

# Solar Geometry Application for Single and Dual Axis Tracking

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**Abstract** - Photovoltaic modules are usually placed and positioned with a tilt angle to enhance solar module's performance along with preventing accumulation of water or dust over panel's surface. The various Solar Geometry Angles namely, Latitude Angle( $\phi$ ), Declination Angle( $\delta$ ), Hour Angle( $\omega$ ), Altitude Angle( $\alpha$ ), Zenith Angle( $\Theta_z$ ), Solar Azimuth Angle( $\gamma$ ) and the Angle of Incidence( $\theta$ ) aid in judging the precise sun's position at any particular time of the day and the determination of required ideal tilt angles for Solar application for any particular location. The determination of optimum tilt angle for Solar panels allows us to ensure the maximum utilization of the incident Solar Irradiance and maximize Power generation through Solar panels. In this paper, with the application of Solar geometry angles, sample calculations for the city of Jaipur has been carried out for real time solar axis tracking.

**Key Words:** Solar Geometry, Photovoltaic Modules, Optimum Tilt Angle, Solar Axis Tracking, Maximum Power Generation, Solar Irradiance

## 1. INTRODUCTION

A renewable energy oriented future is technically possible, and the scope is really good that various renewable energy technologies might compete with conventional energy resources in a short span of time. The solar energy could act as a vital renewable energy form because it is sustainable in nature along with being in harmony with the environment. One of the vital ways of Harnessing Solar energy is through Photo Voltaic (PV) cells. The PV cells are assembled in modules that generates a particular voltage and current when Sunlight falls over it. An improvement in the power production cost can be performed by altering the angle of tilt and angle of orientation which depends upon various factors like day of the year, time, and location of the place. The inclination angle plays an important role in utilizing the maximum solar radiation. Maximizing the potential power generation by the solar panels has been the key engineering by proper positioning and placement of the Solar panels which conforms to the solar geometry angles. Because of the intermittent existence of sun radiation, solar photovoltaic power generation fluctuates as compared with conventional power plants. There are many problems regarding to the use of solar energy. The major is it's a dilute energy source. The solar radiation flux rarely exceeds 1 kW / m<sup>2</sup> even in the

hottest regions on earth, and the total radiation over a day is at best about 7 kWh / m<sup>2</sup>. [4]. These are poor values from a technical point of view. Thus large collecting areas are required in many applications which result in high costs. The second major problem with the utilization of solar energy is that its availability varies greatly over time. Owing to the day-night period and even, seasonally because of earth's orbit around the sun, the variability in availability occurs regularly. Additionally, the variability occurs due to local environmental conditions at a given venue. Consequently, when the sun shines, the energy obtained must be preserved for use during times when this is not possible. The storage requirement also adds significantly to any system's cost. The actual challenge of using solar energy as an alternative energy is economic in nature. One needs to strive to develop cheaper and more effective collection and storage methods so that the large initial investments needed in most applications at present are lesser. There are various solar calculator applications providing information about solar resource and performing rooftop solar PV installation calculations along with cost-benefit analysis pre and post solar PV installations. But the real time calculation for tilt and orientation angle values for any particular time of the day are not provided in these sources. These works lacks the information about the positioning and placement of solar panels in a broad respect and just provides an annual average tilt angle to maximise utilization of solar potential available at the location. They mainly focus over the potential financial savings after installation of solar application rather than producing accurate results about the solar data. Limited information is provided about the solar resource data and the installation placement and positioning of the solar panels for efficient utilisation of solar energy and maximise power generation. Hence significant work could be performed in this area by incorporating various empirical relations to get real time optimum angle suggestion for solar applications. For Jaipur city, with the help of latitude, longitude, date, time, standard time zone as basic inputs, the precise solar geometry angles could be calculated and with the application of the calculated solar geometry angles real time optimum inclination and orientation angles could be suggested for panel applications.

## 2. SOLAR GEOMETRY ANGLES

The various solar geometry angles could be described as follows:

1. Declination angle: It is the angle between the line extending from the center of earth to the center of Sun and the projection of this line over the equatorial plane. The declination in degrees for any day of the year (n) can be calculated approximately by the equation:

$$\delta = 23.45 \sin\left(\frac{360}{365}(n + 284)\right)$$

The rotational Earth axis (the polar axis) is typically inclined from the ecliptic axis at an angle of 23.45 which is perpendicular to the ecliptic plane. The ecliptic plane is the plane of earth's orbit around the sun. Solar declination is the angular distance of the equator's north (or south) rays of the sun, defined as positive by north declination. The angle between the centerline of the sun – earth and the projection of this line on the equatorial plane. Declination that are towards the north of the equator (northern hemisphere summer) are positive, and which are at south are negative. The angle of declination carries a range ranges from 0 to 23.45 at the spring equinox and summer solstice respectively and again 0 for the fall equinox, and the winter solstice has a value of 23.45.

