^{\theta t_1}+\frac{C_2ke^{\theta t_1}}{\theta}$$

$$-\frac{C_2k}{\theta}-C_2a+\frac{C_2k}{\theta}-C_2kt_1$$

$$+\frac{C_3e^{-\delta t_1}}{\delta^2}(a\delta+2k)+\frac{C_3t_1ke^{-\delta t_1}}{\delta}$$

$$+\frac{C_3ke^{-\delta t_1}}{\delta^2}-\frac{C_3T}{\delta^2}\left(a\delta^2+\delta k\right)e^{-\delta t_1}$$

$$-C_3Tkt_1e^{-\delta t_1}+\frac{C_3}{\delta}e^{-\delta t_1}Tk$$

$$+\frac{C_3}{\delta}t_1e^{-\delta t_1}(a\delta+k)-\frac{C_3}{\delta^2}e^{-\delta t_1}(a\delta+k)$$

$$+C_3kt_1^2e^{-\delta t_1}-\frac{C_3}{\delta}e^{-\delta t_1}.2t_1k$$

$$-C_4a-C_4kt_1+\frac{C_4e^{-\delta t_1}}{\delta}(a\delta+k)$$

$$+C_4e^{-\delta t_1}kt_1-\frac{C_4}{\delta}ke^{-\delta t_1}$$

$$=\frac{C_1}{\theta^2}(a\theta-k)e^{\theta t_1}+\frac{C_1kt_1e^{\theta t_1}}{\theta}$$

$$+\frac{C_1}{\theta^2}ke^{\theta t_1}-\frac{C_1a}{\theta}-\frac{C_1kt_1}{\theta}+C_2ae^{\theta t_1}$$

$$-\frac{C_2ke^{\theta t_1}}{\theta}+C_2kt_1e^{\theta t_1}+\frac{C_2ke^{\theta t_1}}{\theta}-C_2a$$

$$-C_2kt_1+\frac{C_3ae^{-\delta t_1}}{\delta}+\frac{2kC_3e^{-\delta t_1}}{\delta^2}+\frac{C_3t_1ke^{-\delta t_1}}{\delta}$$

$$-\frac{C_3}{\delta^2}ke^{-\delta t_1}-C_3Tae^{-\delta t_1}-\frac{C_3Tke^{-\delta t_1}}{\delta}$$

$$-C_3Tkt_1e^{-t_1\delta}+\frac{C_3}{\delta}e^{-\delta t_1}Tk+C_3t_1ae^{-\delta t_1}$$

$$+\frac{C_3}{\delta}t_1ke^{-\delta t_1}-\frac{C_3a}{\delta}e^{-\delta t_1}-\frac{C_3}{\delta^2}ke^{-\delta t_1}$$

$$+C_3kt_1^2e^{-\delta t_1}-\frac{C_3}{\delta}2t_1ke^{-\delta t_1}$$

$$-C_4a\left(1-e^{-\delta t_1}\right)-c_4kt_1\left(1-e^{-\delta t_1}\right)$$

$$=\frac{C_1}{\theta^2}ae^{\theta t_1}+\frac{C_1k}{\theta}e^{\theta t_1}+\frac{C_1k}{\theta^2}t_1e^{\theta t_1}$$

$$+\frac{C_1ke^{\theta t_1}}{\theta^2}-\frac{C_1a}{\theta}-\frac{C_1kt_1}{\theta}+C_2ae^{\theta t_1}$$

$$-\frac{C_2k}{\theta}e^{\theta t_1}+C_2kt_1e^{\theta t_1}+\frac{C_2k}{\theta}e^{\theta t_1}$$

$$-C_2a-C_2kt_1+\frac{C_3a}{\delta}e^{-\delta t_1}+\frac{2kC_3e^{-\delta t_1}}{\delta^2}$$

$$+\frac{C_3t_1ke^{-\delta t_1}}{\delta}-\frac{C_3ke^{-\delta t_1}}{\delta_2}-C_3Tae^{-\delta t_1}$$

$$-\frac{C_3Tke^{-\delta t_1}}{\delta}-C_3Tkt_1e^{-\delta t_1}+\frac{C_3}{\delta}Tke^{-\delta t}$$

$$+C_3t_1ae^{-\delta t_1}+\frac{C_3t_1k}{\delta}e^{-\delta t_1}-\frac{C_3e^{-\delta t_1}a}{\delta}$$

$$-\frac{C_3ke^{-\delta t_1}}{\delta_2}+C_3kt_1^2e^{-\delta t_1}-\frac{C_3}{\delta}2t_1ke^{-\delta t_1}$$

