# Modal Analysis of Standard and Profile Modified Spur Gears by Finite Element Analysis

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**Abstract –** *In this study, efforts were made to study the effect of profile modification on the vibratory behaviour of standard and profile-modified spur gears. Profiles have been modified by the disposition of an addendum with a suitable modification factor. The Finite element method is used to carry out modal analysis of standard and profile modified spur gears. Ansys Workbench v.17 is used as finite element software. Vibratory behaviour has been studied in terms of modal. It is shown that profile modified spur gears have better vibratory behaviour compared to standard gears.*

***Key Words***: **Standard gear, spur gear, profile modification, modal analysis, Profile modified spur, Vibration analysis, Modification factor, Natural frequency, finite element, Ansys.**

## 1. INTRODUCTION

GEARS, or toothed wheels, are the basic machine elements that transmit motion and power by successive engagement of teeth on their periphery. They constitute an economical method for such transmission, particularly if power levels or accuracy requirements are high. Gears are an important element in all types of machinery. Applications of gears are diverse, like automobiles, aerospace, agriculture, precision equipment, robots, machine tools, etc.

Spur gears are used to transfer low and medium power between two parallel axes. The disadvantage of spur gear is that more operating sound will be produced at a higher operating speed. Gears transmit power due to meshing action between the gears. The meshing action of gears occurs due to rolling and sliding action between the gears. Generally, gears fail due to wear, scoring, fatigue, corrosion, spalling, pitting, and interference. This results in alteration of profile modification of teeth and becomes a source for the vibration of gears. Hence, it is necessary to study the profile modification of gears to minimise the vibration and give direct control over the failure of gears.

The main purpose of the gear tooth profile is to transmit power constantly from one shaft to another through the conjugate action of gear teeth. The involute tooth profile is commonly used because of its various advantages. The main source of vibration of the gear is from the profile of the gear tooth, and vibration may increase or decrease due to modification of the profile. Corrosion, pitting, wear, scuffing, etc., will also increase vibration in gears.

Spur gears have their teeth parallel to the axis and are used for transmitting power between two parallel shafts with a basic rack of standard gears having an equal amount of tooth thickness and tooth space on the pitch circle. For standard gears, there is no profile correction and the radius of curvature of the involute at the root is zero. The pressure angle for standard gears may vary from 14.5° to 20°, etc., and the teeth may come in the form of full depth or stub tooth systems. The 20° full depth involute system is widely used.

### 1.1 Vibration Analysis of Spur Gear

Vibration is a virtual property of the system when it is subjected to dynamic loads. A spur gear is a geometrically symmetric structure, and each tooth is a substructure of the gear. Loading on spur gear is unsymmetrical, resulting in unbalanced forces that lead to vibration [1]. The non-conformity of the tooth profile while manufacturing gear results in vibration of the gear.

Hence, vibration analysis vibration of spur gear is very important and it has to be minimised before the actual working environment for long service. Modal analysis, harmonic analysis, and transient analysis are the types of vibration analysis. Modal analysis is the free vibration of the gears under no load conditions. This analysis will give the natural frequency and mode shapes of gear teeth.

### 1.2 Profile Modification

By altering the profile of the gear, the vibration of the gear behaviour can be altered. There are many methods of profile modification, like linear profile modification [2], parabolic profile modification [3], tooth lead crown relief, and profile shift modification [4]. Profile shift modification, also known as addendum modification, involves disposition of addendum and dedendum relative to the pitch circle on both pinion and gear by selecting the portion of the involute with a larger radius of curvature with a suitable modification factor.

A profile modification factor is a fraction related to the selected gear module. Positive profile modification means shifting of the tooth profile away from the centre of the gear and negative profile modification means shifting of the tooth profile towards the centre of the gear. The positive profile modification will have a larger tooth thickness on the pitch circle, and the negative profile modification will have more tooth space on the pitch circle.

There are two types of profile shift modification, i.e., SO and S gear systems. For So gear gearing systems, the sum of the modifications on both pinion and gear must be equal to zero, which means one gear with positive and the other with negative profile modification to be done. Hence, the centre distance will be constant. The S gear system of profile modification must have a positive or negative sum of modification factors on both the pinion and the gear [5].

