

Calculating Fractal Dimension of Grey scale and Color image by using DBC and RCC

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Abstract - Fractal dimension (FD) is the ratio for providing the index of complexity by comparing a image or pattern can change in detail with the scale with which it is measured. It can also be defined as the measure of space filling capacity of a image or pattern that explains how the fractal scales covers the image. In this research work Firstly we have used Differential Box Counting Method (DBC) to calculate the fractal dimension of the gray scale image and the color image as well. Moving on we have also used the Reticular Cell Counting Method for calculating the same. Finally we have concluded the thesis with the fractal dimension values of the gray scale image and the color image by using the above said approaches.

Key Words: DBC, RCC, FD, Grey Scale image, Color image

1. INTRODUCTION

Fractal dimension (FD) is the ratio for providing the index of complexity by comparing a image or pattern can change in detail with the scale with which it is measured. It can also be defined as the measure of space filling capacity of a image or pattern that explains how the fractal scales differently [20]. In this work I have explained the fractal geometry as well as some well-known methods for calculating the FD for a grey scale image. The term fractal was firstly derived by Mandelbrot in 1983[1] to explain the shape and appearance of different images. Fractal images are the self-similarity in nature which can be used to calculate the FD. Fractal geometry is the method to describe the regular and fragmented shapes of the natural objects or complex objects where the Euclidian geometry fails to do the same. The fractal methodologies mainly used for digital image analysis where the Fractal algorithms are used to reduce the difference of the surface roughness by manmade design and natural senses. Fractal geometry provides mathematical representation for many complex objects such as mountain, clouds etc. and for many natural objects also. The fractal analysis has been improved for digital image analysis and mostly applied for texture analysis and segmentation analysis [4-6]. Fractal Dimension (FD) is mainly used to calculate the minimum reducing factor at which the image or objects can be divided into multiple numbers of self-similar objects [4, 6, 7, 8].

2. FD for Grey Scale Image

2.1 Reticular Cell Counting Method:

Baboon image has been taken calculating FD by using some defined method called as reticular cell counting method and Differential Box Counting (DBC) method for grey scale image and color image as well.

According to Gangepain and Roques-Carmes

$$L' = L \left(\frac{G}{M} \right)$$

L stands for length of box of an image, M stands for the length of image and G is the grey level of the referred image. L' is the reduced gray level as per the reduction factor of the given image. Here we have taken a reduced box of size L X L hence the length of the box is L. We need to calculate the reduction factor to calculate the FD. In mathematical term the reduction factor can be written as follows.

$$r = L/M$$

So reduction factor $1/r = M/L$, where M shows the image length and L shows the box length.

Baboon image of length 64 X 64 has been taken for the experiment. Firstly we have performed the experiment in gray level. From this we have found that there are several intensity values at each pixel. Next we have reduced the whole image into different boxes with the condition that each box will contain at least 1 gray level of intensity. We have considered different boxes of same size for different gray level intensity. Then all intensities have been added to find out the gray level intensity of the whole image. Now the for calculating the FD equation 1 has been applied.

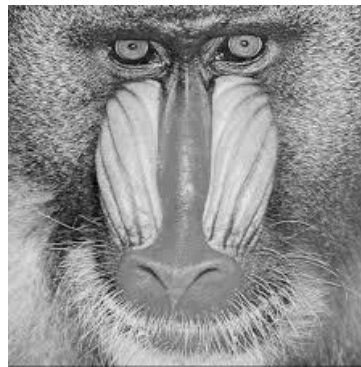


Fig-1: Gray-level Baboon image

Table-1: Box Counts

Box size(L)	log (N)	Reduction factor log (1/r)	Dimension D
4	7.1802	2.7926	2.5711
6	6.2621	2.3471	2.6710
8	5.5891	2.0712	2.6984
10	5.2103	1.8983	2.7447
12	4.8645	1.6846	2.8747
14	4.3989	1.5112	2.8833
16	3.7912	1.3853	2.7367
18	3.7236	1.2585	2.9587
20	3.7676	1.1612	3.2446
22	3.1812	1.0712	2.9697
24	3.1781	0.9782	3.2403
26	2.9972	0.9012	3.3258
28	2.8802	0.8182	3.5202
30	2.8901	0.7512	3.8473
32	2.0812	0.6912	3.0110

