

Investigating Effects of Geometrical Parameters of Arrowhead Shaped Auxetic Structure on Negative Poisson's Ratio

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Abstract - Auxetic structures are class of non-conventional structures having negative Poisson's ratio. They expand laterally when axially stretched and laterally contract when axially compressed which is different from conventional structures. Light weight, high strength, impact damping capabilities and stiffness offers potential applications in the field of aerospace, automobile, military protection equipment, textile, suspension mount. Although different auxetic structures are studied and analyzed because of their strength to weight ratio and negative Poisson's ratio, there is need of development of new structures to improve these properties. The present work is focused on design of arrowhead shaped auxetic structure and to explore influence of the geometrical parameters of arrowhead shaped auxetic structure on negative Poisson's ratio. In this work, the equation of Poisson's ratio for arrowhead shaped structure is derived. By changing the values of angle α , wall thickness and length of small wall of unit cell; the value of Poisson's ratio is analytically determined and it is verified with Finite Element Analysis. It is concluded that Poisson's ratio of arrowhead shaped structure is a function of angle α , wall thickness and length of small wall of unit cell. Here the angle α plays a major role.

Key Words: Auxetic, Negative Poisson's Ratio, Arrowhead shape, Auxetic modelling, Cellular structure, Auxetic simulation.

1. INTRODUCTION

Materials with negative Poisson's ratio (ν) are known as auxetic materials. Auxetic materials are structures arranged in such a way that they exhibit certain peculiar behaviour of lateral expansion when they are stretched and lateral contraction when they are compressed. This unusual behaviour of the materials under deformation gives rise to negative Poisson's ratio (ν) and enhanced mechanical properties of the materials. It also increases strength to weight ratio of the materials which is the need of hour. Almost in all analysis of auxetic materials, the geometrical structure plays major role [1]. Auxetic materials have some improved mechanical properties like increased indentation resistance [2] where the material does not dent very easily, higher energy absorption capabilities [3], increased shear modulus [4], better fracture toughness [5].

During the past three decades, the researchers have developed numerous different auxetic structures like a re-entrant (honeycomb, triangle, S-shape, star-3, star-4, star-6) structures, chiral structures, rotating rigid (rectangle, triangle, rhombus) structures that express their individual impact on negative Poisson's ratio as well as on different mechanical properties have been reported [6][7]. It has widely spread applications in ballistic resistance, medical bandage, sports helmet, protective equipment [8][9].

Wenwang Wu et al. [10] have developed a stent using an anti-tetrachiral auxetic structure. The author proposed two innovative design i.e. anti-tetrachiral stent and hierarchical anti-tetrachiral stent with circular and elliptical nodes. To extract mechanical property, above structures investigated analytically and the tensile test is carried out to the above specimens for verification. Here designing for both anti-tetrachiral and hierarchical anti-tetrachiral stent with circular and elliptical nodes was explored. Geometrical influence on tensile mechanical behaviour was elaborated using finite element analysis. It is found that these properties were mostly depend on geometrical parameters of the unit cell.

Yiru Ling [11] has carried out a study of the geometrical influence of the re-entrant honeycomb structure on Poisson's ratio. Different parameters like rib length, wall thickness and rib angle of the structure were taken into account. To extract results for the above parameters finite element analysis was done and these results were compared with experimental results. To extract data experimentally, tension test was carried out on the 3D printed specimen on ARAMIS. Yichao Tang et al. [12] have designed cut unit geometry in hierarchical kirigami to increase the expansion and compression ability of a matter. The cut unit structure was designed with a combination of a line cut, cutout and hierarchy of structure. Expansion as well as compression ability of combined architecture design were explored through experiments together with geometrical modelling and finite element analysis. From results, it was found that solid rectangular structure has expansion ability from 41% to 124% without hierarchy and 62% to 156% with hierarchy. Also, structure with gaps have extreme compressibility up to 81%.

Kusum Meena et al. [13] have designed and developed a new structure to reduce the stress

concentration effect on sharp edges. In this paper, derivation of Poisson's ratio, Young's modulus of the new structure was established analytically. A comparative study of stress concentration effect has been carried out for re-entrant hexagon and newly developed auxetic structure by using finite element analysis and results were compared with experimental analysis. The new structure has proven to be less stress concentration effect than re-entrant hexagon and also proved that auxetic limit of the structure is up to 15% strain and maximum Poisson's ratio is up to -2.5.