$$-C_4a\left(1-e^{-\delta t_1}\right)-C_4kt_1\left(1-e^{-\delta t_1}\right).$$

$$=\frac{C_1ae^{\theta t_1}}{\theta}+\frac{C_1kt_1e^{\theta t_1}}{\theta}-\frac{C_1a}{\theta}-\frac{C_1kt_1}{\theta}$$

$$+C_2ae^{\theta t_1}+C_2kt_1e^{\theta t_1}-C_2a-C_2kt_1$$

$$-C_3Tae^{-\delta t_1}-C_3Tkt_1e^{-\delta t_1}+C_3t_1ae^{-\delta t_1}$$

$$+C_3kt_1^2e^{-\delta t_1}-C_4\left(a-e^{-\delta t_1}\right)\left(a+kt_1\right)$$

$$\frac{C_1a}{\theta}\left(e^{\theta t_1}-1\right)+\frac{C_1kt_1}{\theta}\left(e^{\theta t_1}-1\right)$$

$$+C_2kt_1\left(e^{\theta t_1}-1\right)-TC_3e^{-\delta t_1}\left(a+kt_1\right)$$

$$C_3t_1e^{-\delta t_1}\left(a+kt_1\right)-C_4\left(1-e^{-\delta t_1}\right)\left(a+kt_1\right)$$

$$=\left(e^{t_1\theta}-1\right)\left(\frac{C_1a}{\theta}+\frac{C_1kt_1}{\theta}+C_2a+C_2kt_1\right)$$

$$+C_3e^{-\delta t_1}\left(t_1-T\right)\left(a+kt_1\right)-C_4\left(a+kt_1\right)\left(1-e^{-\delta t_1}\right)$$

$$=\left(e^{\theta t_1}-1\right)\left[\frac{C_1}{\theta}+\left(a+kt_1\right)+C_2\left(a+kt_1\right)\right]$$

$$+C_3e^{-\delta t_1}\left(t_1-T\right)\left(a+kt_1\right)-C_4\left(a+kt_1\right)\left(1-e^{-\delta t_1}\right)$$

$$=\left[\left(e^{\theta t_1}\right)\left(\frac{C_1}{\theta}+C_2\right)+C_3e^{-\delta t_1}\left(t_1-T\right)\right.$$

$$\left.+C_4e^{-\delta t_1}-C_4\right]\left(a+kt_1\right)$$

Now putting $\frac{\partial k}{\partial t_1}=0$, we get

$$\frac{\partial k}{\partial t_1}=0-\left[\left(e^{\theta t_1}-1\right)\left(\frac{C_1}{\theta}+C_2\right)\right.$$

$$\left.+C_3e^{-\delta t_1}\left(t_1-T\right)+C_4e^{-\delta t_1-C_4}\right]\left(a+kt_1\right)$$

$$\left(e^{\theta t_1}-1\right)\left(\frac{C_1}{\theta}+C_2\right)$$

$$+e^{-\delta t_1}\left\{C_3\left(t_1-T\right)+C_4\right\}-C_4=0 \quad \text{...(2.14)}$$

Now for $\frac{\partial k}{\partial T}=0$

Since $k=\frac{R}{T}$

$$\Rightarrow \quad \frac{\partial k}{\partial T}=\frac{\partial}{\partial T}\left(\frac{R}{T}\right)=-\frac{R}{T^2}+\frac{1}{T}\left(\frac{\partial R}{\partial T}\right)$$

$$\frac{\partial k}{\partial T}=-\frac{R}{T^2}+\frac{1}{T}\left[-\frac{C_3e^{-\delta T}}{\delta^2}\left(a\delta+2k\right)\right.$$

$$-\frac{C_3kT}{\delta}e^{-\delta T}+\frac{C_3}{\delta^2}ke^{-\delta T}+\frac{C_3}{\delta^2}\left(a\delta+k\right)e^{-\delta t_1}$$

$$+\frac{C_3}{\delta}t_1ke^{-\delta t_1}+C_4a+C_4kT-\frac{C_4}{\delta}e^{-\delta T}$$

$$\left.\left(a\delta+k\right)-C_4kTe^{-\delta T}+\frac{C_4}{\delta}e^{-\delta T}k\right]$$

$$=-\frac{R}{T^2}+\frac{1}{T}\left[C_3e^{-\delta T}\left\{-\frac{a}{\delta}-\frac{2k}{\delta^2}-\frac{kT}{\delta}+\frac{k}{\delta^2}\right\}\right.$$

$$+C_3e^{-\delta t_1}\left\{\frac{a}{\delta}+\frac{k}{\delta^2}+\frac{t_1k}{\delta}\right\}$$