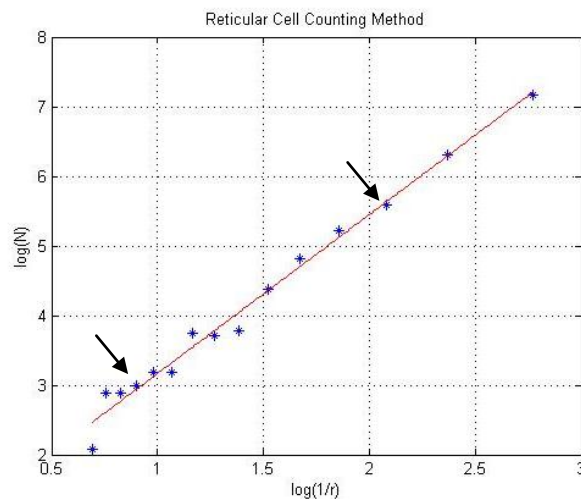


Fig-2: Best fit line of gray-scale image using reticular cell counting method

By using equation (3) and best fit line on the graph as shown in figure, we can find out fractal dimension $D=2.2771$

Two Most nearest points to best-fit line in the graph are taken. Most approximate Fractal dimension using best fit line

$$= (y_2 - y_1) / (x_2 - x_1)$$

$$= \frac{(6.2621 - 4.3989)}{(2.3471 - 1.5112)} = 2.2289$$

2.2 Differential Box Counting Method

We have taken the same Baboon image of size 64 X 64 length for conducting the experiment by using the above said method. The box counts of the specifies image using DBC method are given in Table-II.

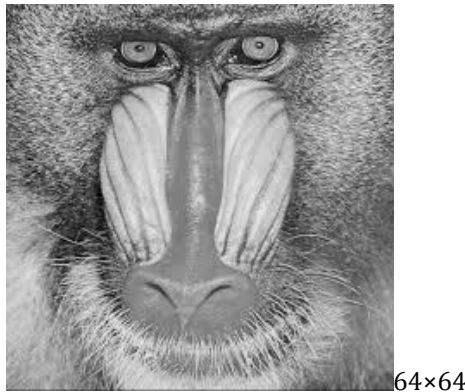


Fig-3: Gray-level Baboon image 64 x 64

Table-2: Box Counts for DBC

Box size(L)	Log (N)	Reduction factor log (1/r)	Dimension D
4	7.1778	2.7721	2.5893
6	6.2979	2.3681	2.6594
8	5.6111	2.0694	2.7115
10	5.2404	1.8663	2.8079
12	4.8201	1.6750	2.8776
14	4.3821	1.5202	2.8826
16	3.9522	1.3865	2.8511
18	<u>3.7843</u>	<u>1.2686</u>	2.9830
20	3.7136	1.1632	3.1926
22	<u>3.2189</u>	<u>1.0688</u>	3.0117
24	3.1785	0.9818	3.2374
26	2.9952	0.9006	3.3258
28	2.8908	0.8265	3.4976
30	2.8902	0.7567	3.8198
32	1.3866	0.6941	1.9977

In the graph given below X-axis denotes the reduction factor and Y-axis denotes the number of boxes required to fill the image. After that we have taken two points which more closest to the best fit line in the graph. By using these points we have calculated the FD.

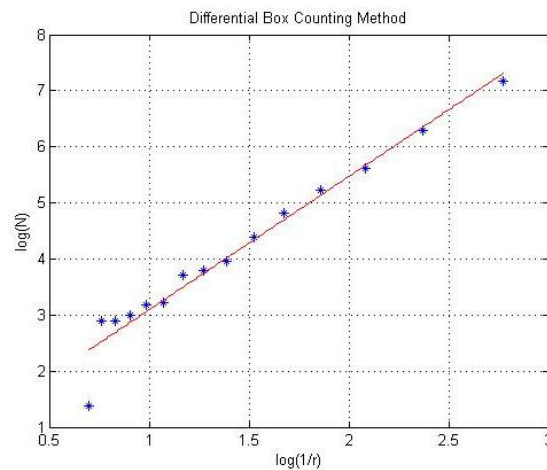


Fig-4: Best-fit line of various box size of Gray-scale image using DBC method

By using equation (1) and best fit line on the graph as shown in the figure we found out fractal dimension $D=2.9973$
Two Most nearest points to best-fit line in the graph are taken.