Arrowhead shaped auxetic structure has wide applications in medical, sports, textile and military equipment. Affecting parameters of arrowhead shaped structure are angle α , wall thickness and length of small wall of unit cell.

Previous researchers have focused their studies on the structural behaviour of arrowhead shaped structure after loading, less work was done on effect of parameters like angle α , wall thickness and length of small wall of unit cell. These parameters also affect Poisson's ratio hence it is need of hour.

In this paper, the arrowhead shaped structure is designed and modelled with wall thickness, angle α , and length of small wall of unit cell consideration. The equation of Poisson's ratio is analytically determined with initial assumptions and simple trigonometric relations. Data for Poisson's ratio is collected separately for each parameter both by analytically and by finite element analysis. The graphs are plotted for Poisson's ratio concerning angle α , wall thickness and length of small wall of unit cell.

2. ANALYTICAL DERIVATION

To determine equation of Poisson's ratio for arrowhead shaped auxetic structure, the unit cell of arrowhead structure is considered and have made certain assumptions to carried out analytical modelling.

Assumptions made for analytical modelling are:

- Under deformation of arrowhead shaped structure
- Wall 'FP' and 'GP' of unit cell will always rotate around center 'P'.
 - Node E and P always follows straight path along Y-axis.
 - Individual wall length will remain constant.
 - Wall thickness of unit cell will remain constant.

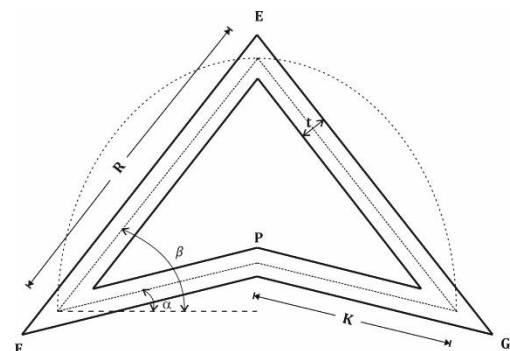


Fig -1: Arrowhead shaped unit cell

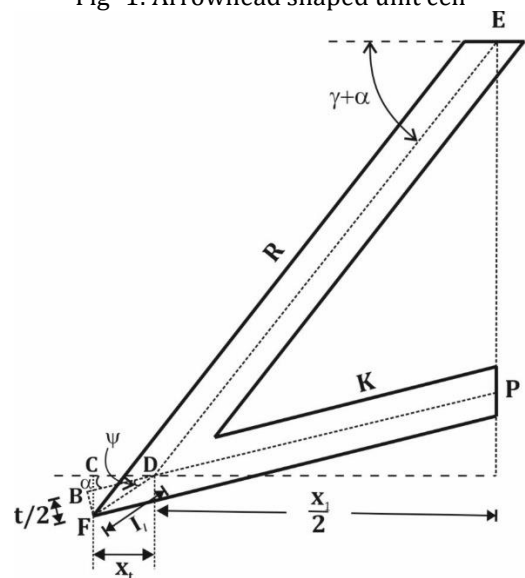


Fig -2: Enlarged portion of arrowhead shaped unit cell

Fig -1 shows arrowhead shaped unit cell and fig -2 shows enlarged portion of fig-1 with wall thickness consideration. From fig -1, in the unit cell structure, α is the angle between FP and horizontal axis and β is the angle between FE and horizontal axis. Change in angle β i.e. ($d\beta$) with respect to change in angle α i.e. ($d\alpha$) is denoted by Z i.e. $Z = d\beta / d\alpha$. Fig. (2) is used to determine overall length (X) of unit cell structure in X-direction. Here overall length (X) in X-direction is expressed as,

$$X = 2x_t + x_1 \quad (1)$$

Where, x_1 is X-direction length of unit cell without thickness consideration and x_t is an extra length formed due to thickness consideration.

$$x_t = I_t \cos(\psi + \alpha) \quad (2)$$

$$x_1 = 2K \cos \alpha \quad (3)$$

In fig -2, assuming $\alpha = 0$, from which γ can be determined, which will remain constant before and after deformation as shown in fig -3.