## 1.3 The Concept of Profile Modification

Figure 1 represents the nature of the involutes of a circle. In a standard pinion, the radius of curvature of involutes at root is zero and will have a definite radius of curvature at the tip circle diameter. Figure 1.2 distinguishes the curvature radius at the tip circle diameter for standard and profile modified gears. The increase in the profile shift coefficient yields a specific portion of involute with a larger radius of curvature.

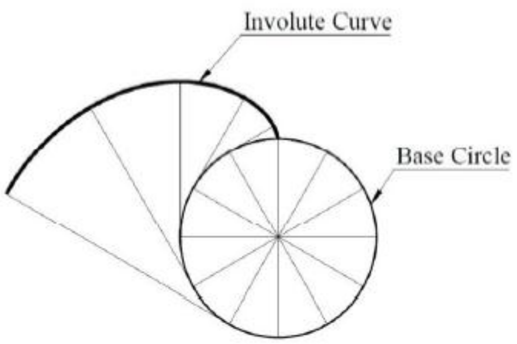



**Fig-1:** Involute of a circle

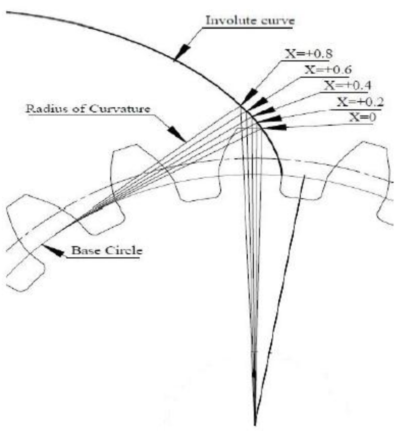



**Fig-2:** Involute curve with different modification and radius of curvature

Figure 3 and 4 gives information on addendum modification with suitable modification factor for both pinion and gear. Profile modifications also have the following advantages. Positively corrected pinions will have an increase in the strength at the root of the pinion as tooth thickness on the pitch circle is increased, sliding and contact relations are improved, and contact stresses are decreased, resulting in better surface durability.

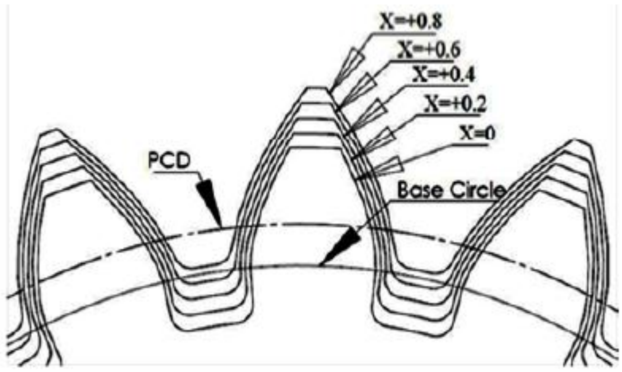



**Fig-3:** Addendum modified pinion with positive (+) profile modification

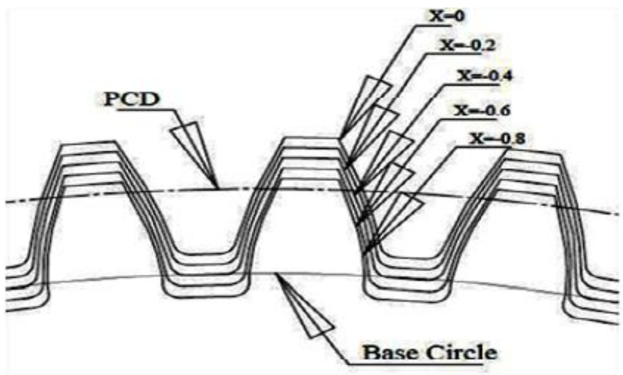



**Fig-4:** Addendum modified gear with negative (-) profile modification

## 1.4 Literature Review

The following main papers have been gone through, and a brief overview of those papers is given here. S.S. Ghosh and G. Chakraborty [2] have studied the effects of gear tooth profile modification on the overall dynamics of a spur gear system. They have been studied with the objective of achieving the best profile which will lead to a low vibration level.