$$\left.+C_4\left(a+kT\right)+C_4e^{-\delta T}\left(-a-\frac{k}{\delta}-kT+\frac{k}{\delta}\right)\right]$$

$$=-\frac{R}{T^2}+\frac{1}{T}\left[C_3e^{-\delta T}\left(-\frac{a}{\delta}-\frac{k}{\delta^2}-\frac{kT}{\delta}\right)\right.$$

$$+C_3e^{-\delta t_1}\left(\frac{a}{\delta}+\frac{k}{\delta^2}+\frac{t_1k}{\delta}\right)+C_4$$

$$\left.+C_4e^{-\delta T}\left(-a-kT\right)\right]$$

$$=-\frac{R}{T^2}+\frac{1}{T}\left[\frac{C_3}{\delta^2}\left\{e^{-\delta t_1}\left\{a\delta+k\left(1+t_1\delta\right)\right\}\right.\right.$$

$$\left.\left.-e^{-\delta T}\left\{a\delta+k\left(1+T\delta\right)\right\}\right\}+C_4\left(a+kT\right)\left(1-e^{-\delta T}\right)\right]$$

Putting $\frac{\partial k}{\partial T}=0$, we get

$$\frac{\partial k}{\partial T}=-\frac{R}{T^2}+\frac{1}{T}\left[\frac{C_3}{\delta^2}\left\{e^{-\delta t_1}\left\{a\delta+k\left(1+t_1\delta\right)\right\}\right.\right.$$

$$\left.-e^{-\delta T}\left\{a\delta+k\left(1+T\delta\right)\right\}\right\}$$

$$\left.+C_4\left(a+kT\right)\left(1-c^{-\delta T}\right)\right]=0$$

$$\frac{C_3}{\delta^2}\left[\left\{a\delta+k\left(\delta t_1+1\right)\right\}e^{-\delta t_1}\right.$$

$$\left.-\left\{a\delta+k\left(\delta T+1\right)\right\}e^{-\delta T}\right]+C_4\left(a+kT\right)$$

$$\left(1-e^{-\delta T}\right)-\frac{R}{T}=0 \quad \text{...(2.15)}$$

The above equations $\frac{\partial k}{\partial t_1}=0$ and $\frac{\partial k}{\partial T}=0$ provided, they satisfy the following conditions :

$$\left.\begin{aligned}&\frac{\partial^2k}{\partial t_1^2}>0,\ \frac{\partial^2k}{\partial T^2}>0\\&\left(\frac{\partial^2k}{\partial t_1^2}\right)\left(\frac{\partial^2k}{\partial T^2}\right)-\left(\frac{\partial^2k}{\partial t_1\partial T}\right)^2>0\end{aligned}\right\} \quad \text{...(2.16)}$$

Equation (2.14) and (2.15) are highly nonlinear equations. Therefore numerical solution of these equation obtained by using the software MATLAB 7.0.1.

4. CONCLUSIONS

This research work develops an inventory model with price, stock, and time-dependent demand. The physical deterioration and condition of freshness degradation over time are both considered, and zero-ending inventory is assumed. When working with perishable products, a salvaged value and a deterioration cost are considered in the entire cycle. A nonlinear time-dependent holding cost is included, specifically with a quadratic-type function.

Through an algorithm, the inventory model determines the optimal values for price, the inventory cycle time, and the order quantity. Some numerical examples are provided, and a sensitivity analysis is presented for all the input parameters. By observing the behavior of the decision variables and total profits, it was found that an increase in the ordering cost , purchasing cost , and shelf-life results in a similar pattern in the selling price, the inventory cycle time, the quantity to order, and the total profit. Furthermore, an increase in the value of the shelf-life results in an increment in price, inventory cycle time, quantity ordered, and profits generated for all functions. In addition, as the ordering cost increases, price, the inventory cycle time and quantity ordered also increase for all functions. Nonetheless, the profits show a decreasing trend. Finally, by escalating the purchasing cost for all functions, there is an increase in both the price and the inventory cycle time; however, the quantity to order and total profits tend to decrease.

This research work extends and widely contributes to the state-of-the-art on the inventory field, with focus on perishable items with price-stock-time-dependent demand. The inventory model studied here has some limitations from where several directions for extension and further research are highlighted. First, an inventory model can be built with the same characteristics and demand pattern, but including the sustainability elements, so the effects of the carbon-tax and cap-and-trade mechanisms can be assessed. Second, a model that allows shortages with full or partial backlogging should be explored. Third, the trade-off and benefits of investing on preservation technology should be also studied. Fourth, the noninstantaneous item's freshness degradation can be integrated into the proposed inventory model. Finally, other components such as incorporating discount policies or advertising efforts can also be investigated.

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