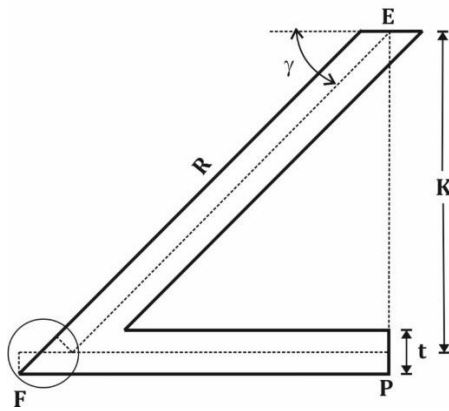


Fig-3: When angle $\alpha = 0$

From fig -3,

$$\gamma = \sin^{-1} \left(\frac{K}{R} \right) \tag{4}$$

From fig -2,

$$R = \sqrt{[(K \sin \alpha + K)^2 + (K \cos \alpha)^2]} \tag{5}$$

By simplifying we get,

$$\frac{K}{R} = \left(\frac{1}{\sqrt{2(1 + \sin \alpha)}} \right) \text{ or } R = K \sqrt{2(1 + \sin \alpha)} \tag{6}$$

$$\gamma = \sin^{-1} \left(\frac{1}{\sqrt{2(1 + \sin \alpha)}} \right)$$

By assuming, $\sqrt{2(1 + \sin \alpha)} = M$ gives,

$$\gamma = \sin^{-1} \left(\frac{1}{M} \right) \tag{7}$$

Fig. (4) shows enlarged portion of fig. (3).

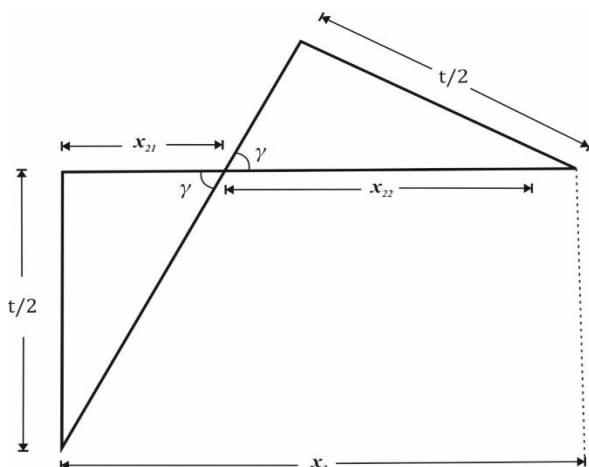


Fig-4: Enlarged portion of fig -3 to find the angle γ

From fig -4, we get the value of x_2 .

$$x_2 = \frac{t}{2} \left(\frac{1 + \cos \gamma}{\sin \gamma} \right) \tag{8}$$

From fig -2, consider Δ FBD as shown in fig -5.

$$\tan \psi = \frac{t/2}{x_2}$$

$$\psi = \tan^{-1} \left(\frac{\sin \gamma}{1 + \cos \gamma} \right) \tag{9}$$

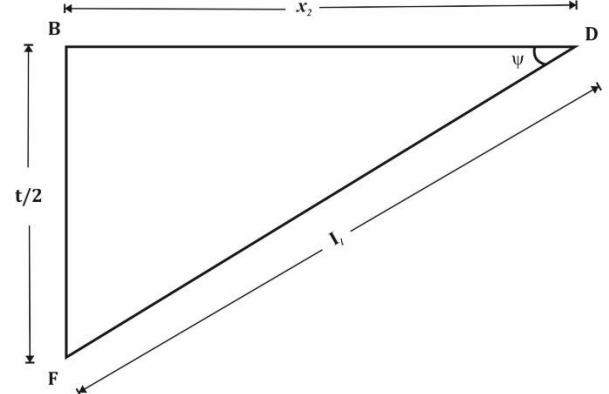


Fig-5: Enlarged portion of Δ FBD

And inclined length is,

$$I_1 = \left(\frac{t}{2 \sin \psi} \right) \tag{10}$$

Here inclined length will remain constant during deformation. Hence equation (1) become

$$X = 2 I_1 (\cos \psi \cos \alpha - \sin \psi \sin \alpha) + 2K \cos \alpha \tag{11}$$

To calculate overall length in Y-direction, thickness does not play any role. Hence, thickness is not considered here.