Hsiang Hsi Lin [3] has studied the effect of tooth profile modification on the dynamic performance of spur gear systems. Parabolic tooth profile modification is generally preferred for low dynamic response in gear, which operates at low operating over range of loading conditions.

The author, Hui Man [5], has carried out a study on optimum profile modification. Based on a mesh stiffness model of profile shifted gears with addendum modification and tooth profile medications (TPM) proposed in this paper, time varying mesh stiffness (TVMS), load sharing factor (LSF), load sharing and transmission error (LSTE), and net load sharing and transmission error (NLSTE) of the profile shifted gear with different amounts of profile modification corresponding to four types of profile modification are analyzed.

ZehuaHu[5], focuses on the analysis of the effects of the tooth profile modification on the dynamic responses of a high-speed gear-rotor-bearing transmission system.

The authors M. Divandari [6] and others discuss the effects of tooth profile modification and its effects on the gear pair vibration. They have tried to conclude effect on localized tooth defects due to tooth profile modification by calculating tooth mesh stiffness. The mesh stiffness of gear teeth varies due to the changes in the profile of gear teeth.

### 1.5 Problem Definition

Based on the above, the problem for this work is defined as "Modal Analysis of Standard and Profile Modified Spur Gears by Finite Element Analysis."

### 1.6 Methodology

To realise the objectives outlined above, the following methodology has been adopted.

1. Estimation of the module using Lewis theory and determining the dimensions of standard and profile modified spur gear for a specific loading condition.

2. Geometric modelling of standard and profile-modified spur gears

3. Modal analysis of the gear tooth by using the Finite Element Method (FEM)

4. Comparison of modal analysis results of standard and profile-modified spur gear

## 2. MODAL ANALYSIS

Modal analysis is the undamped free vibration of the system to know the dynamic characteristics of the system. Natural frequency, modal behaviour, and damping coefficients are dynamic characteristics of the system.

The methods of determination of natural frequencies for continuous systems are explained here [7]. 1. Mass and stiffness method 2. Rayleigh's method and 3. Numerical (FEM) method In this work, a numerical method is adopted to find the natural frequency of standard and profile modified spur gear teeth.

### 2.1 General Procedure for Modal Analysis Using FEA

Analysis work has been carried out in Ansys Workbench 17.0. These are steps to be followed in any FEM based analysis software, and Figure 5 shows the flow chart of FEM steps.

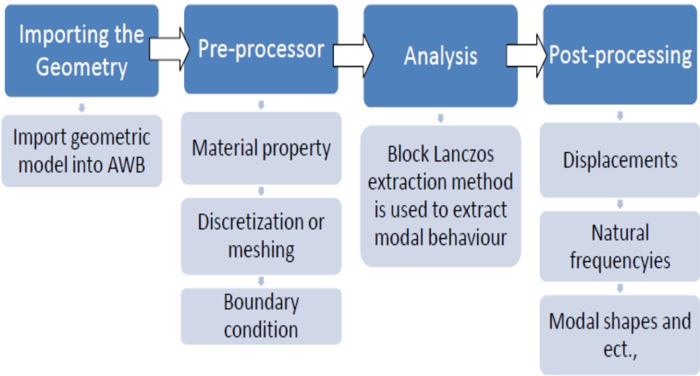



**Fig- 5:** General procedure for modal analysis using FEA

### 2.2 Modal Analysis of Standard and Profile Modified Pinion

Figure 6. A gear model of pinion consisting of 3 teeth sections subtended by an angle of 60 degrees is shown in Figure 6. The total number of teeth is 18, with a module of 5 mm. The three-tooth model is considered [6] for the modal analysis because the gear has a cyclically symmetric structure and each tooth is a sub-structure of the gear. One tooth may be considered for analysis and applied boundary condition, but it will not be similar to the actual one. Hence, adjacent teeth are also considered for the analysis. The boundaries of adjacent teeth are constrained. Thus, displacement of the central tooth will be the same as in actual gear.

