TOTAL CO-INDEPENDENT DOMINATION NUMBER IN JUMP GRAPH

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1. Introduction: All graphs in this paper will be finite and undirected without loops and multiple edges. As usual p = |V| and q=|E| denote the number of vertices and edges of a jump graph J(G) respectively 8in general, we use X denote the sub graph number induce by the set of vertices X. N(v0)and N[v] denote the open and closed neighborhood of vertex v, respectively. A set D of vertices in J(G)is a denoted set if every vertex in V − D is adjacent to some vertex in D. The domination number γ (J(G)) is the minimum cardinality of a dominating set of J(G). If J(G) is connected graph then a vertex cut of J(G) is a subset R of V with the property that the sub graph of J(G) induced by J(V) − R is disconnected. If J(G) is not a complete graph then the vertex connectivity number κ (J(G)) is the minimum cardinality of a vertex cut . If J(G) is complete graph J(kp) it is known that κ(J(G)) = p-1. For terminology and notations not specifically defined hence we refer reader to [4] For more details a cut domination number and its related parameters, we refer to [5] [11] and [13]. A dominating set S of J(G) is called a connected dominating set if the induced sub graph S is connected. The minimum cardinality of a connected dominating set of J(G) and is denoted by γc(J(G))[12]. A dominating set S of J(G) is called non split dominating set if he induced sub graph J(V)-S is connected . The minimum cardinality of a non-split dominating set S of J(G)and 1 denoted by γns (J(G)) [8]. A dominating set S of J(G) called total dominating set of J(G) is called total domination number of J(G) is denoted by γ(J(G)) [5]. Many application of domination in graphs can be extended to the co-independent domination. For example the routing protocols in such networks are typically based on the con ept of a virtual backbone in ad hoc wireless network. This motivates us to introduce the concept of total co-independent domination in a jump graph.

2. Total Co-independent Domination Number

Definition: A total dominating set S of a graph J(G)= (V,E) is called total co-in pendent dominating set, if the induced sub graph V-S has no edge and has at least one vertex. The minimum cardinality of a co-independent dominating set of J(G) is called the total co-independent J(G) is denoted by γtoc (J(G)). A Total co-independent dominating set S is said to be minimal if no proper subset of S is total co-independent dominating set.

Observation 2.1: A non empty graph J(G) is without isolated vertices if and only if it admits a total co-independent dominating set.

Observation 2.2: A total co-independent dominating set D of a jump graph J(G) is minimal irf and only if for each vertex v∈D, one of the following condition is satisfied.

9) There exists a vertex u∈V such that N(u) ∩ D={v}
(ii) V −(D −{v}) is independent set. Therefore D−{v} is total co-independent dominating set of J(G) a contradiction. Hence one of the given conditions is satisfies. The converse is straight forward.

The following observations are immediate. Observation 2.3: For any cycle C_p, γ toc(C_p)= p − p|3

Observation 2.4: For any path P_p, γtoc (P_p) = p − |3

Observation 2.5: For any wheel W_p with p vertices γ toc(W_p) = 1+ p-1.2

Observation 2.6: For any complete graph K_p, γ toc(K_p) = p − 1.

Observation 2.7: For any complete bipartite graph K_{rs} where r≤ s γ toc(K_{rs}) = r+1.

Proof: Let J(G) be a graph with p ≥ 3 vertices and has isolated vertices, then
2 ≤ γ toc(J(G)) ≤ p − 1. Further the equality of upper bound is attained if J(G) is P_3 or J(G) is two star, and the upper bound attained if J(G) is complete graph K_p or G = P_3 U P_3.

Proof: Let J(G) be graph with p ≥ 3 vertices and has no isolate4d vertices and D total co-independent dominating set of J(G).Then obviously D is total dominating set

Hence
Proof: Let $J(G)$ be a jump graph with no isolated vertices. Suppose that $S \subseteq V$ is any minimum total co-independent dominating set of $J(G)$. Since for any graph $J(G)$ any total co-independent dominating set $S$ is also split dominating set and every dominating set is also dominating set. Hence $\gamma(J(G)) \leq \gamma_{t}(J(G)) \leq \gamma_{t, col}(J(G)).$ Similarly we can prove (ii)

**Proposition 2.10:** If $J(G)=(V,E)$ is a jump graph with no isolated vertices and $|V| \geq 3$ and $H$ is spanning sub graph with no isolated vertices and has vertices greater than $2 \leq \gamma_{t, col}(J(G)) \leq \gamma_{t}(J(G)).$

**Proof:** Let $S$ be any minimum total co-independent dominating set of $H$. Then obviously from the definition of the total co-independent domination, $S$ is also total dominating set of $H$ and therefore $S$ is total dominating set of $J(G)$. Hence $\gamma(J(G)) \leq \gamma_{t}(J(G)) \leq \gamma_{t, col}(J(G)).$

**Proposition 2.11:** Let $J(G)$ be a graph with $D$ is minimal total co-independent dominating set then $V-D$ is independent set of $J(G)$.

**Proof:** Let $D$ be a minimal total co-independent dominating set of $J(G)$. Suppose that $V-D$ is not independent dominating set of $J(G)$. Since $D$ is total co-independent dominating set of $J(G)$, then there exist a vertex $u$ such that $u$ is not dominated by any vertex in $V-D$. Since $J(G)$ has total co-independent dominating set, then $J(G)$ has no isolated vertices, therefore $u$ is dominated by at least one vertex in $D \cup \{u\}$. Thus $D \cup \{u\}$ is total co-independent dominating set of $J(G)$ which contradicts the minimality of $D$. Thus every vertex in $D$ is adjacent with at least one vertex in $V-D$ and $V-D$ is independent dominating set of $J(G)$.

**Proposition 2.12** If $J(G)=(V,E)$ is a jump graph with no isolated vertices and $|V| \geq 3$ and $H$ is spanning sub graph with no isolated vertices and has vertices greater than $2 \leq \gamma_{t, col}(J(H)) \leq \gamma_{t}(J(G)).$

**Proof:** As the number of independent vertices may increase in any connected spanning sub graph $J(H)$ of $J(G)$, we can still maximize the set $V-D$ which results in the decrease of the value $V \gamma_{t, col}(J(H))$. Hence $\gamma_{t, col}(J(H)) \leq \gamma_{t, col}(J(G))$.

**Observation 2.13:** For any jump graph $J(G)$ any total co-independent dominating set of $J(G)$ contains all the support vertices.

**Proof:** Suppose the graph $J(G)$ has a total co-independent dominating set $D$ and let $v$ be support vertices does not belong to $D$ then clearly the pendent vertex which adjacent to $v$ can not belong to $D$ from the definition of total co-independent dominating set. Hence $D$ is not a dominating set which is contradiction.

**References:**