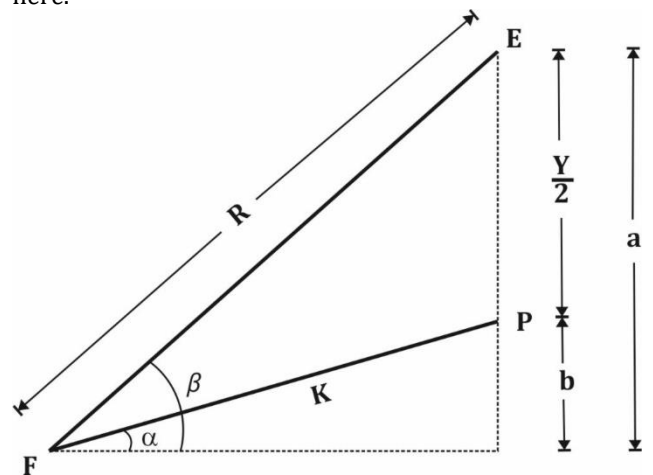


Fig-6: Y-axis symmetric line diagram

Fig -6 shows line diagram of arrowhead shaped structure to determine overall length of arrowhead shaped structure in Y-direction.

From Fig -6,

$$Y = 2(a - b) \tag{12}$$

$$a = R \sin\beta, \quad b = K \sin\alpha$$

$$\cos\beta = \left(\frac{K \cos\alpha}{R}\right)$$

$$\beta = \cos^{-1}\left(\frac{\cos\alpha}{\sqrt{2(1 + \sin\alpha)}}\right) \tag{13}$$

$$Y = 2R[\sin\beta - (K/R)\sin\alpha]$$

Hence Y- axis length is given by,

$$Y/2 = R[\sin\beta - (K/R)\sin\alpha] \tag{14}$$

Poisson's ratio is predefined as,

$$\nu_{lt} = -\left(\frac{\epsilon_{\text{transverse}}}{\epsilon_{\text{longitudinal}}}\right)$$

Where, $\epsilon_{\text{transverse}}$ and $\epsilon_{\text{longitudinal}}$ are strains in transverse and longitudinal direction respectively.

But as we are giving small compressive displacement to specimen, Poisson's ratio will be,

$$\nu_{lt} = -\left(\frac{d\epsilon_t}{d\epsilon_l}\right) \tag{15}$$

Where, $d\epsilon_t$ and $d\epsilon_l$ are the small incremental strains in the transverse and longitudinal directions respectively, which may be expressed as,

$$d\epsilon_t = \frac{dX}{X} = \frac{1}{X} \frac{dX}{d\alpha} d\alpha \tag{16}$$

$$d\epsilon_l = \frac{dY}{Y} = \frac{1}{Y} \frac{dY}{d\alpha} d\alpha \tag{17}$$

Now by differentiating equation (11) and (14) respectively,

$$\frac{dX}{d\alpha} = -[2I_l(\sin\alpha \cos\psi + \sin\psi \cos\alpha) + 2K \sin\alpha] \tag{18}$$

And

$$\frac{dY}{d\alpha} = K \cos\alpha (Z - 1) \tag{19}$$

Here, Value of Z for every α is calculated using sketch in CAD software and graph of Z Vs α is plotted and after curve fitting equation for Z in terms of α is generated.

$$Z = [-2.07(\alpha)^4 \times 10^{-8} + 3.401(\alpha)^3 \times 10^{-6} - 2.945(\alpha)^2 \times 10^{-4} + 1.737(\alpha) \times 10^{-2} + 0.01918]$$

Here by substituting equations (16) and (17) in equation (15), it gives expression for negative Poisson's ratio of unit cell of arrowhead shaped auxetic structure when loaded in Y-direction as follows.

$$\nu_{lt} = \nu_{yx} = \frac{2I_l(S_\alpha C_\psi + S_\psi C_\alpha) + 2K S_\alpha}{2I_l(C_\psi C_\alpha - S_\psi S_\alpha) + 2K C_\alpha} \times \frac{(MS_\beta - S_\alpha)}{C_\alpha(Z-1)} \tag{20}$$

Where, $S_\alpha = \sin\alpha$, $C_\psi = \cos\psi$, $S_\psi = \sin\psi$, $C_\alpha = \cos\alpha$, $S_\beta = \sin\beta$

Similarly, we can find expression for negative Poisson's ratio of unit cell of arrowhead shaped auxetic structure when loaded in X-direction.