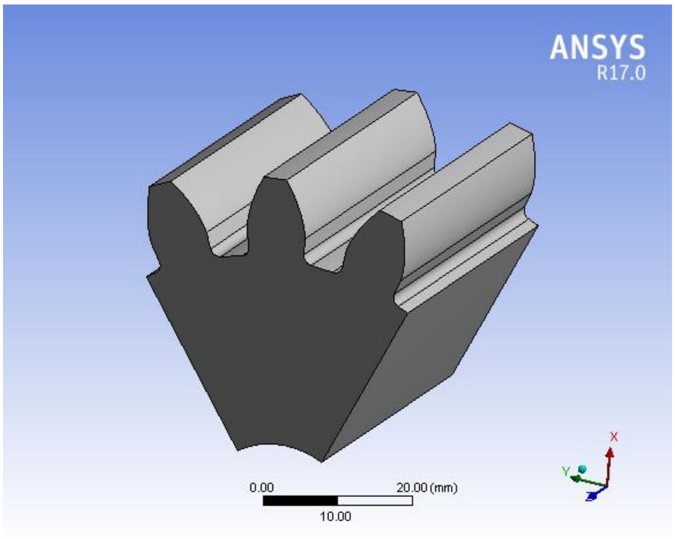



**Fig-6:** Pinion model for FEM analysis.

The imported solid model has been divided into small elements, as shown in Figure 3.3. The element type SOLID186 will be best suited for the analysis of solid models in AWB. This model consists of 34,760 nodes and 7464 elements. Figure 7 shows a discretized model of the pinion.

## 2.3 Boundary Condition:

The total angle subtended by the three (3)-tooth gear sector [7] of the 18-tooth model is about 60 degrees. The following boundary conditions have been chosen for modal analysis of standard pinion. A gear is a cyclically (axisymmetric) symmetric mechanical element. Therefore, there will be no deformation in the tangential (rotational) direction. Hence, the outer radial lines of the three tooth sectors and rim are constrained in all directions as shown in Figure 3.4. The teeth's outer and inner surfaces have been left untouched.

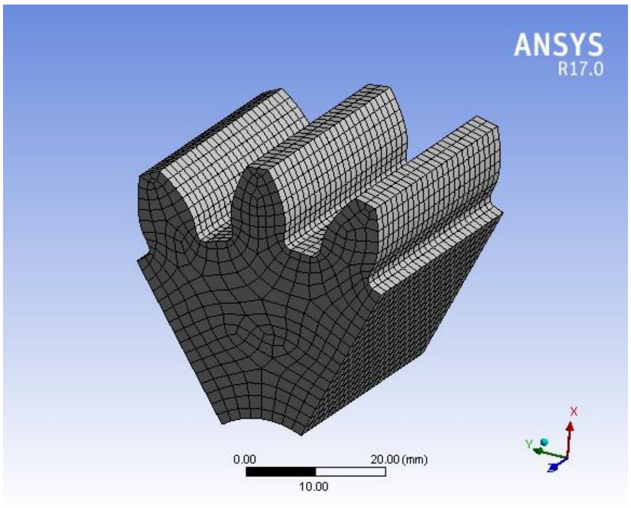



**Fig – 7:** Discretized model of pinion

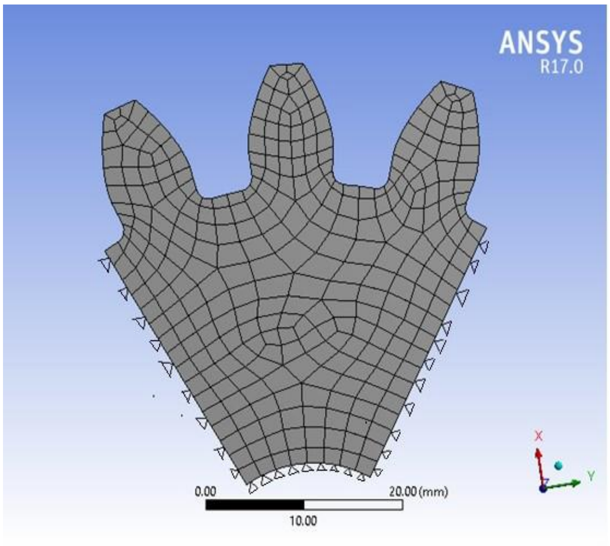



**Fig – 8:** Boundary constrained model of standard pinion

## 2.4 Natural Frequencies of Standard and Profile Modified Pinion.

The results of natural frequencies of modal analysis are given in Table 1. The natural frequencies of the first six modes of vibration are tabulated because the first mode and up to the third mode of vibration can be observed in practise while the rest of the modes of vibration cannot be observed due to mode superposition [7]. In Table 3.1, X = 0, X = +0.2, and so on represent the profile modification factor, and 1, 2, and so on represent the number of vibration modes. Each cell represents the natural frequency of modes of vibration with a corresponding modification factor X.