2. Hour Angle: The hour angle for any point on surface of earth is described as the angle by which the earth must rotate to position the meridian of the point directly aligned with the sun. The point hour angle is the angle determined on the equatorial plane of the earth between the projection of sun rays and the imposition of the sun – earth center line on equatorial plane. Around local solar noon, the hour angle is zero, with each 360/24 or 15 hour longitude equal to 1 hour, with positive value defined afternoon hours.

Observing the sun from earth, the angle of the solar hour is an indicator of time from the solar noon, measured in angular scale, usually in degrees. At solar noon the angle of the hour is zero degree, with the time before the solar noon being expressed as negative degrees, and the local time after the solar noon being expressed as positive. The calculations of the hour angle ( $\omega$ ) involve the Local solar time(LST) which incorporates the Standard time, standard time longitude( from time zone), Longitude of location and the equation of time correction(EOT) as:

$$EOT = 0.2292(0.075 + 1.868 \cos N - 32.077 \sin N - 4.615 \cos 2N - 40.89 \sin 2N)$$

$$LST = \text{Standard time} + / - 4(\text{Standard Time longitude} - \text{Longitude of location}) + EOT$$

$$\text{Hour angle } \omega = 15(12 - \text{Local Solar Time}).$$

3. The Altitude angle: It is the vertical angle between the direction of sun rays and its horizontal projection over the surface of earth. The altitude angle referred to as the solar elevation angle, describes how high the sun appears in the sky. The altitude angle ( $\alpha$ ) could be calculated as:

$$\sin \alpha = \sin \delta \sin \phi + \cos \delta \cos \omega \cos \phi$$

The angle is determined between an imaginary line between the sun and the observer, and the horizontal plane on which the observer sits. The angle of altitude is negative if the sun descends below the horizon.

4. The Zenith Angle: The zenith angle is the angle between the zenith and the center of the Sun's disc. The solar elevation angle is the altitude of the Sun, the angle between the ray direction of sun and its projection on horizontal plane. As the altitude and zenith angles are complementary to each other, the cosine of one equals the sine of the other.

$$\cos \theta_z = \sin \delta \sin \phi + \cos \delta \cos \omega \cos \phi$$

$$\theta_z = (90 - \alpha)$$

They can be calculated using the same formula, using spherical trigonometric results. The zenith angle at solar noon is at its least and is equal to the subtraction of latitude from the angle of solar declination.

5. The Azimuth Angle: It is the angle measured , in horizontal plane, from the North Meridian to meridian position of the horizontal projection of the sun rays. Azimuth angle ( $\gamma$ ) could be calculated using the latitude angle , the hour angle, the declination angle and the altitude angle as:

$$\cos \gamma_s = \sec \alpha (\cos \phi \sin \delta - \cos \delta \sin \phi \cos \omega)$$

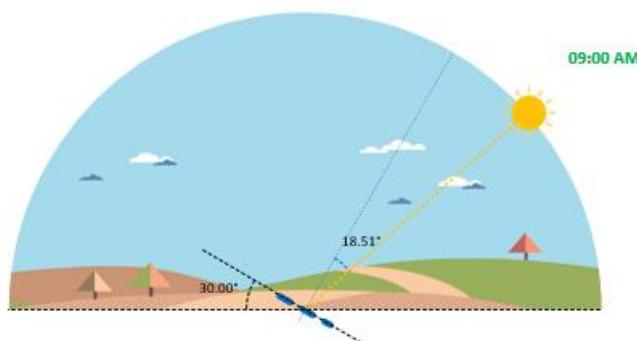
The angle of azimuth is the direction of compass from where the sunlight is incident over surface. At solar noon, the sun in the northern hemisphere is always directed towards south, and in the southern hemisphere is directed towards north. The sun rises directed towards east at the equinoxes and sets directed towards west regardless of the latitude, making the azimuth angles 90 ° at sunrise, and 270 ° at sunset. Overall though, the angle of azimuth varies with the latitude and time of year.