3. FINITE ELEMENT ANALYSIS (FEA)

To validate analytical results and to check the mechanical behaviour of the auxetic structure, finite element simulation was carried out in ANSYS software. Here parametric simulation was carried out to take various parameters in the account.

The 3D CAD model of the arrowhead shaped unit cell structure were designed in the ANSYS design Modeler. The designed structural model was imported into ANSYS workbench for numerical simulation. As auxetic property was depend on geometrical parameters, it is less depending on material used. Here, material used for analysis was structural steel. The hex mesh was employed all over the structure using the sweep mesh method as this body is in sweep able condition as shown in fig -7. Mesh convergence analysis was carried out and element size was reduced up to 0.5 mm.

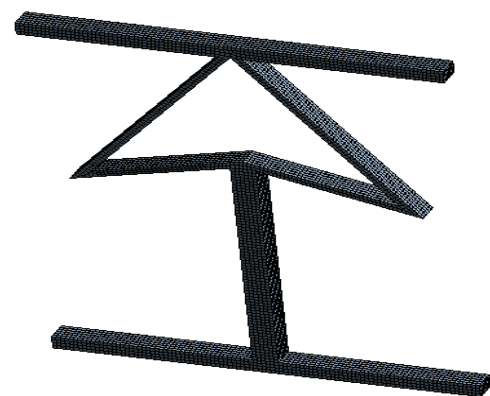


Fig -7: Meshing of arrowhead structure

Boundary conditions were decided by acknowledging some assumptions made while analytical modelling of arrowhead shape unit cell structure as shown in fig -8. Compressive displacement was chosen to apply on structure. Hence face C and face D was selected for the same. -0.75 mm of remote displacement was applied on face C and +0.75 mm of remote displacement was applied on face D, while displacement in X, Z and rotation about X, Y and Z-axis was restricted.

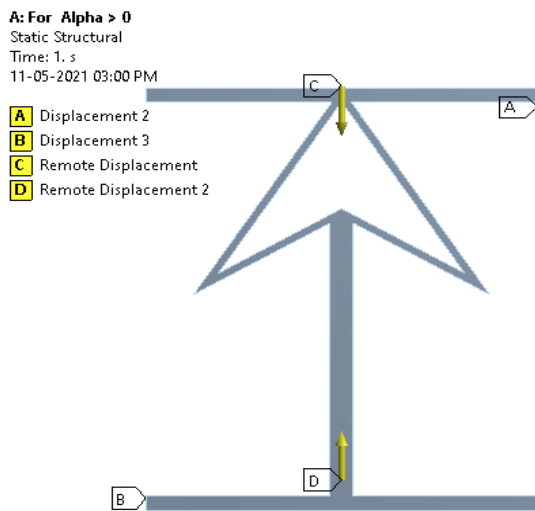


Fig-8: Boundary conditions of arrowhead unit structure

To allow the structure to move only in loading direction displacement was set free in the Y-direction for all four faces of A and B and displacements in other two directions i.e., X, Z were restricted.

Fig -9 represents directional deformation of structure along X-axis and fig -10 represents directional deformation of structure along Y-axis.

As we know that ANSYS does not give direct Poisson's ratio or there is no availability of any tool which will give Poisson's ratio value directly. So here we need to check the deflection of required point and then we have to calculate Poisson's ratio manually. So here directional deformation of point F, G in horizontal direction and point E, P in vertical direction was collected. So, it will give total deflection of dX and dY respectively.