**Table -1:** Natural frequency of the standard and profile modified pinion.

| Mode Number | X=0 | X=+0.2 | X=+0.4 | X=+0.6 | X=+0.8 |
|---|---|---|---|---|---|
| 1 | 31489 | 31281 | 31178 | 31107 | 31029 |
| 2 | 33197 | 35580 | 37865 | 39770 | 41209 |
| 3 | 34797 | 37591 | 40236 | 42074 | 43243 |
| 4 | 35022 | 37745 | 40369 | 42557 | 43300 |
| 5 | 37916 | 39924 | 41490 | 42800 | 44807 |
| 6 | 39627 | 42038 | 44318 | 46413 | 48222 |

The variation of the first six natural frequencies for standard and profile modified pinions with different modification factors X is shown in chart 1. The natural frequencies (NFs) of standard pinions are lower than NF of profile-modified pinions. The NFs of profile modified pinions increase as the profile modification factor increases. As observed in geometric modelling, the width of the pinion tooth increases at pitch circle diameter. Due to this, the NFs of profile modified pinion increases as the modification factor increases.

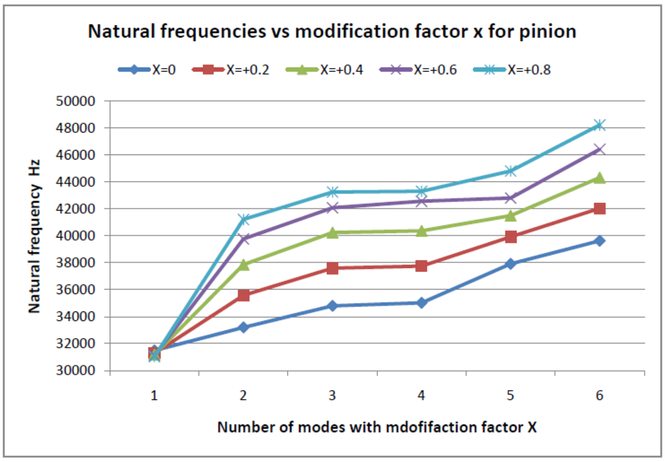
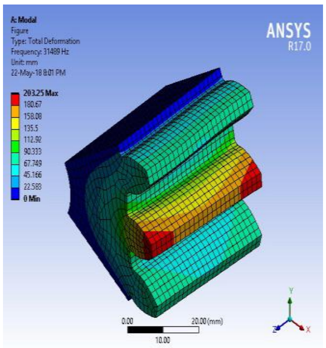