6. The Angle of Incidence: It is the vertical angle made by the normal to any surface over the earth and the direction of rays of sun. In case of a horizontal surface, the incidence angle,  $\theta$ , and the zenith angle,  $\Phi$ , have the identical values. The angle of incidence is calculated as:

$$\begin{aligned} \cos \theta_i &= \sin \phi (\sin \delta \cos \beta + \cos \delta \cos \gamma \cos \omega \sin \beta) + \\ &\quad \cos \phi (\cos \delta \cos \omega \cos \beta - \sin \delta \cos \gamma \sin \beta) + \\ &\quad \cos \delta \sin \gamma \sin \omega \sin \beta \end{aligned}$$

where,  $\beta$  is the tilt angle.

The angle of a sun's ray creates a line perpendicular to a surface; for instance, a surface directly facing the sun has an angle of zero incidence, and a surface parallel to the sun (such as a sun rays striking a horizontal rooftop at sun rise) has an angle of 90 °. Sunlight tends to be absorbed with an incident angle of 90 °, while lower angles are mirrored.



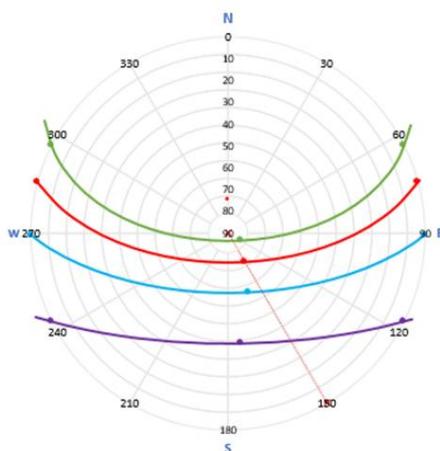
**Fig -1:** Tilt and Incidence angle representation.

### 3. SUN POSITIONING

The Sun Path Diagram is the diurnal locus or the path traversed by the sun during the whole length of the day from sunrise time to sunset time, defined by the combination of solar geometry angles mentioned as the Altitude and Azimuth Angles.

The precise Sun's position for any particular time of the day for any location all over the world could be obtained using the sun path diagram that is developed using the Solar Altitude and Azimuth Angles. The diurnal and annual variations of the sun path could also be inferred from sun path diagram.

The Sun Path Diagram consist of radial lines and concentric circles. The radial lines represent the Solar Azimuth angle value while the concentric circles represent the value of Altitude angles. The north direction radial line represent the zero Azimuth angle and as one moves in the clockwise direction from north meridian the value of the azimuth angle increases as 90 degrees at east direction, 180 degrees at south meridian, 270 degrees at west direction and 360 degrees back again at north. Intermediate values of Solar Azimuth angles is also mentioned as 30°, 60°, 120°, 150°, 210°, 240°, 300° and 330° for the convenience of judging the value of azimuth angle of the sun at any particular time from the Sun Path diagram. The concentric circles representing the altitude angles, have the maximum value at the center of the sun path diagram 90° altitude angle and it decreases as one moves towards the periphery with an interval of 10 degrees. The peripheral circle represents the 0° altitude angle. Now with the combination of the value of Altitude and azimuth angles the precise sun's position could be located easily and it's extended pattern for the whole length of the day represent the sun path diagram.



**Fig -2:** Sun Path Diagram for Jaipur city.

The radial lines represent the solar azimuth angles and the concentric circles represent the Altitude angles. The Sun Path Diagram graph is developed in form of four lines, each indicated by the color coding legends: green line represents the Summer solstice day sun path, purple line represent the winter solstice day sun path, blue line represent the Sun path for both the yearly equinoxes day, while the red line represent the Sun path for the date of 7<sup>th</sup> August.

### 4. OPTIMUM TILT ANGLES

When the surface over the earth is placed normal to the direction of the sun rays, the angle of incidence becomes zero and the Solar irradiance becomes maximum as:

$$\theta_i = 0$$

$$\therefore I = I_0$$

This introduces us to the Ideal case for the placement and positioning of tilt of solar modules. In this case the solar modules are exactly aligned with the direction of the sun rays and there occurs the maximum utilization of the incident Solar power.