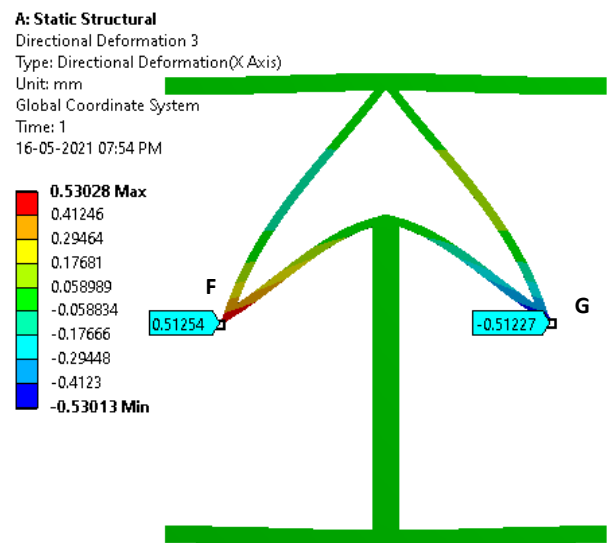


Fig -9: Directional deformation along X-axis

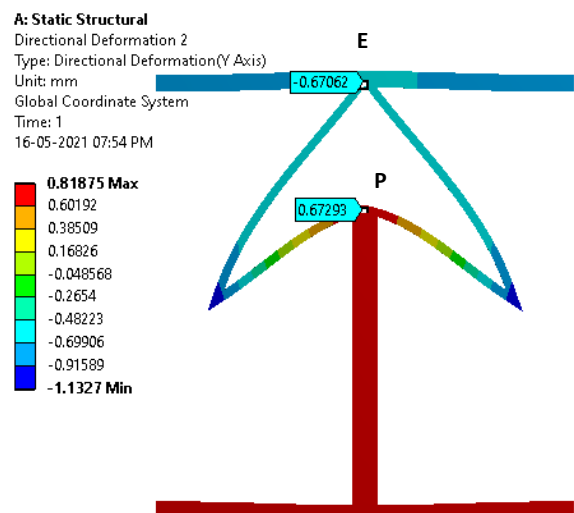


Fig -10: Directional deformation along Y-axis

To measure displacement along the longitudinal direction, directional deformation along the Y-axis i.e., loading direction was considered and to measure displacement along the lateral direction, directional deformation along the X-axis i.e., other than loading direction was considered.

$$dX = (\text{deformation at F} - \text{deformation at G})_{\text{transverse}}$$

$$dY = (\text{deformation at P} - \text{deformation at E})_{\text{longitudinal}}$$

Then, the value of ϵ_t and ϵ_l becomes

$$\epsilon_t = \frac{dX}{X}$$

$$\epsilon_t = \frac{dY}{Y}$$

Then, the Poisson's ratio is,

$$\nu_{lt} = - \left(\frac{\epsilon_{\text{transverse}}}{\epsilon_{\text{longitudinal}}} \right)$$

It was observed that FEA results were approximately same as that of the analytical results and the percentage of error is in between 4 to 8 %.

4. COMPARISION OF ANALYTICAL AND FEA

To collect the data of Poisson's ratio analytically for various parameters, Microsoft Excel was used. In the Excel, the formula was developed for an equation which was derived analytically. The ideal size of a specimen was assumed as length of small wall of unit cell (K)= 20 mm, wall thickness (t) =1 mm and angle (α) = 15 degree. The graphs of variation of Poisson's ratio with angle (α), length of small wall of unit cell (K) and wall thickness (t) were plotted.

From ANSYS Poisson's ratio data for each deformation was extracted in Excel and the graphs were plotted for FEA. Both analytical and FEA graphs was superimposed and compared with each other.

5. RESULTS AND DISCUSSION

Chart -1 Shows variation of Poisson's ratio (ν) with angle (α). The range for angle α was considered as -30 degree to +30 degree. From the graph, it is observed that the nature of the graph is quartic, and when α is positive, Poisson's ratio tends to be negative and vice versa.

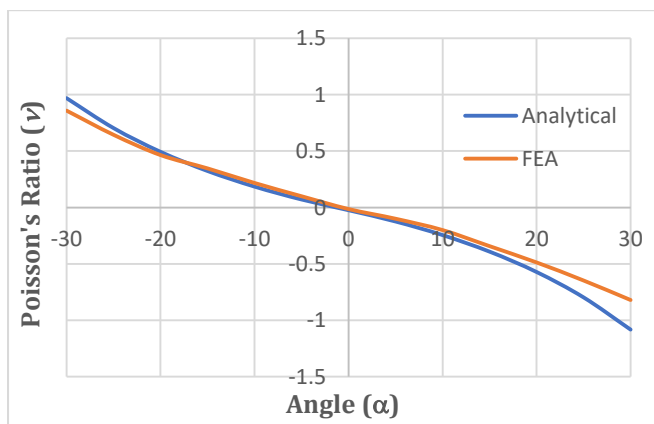


Chart -1: Variation of Poisson's ratio with angle (α)

Chart -2 Shows variation of Poisson's ratio (ν) with wall thickness (t). The range for the thickness was taken from 1 mm to 5 mm. From the graph, it is observed

that the nature of the graph is quadratic and Poisson's ratio decreases by increasing the wall thickness.