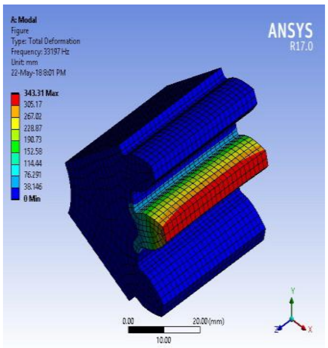

**Chart -1**: Variation of Natural frequencies for pinion with modification factor X

## 2.5 The First Six Mode Shapes of Standard Pinion Modal Analysis

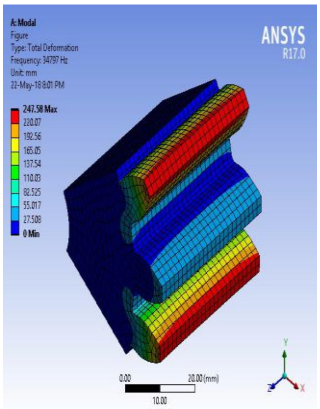
The modal behaviour of a profile-modified pinion remains the same as a standard pinion. Brief discussions on the modal behaviour of standard pinion teeth are presented here. The first mode of vibration occurs at a natural frequency of 31849 Hz. A pinion tooth vibrates in an axial direction. The second mode of vibration occurs at a natural frequency of 33197 Hz. Pinion tooth bending occurs in a tangential direction. The third mode of vibration occurs at a natural frequency of 34797 Hz. The bending of adjacent teeth occurs with the same amplitude in the opposite direction tangential to the pinion axis of rotation. The fourth mode of vibration occurs at a natural frequency of 35022 Hz. The bending of adjacent teeth occurs with the same amplitude in the same direction radial to the pinion axis of rotation. The fifth mode of vibration occurs at a natural frequency of 37916 Hz. The twisting pinions tooth each other in the same direction along the axis of rotation pinion. The sixth mode of vibration occurs at a natural frequency of 39627 Hz. The twisting pinions tooth each other in opposite directions along the axis of rotation pinion. The first six modes of vibration pinion are shown in figure 9.



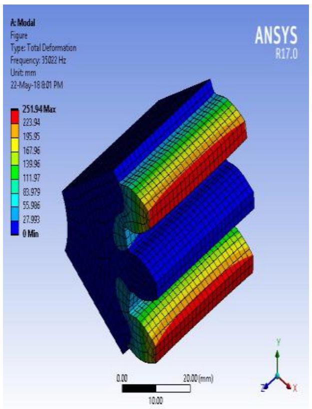

Mode 1-Axial



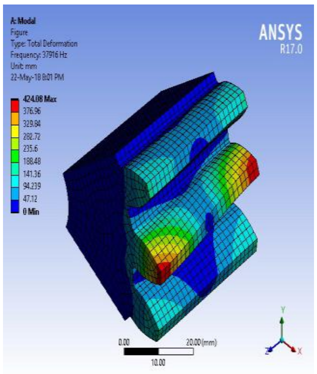

Mode 2-Tangential (Bending)



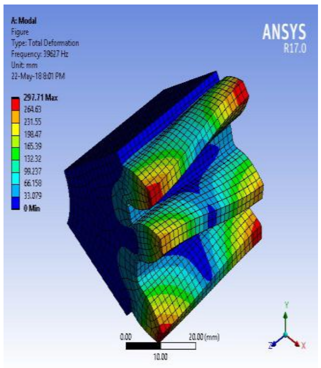

Mode 3- Tangential (Same direction)




Mode 4- Radial (opposite Direction)




Mode 5- Twisting (same direction)




Mode 6 – Twisting (opposite direction)

**Fig - 9:** First six modal behaviour of standard pinion

## 2.6 Natural Frequencies of Standard and Profile Modified Gear

Figure 10. A gear model of pinion consisting of 3 teeth sections subtended by an angle of 20 degrees is shown in Figure 10. The total number of teeth is 54 with a module of 5 mm. The three-tooth model is considered [7] for the modal analysis because the gear has a cyclically symmetric structure and each tooth is a sub-structure of the gear. One tooth may be considered for analysis and applied boundary condition, but it will not be similar to the actual one. Hence, adjacent teeth are also considered for the analysis. The boundaries of adjacent teeth are constrained. Thus, displacement of the central tooth will be the same as in actual gear.

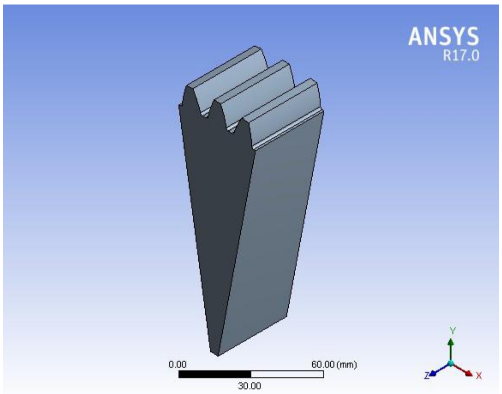



**Fig - 10:** Imported model of three teeth standard gear.

The model has been discretized and boundary conditions have been applied as in the case of pinion. The results of natural frequencies of modal analysis standard and profile modified gear are given in Table 2. The natural frequencies of the first six modes of vibration are tabulated because the first mode and up to the third mode of vibration can be observed in practise while the rest of the modes of vibration cannot be observed due to mode superposition [7]. In Table 2 X=0, X= -0.2, etc., represents the negative profile modification factor and 1, 2, 3 etc., represents the number of modes of vibration. Each cell represents the natural frequency of modes of vibration with a corresponding modification factor X.