$$\begin{aligned} \text{Fractal dimension} &= (y_2 - y_1) / (x_2 - x_1) \\ &= \frac{(3.7843 - 3.2189)}{(1.2686 - 1.0688)} \\ &= 2.8298 \end{aligned}$$

3. FD for Color Image

3.1 Reticular Cell Counting Method

A color image always represented by three color levels i.e. RGB. So using experimental tools we separate the different color intensity level from an image and then count the box at different levels. Accordingly the color Baboon image of size 64×64 is considered, after that the red, green, and blue intensity values of that image are found separately using Matlab as shown in **figure.4**. Then the average box counts are made from red, green and blue and fractal dimensions found as per the Reticular Cell Counting Method for color image. The red level, green level and blue level of color image of baboon as shown in **figure 5** For convenience, we have considered the same Baboon image of size 64×64 . Table VII, VIII and IX shows the box counts of the red, green and blue levels of Baboon image. Table X shows the average box count for Baboon image.

Table-3: Box Count in Red- Intensity Level

Box size(L)	Total no. of box (N_r)	Log-log plot of N_r	Reduction factor (1/r)	D
4	1344	7.2036	2.7729	2.5978
8	285	5.6526	2.0796	2.7186
16	56	4.0257	1.3865	2.9031
32	8	2.0796	0.6933	3.0001

Table-4: Box Count in Green- Intensity Level

Box size(L)	Total no. of box (N_r)	Log-log plot of N_r	Reduction factor (1/r)	D
4	1345	7.2042	2.7728	2.5987
8	293	5.6804	2.0796	2.7417
16	54	3.9891	1.3863	2.8674
32	8	2.0797	0.6936	3.0005

Table-6: Box Count in Blue- Intensity Level

Box size(L)	Total no. of box (N_r)	Log-log plot of N_r	Reduction factor (1/r)	D
4	1460	7.2891	2.7732	2.6276

8	316	5.7565	2.0763	2.7685
16	56	4.0258	1.3873	2.9033
32	8	2.0794	0.6936	3.0001

Table-7: Average Box Count for Baboon image

Box size(L)	Total no. of box (N _r)	Log-log plot of N _r	Reduction factor (1/r)	D
4	1383	7.2321	2.7732	2.6081
8	298	5.6974	2.0796	2.7393
16	55.33	4.0133	1.3865	2.8943
32	8	2.0797	0.6935	3.0001

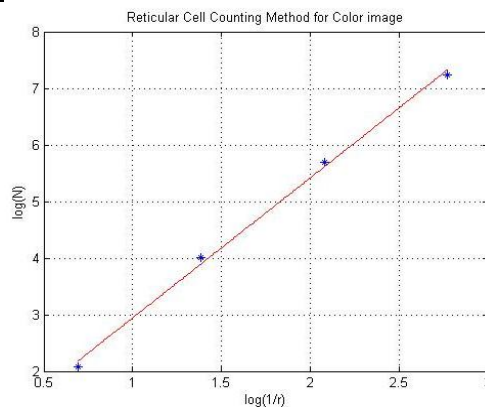


Fig-5: Best-fit line of color image using Cell Counting method

By using equation (1) and best fit line on the graph as shown in the figure we found out fractal dimension D=2.2812. Two Most nearest points to best-fit line in the graph are taken.

$$\begin{aligned}
 \text{Fractal dimension} &= (y_2 - y_1) / (x_2 - x_1) \\
 &= \frac{(7.2312 - 5.6975)}{(2.7728 - 2.0792)} \\
 &= 2.2146
 \end{aligned}$$