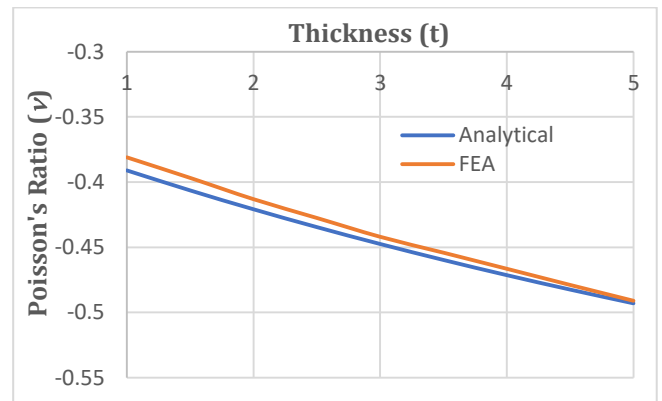


Chart -2: Variation of Poisson's ratio with wall thickness

Chart -3 Shows variation of Poisson's ratio (ν) with length of small wall of unit cell (K). The range for the length was taken from 5 mm to 30 mm. From the graph, it is observed that the Poisson's ratio increases quadratically, up to 25 mm and it further remain constant.

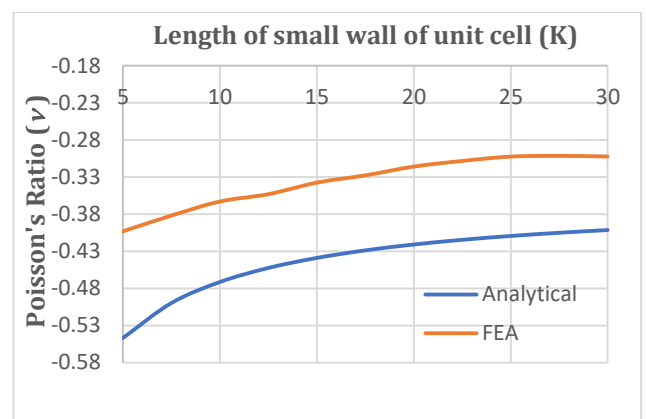


Chart -3: Variation of Poisson's ratio with length of small wall of unit cell (K)

The above result shows that Poisson's ratio depends on K, α and t. But it has less impact of length of small wall of unit cell (K) and wall thickness (t) while angle (α) plays a major role in it.

6. CONCLUSION

In this paper, negative Poisson's ratio of arrowhead shaped auxetic structure have been measured for different geometrical parameters and good accordance has been found between analytical and FEA values.

The equation of Poisson's ratio for arrowhead shaped auxetic structure was analytically derived,

modelled and simulated with FEA. Analytical and numerical results proved that geometrical parameters of arrowhead shaped auxetic structure has an influence on Poisson's ratio. The following are specific conclusions:

Negative Poisson's ratio of arrowhead shaped auxetic structure is

- Inversely proportional to angle (α) and Poisson's ratio follows quartic behaviour.
- Independent of material properties as geometrical parameters play major role.
- Reduces quadratically with thickness (t).
- Proportional to length of small wall of unit cell (K) in a quadratic manner where negative Poisson's ratio increases quadratically up to 25 mm and further it remains constant at -0.30.

For arrowhead shaped auxetic unit cell negative Poisson's ratio can be achieved up to -0.8.

To achieve negative Poisson's ratio angle ' α ' must be greater than zero. Hence it is concluded that Poisson's ratio of arrowhead shaped auxetic structure is function of angle (α), wall thickness and length of small wall of unit cell and angle plays major role.

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BIOGRAPHIES



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