**Table 2:** Natural frequency of the standard and profile modified gear

| Mode number | X= - 0 | X= - 0.2 | X= - 0.4 | X= - 0.6 | X= - 0.8 |
|---|---|---|---|---|---|
| **1** | 27823 | 27985 | 28015 | 28174 | 28213 |
| **2** | 34862 | 34885 | 34556 | 34266 | 33463 |
| **3** | 38169 | 38165 | 36631 | 35520 | 34008 |
| **4** | 41515 | 41065 | 39550 | 38274 | 36376 |
| **5** | 41572 | 41611 | 41387 | 40765 | 39188 |
| **6** | 42023 | 41968 | 41485 | 40914 | 39878 |

The variation of the first six natural frequencies for standard and profile modified gear with different modification factors X is shown in chart 2. The natural frequencies (NFs) of standard gear are higher than the NF of profile modified gear. The NFs of profile modified pinions decrease as the profile modification factor increases. As observed in geometric modelling, the width of the gear tooth decreases at pitch circle diameter. Due to this, the NFs of profile modified pinion decreases as the modification factor increases.

## 2.7 The First Six Mode Shapes of Standard Gear Modal Analysis

The result of the first six modal behaviour of standard gear is shown in Figure 11. The modal behaviour of profile-modified gear remains the same as standard gear. Hence, the modal behaviour of standard gear is presented in this work. Brief discussions on the modal behaviour of standard gear teeth are presented here. The first mode of vibration occurs at a natural frequency of 27867 Hz. Gear teeth vibrate in an axial direction. The second mode of vibration occurs at a natural frequency of 34862 Hz. The bending of adjacent teeth occurs with the same amplitude in the opposite direction radial to the gear axis of rotation. The third mode of vibration occurs at a natural frequency of 38169 Hz. Gear tooth bending occurs in a tangential direction. The fourth mode of vibration occurs at a natural frequency of 41515 Hz. The bending of adjacent teeth occurs at the same amplitude in the same direction tangential to the gear axis of rotation. The fifth mode of vibration occurs at a natural frequency of 41572 Hz. The twisting gears tooth each other in the same direction along the axis of the rotation gear. The sixth mode of vibration occurs at a natural frequency of 42023 Hz. The twisting gears tooth each other in opposite directions along the rotation gear's axis.

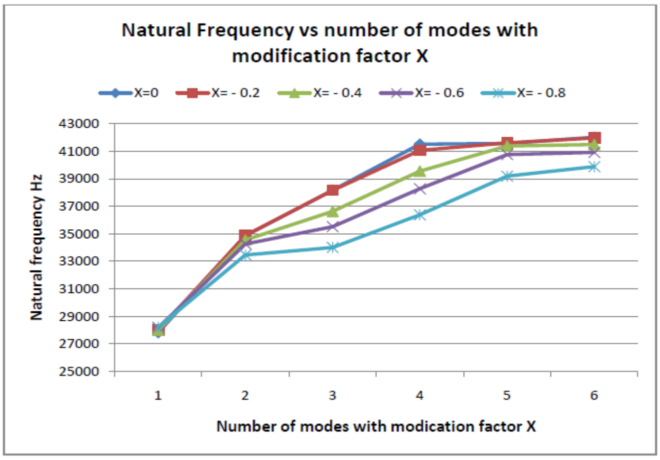
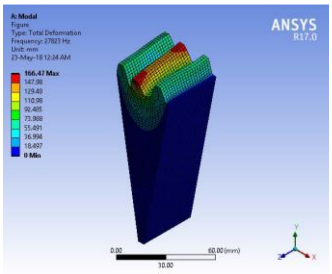
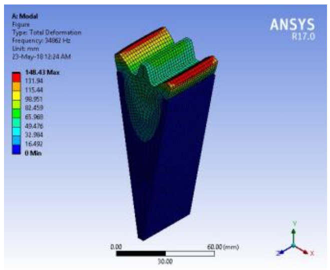