The tilt of the Solar panels could be defined for any particular season that is the Winter season, Summer season and the Spring, Fall season. The tilt angle for the mentioned season cases could be calculated on the basis of latitude angle as:

#### Season-wise

Winters :

$$\beta = (\phi \times 0.9) + 29^\circ$$

Summers :

$$\beta = (\phi \times 0.9) - 23.5^\circ$$

Spring, Fall :

$$\beta = \phi - 2.5^\circ$$

#### Example Calculations for Jaipur City

Latitude- 26.9°N

Longitude- 75.78°E

Day- 7 August (219th day)

Time- 10:30 hours

Standard time longitude = 82.5°E

Declination angle:

$$\delta = 23.45 \sin((360/365)*(219+284))$$

$$= +16.25^\circ$$

$$N = (n-1)*(360/365)$$

$$= (219-1)*(360/365)$$

$$= 215.013$$

Equation of time correction:

$$\begin{aligned} EOT &= 0.2292 * (0.075 + 1.868 * \cos(215.013) - \\ &32.077 * \sin(215.013) - 4.615 * \cos(2 * 215.013) - \\ &40.89 * \sin(2 * 215.013)) \\ &= -5.2856 \end{aligned}$$

Local Solar Time:

$$\begin{aligned} LST &= 10:30 - 4 * (82.5 - 75.78) - 5.2 \\ &= 9:57:50.16 \end{aligned}$$

Hour Angle:

$$\begin{aligned} \omega &= 15 * (12 - LST) \\ &= 30^\circ 32' 27.6'' \end{aligned}$$

Zenith angle:

$$\begin{aligned} \cos \theta &= \cos(26.9) * \cos(30^\circ 32' 27.6'') * \\ &\cos(16.25) + \sin(26.9) * \sin(16.25) \end{aligned}$$

$$\cos \theta = 0.86399$$

$$\theta = 30.23^\circ$$

Altitude angle:

$$\alpha = 90 - \theta = 90 - 30.23^\circ = 59.76^\circ$$

Solar Azimuth Angle:

$$\begin{aligned} \cos \gamma &= \sec(59.76^\circ) * ((\cos(26.9) * \sin(16.25)) - \\ &(\cos(16.25) * \sin(26.9) * \cos(30^\circ 32' 27.6''))) \end{aligned}$$

$$\gamma = 104^\circ$$

Angle of incidence:

For a surface in horizontal plane (tilt angle  $\beta=0$ ),

$$\begin{aligned} \cos \theta_i &= \sin 26.92 * (\sin(16.257) * \cos(0) + \\ &\cos(16.257) * \cos(104) * \cos(30^\circ 32' 27.6'') * \sin(16.257) + \cos(26.92) * ((\cos(16.257) * \cos(0) * \cos(30^\circ 32' 27.6'')) * (\sin(16.257) * \sin(0) * \cos(104))) + (\cos(16.257) * \sin(104) * \sin(30^\circ 32' 27.6'') * \sin(0))) \\ \theta_i &= 33.23^\circ \end{aligned}$$

Optimum tilt Angles:

For zero angle of incidence, the tilt angle of the panel becomes equal to zenith angle i.e.

$$\beta = \theta = 30.23^\circ$$

This is the most optimum angle for this particular time as the sun rays are directly normal to the panel.

For season wise,

For winters:

$$\begin{aligned} \text{Tilt angle} &= (\text{Latitude angle} * 0.9) + 29^\circ \\ &= (26.92 * 0.9) + 29^\circ \\ &= 53.23^\circ \end{aligned}$$

For Summer:

$$\begin{aligned} \text{Tilt angle} &= (\text{Latitude angle} * 0.9) - 23.5^\circ \\ &= (26.92 * 0.9) - 23.45^\circ \\ &= 1^\circ \end{aligned}$$

For Spring and fall:

$$\begin{aligned} \text{Tilt angle} &= \text{Latitude angle} - 2.5^\circ \\ &= 26.92 - 2.5^\circ \\ &= 24.42^\circ \end{aligned}$$

The Horizontal axis tilt angle or the orientation angle of the panel could be derived from the solar azimuth angle i.e. for this particular case a horizontal angle of 104° from north meridian would be the horizontal axis tilt angle for panel.

## 5. CONCLUSIONS

The various solar geometry angles have been studied along with their relevant expressions and applicability. The application of the solar geometry angles have been derived in terms of the optimum tilt angles for the panels, so that there is maximum utilization of incident solar power. The case of Jaipur city has been studied with the basic inputs of latitude, longitude, date, time, standard time zone and Vertical axis and Horizontal axis tilt angles as the output for Single and dual axis tracking systems for solar panels. For the particular case considered, the Vertical axis tilt angle for the panel should be 30.23° and the Horizontal axis tilt angle should be 104° from north meridian. In the similar manner, Horizontal axis and vertical axis tilt angles for any particular location and at any particular date and time could be evaluated.

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