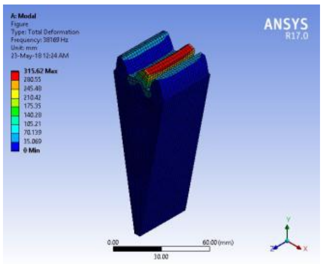

**Chart -2**: Variation of Natural frequencies for gear with modification factor X



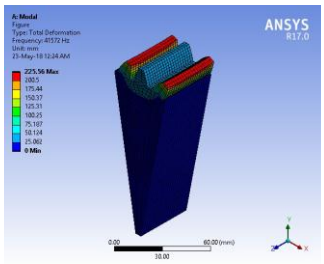

Mode 1-Axial



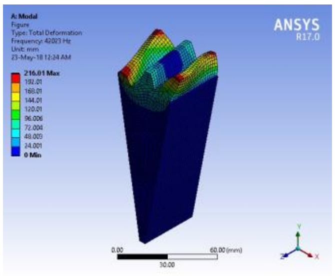

Mode 2- Radial (opposite Direction)



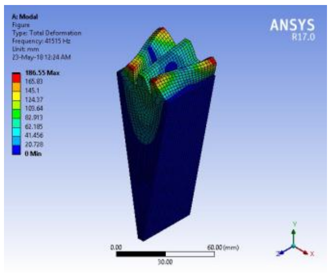

Mode 3-Tangential (Bending)




Mode 4- Tangential (Same direction)




Mode 5- Twisting (Same direction)




Mode 6 – Twisting (Opposite direction)

**Fig-11:** First six modes of vibration of standard gear

## 2.8 Comparison, Results and Discussions

Modal analysis of standard and profile modified pinions (SPMP) and standard and profile modified gears (SPMG) has been done. There is variation in natural frequency in both SPMP and SPMG with respect to profile modification adapted (positive or negative) to pinion and gear is shown in Figure 3.12. Natural frequency variation with respect to SPMP is always increased with increasing positive profile modification factor X due to an increase in thickness at the pinion tooth's root section. Similarly, as the negative profile modification factor X increases, the variation of natural frequency with respect to SPMG decreases due to a decrease in thickness at the root section of the gear tooth.

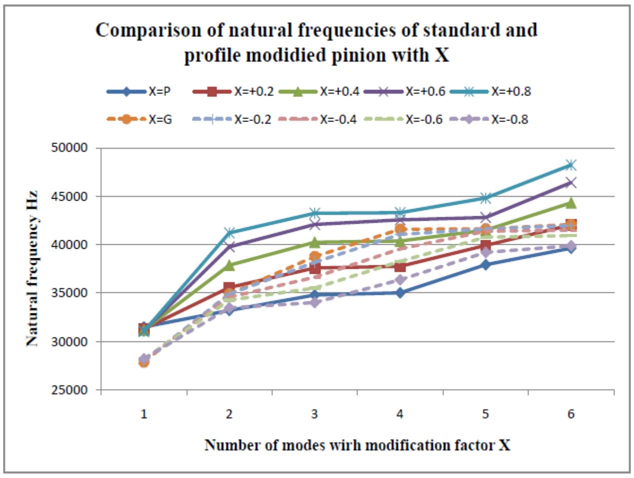



**Chart -3**: Comparison of natural frequencies between standard and profile modified pinion and gear with different modification factor X.

1. The following points were observed.
2. The natural frequencies of standard pinions are initially lower than those of standard gears, but as the modification factor increases, the natural frequencies of pinions are higher than standard gears.
3. Due to the increased natural frequency of the profile-modified pinion with respect to modification factor, the pinion can be operated at higher speeds than the standard pinion.

## 3. CONCLUSIONS

Natural frequencies of positively corrected pinions increase with increasing modification factor, but they decrease with increasing modification factor of negatively corrected gear teeth. Hence, both extreme conditions are not suitable for practical application because the pinion will have a sharp tip and the gear tooth will be thin.

Due to the increase in natural frequency of the profile-modified pinion with respect to modification factor, the pinion can operate at higher speeds than the standard pinion between modification factors of x = +0.2 and x = +0.8. Therefore, optimised profile modification is required for better performance of the gear.

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