3.2 Differential Box Counting Method

A color image always represented by three color levels i.e. RGB. So using experimental tools we separate the different color intensity level from an image and then count the box at different levels. Accordingly the color Baboon image of size 64x64 is considered, after that the red, green, and blue intensity values of that image are found separately using Matlab as shown in figure 4.4. Then the average box counts are made from red, green and blue and fractal dimensions found as per the DBC method for color image

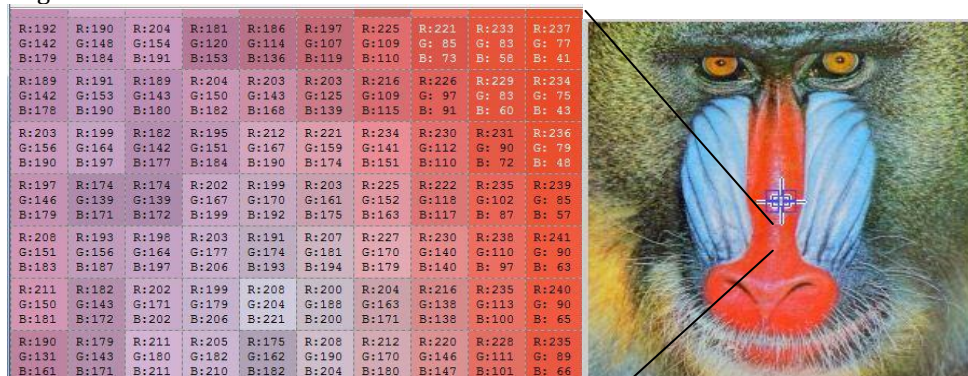


Fig-6: Intensity values of color image

The red level, green level and blue level of color image of baboon as shown in **figure 6**. For convenience, we have considered the same Baboon image of size 64×64.

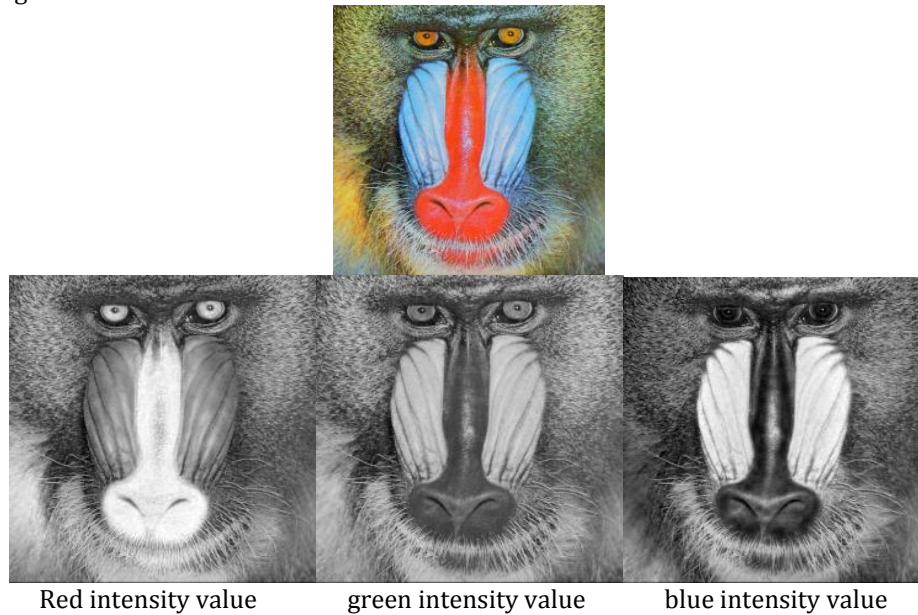


Fig- 7: Separation of color Baboon image into different color levels

Table 8, 9 and 10 shows the box counts of the red, green and blue levels of Baboon image. Table 11 shows the average box count for Baboon image.

Table-8: Box Count in Red-Intensity Level

Box size(L)	Total no. of box (N _r)	Log-log plot of N _r	Reduction factor (1/r)	D
4	1339	7.1991	2.7732	2.5959
8	286	5.6572	2.0781	2.7223
16	56	4.0361	1.3872	2.9095
32	8	2.0787	0.6923	3.0026

Table-9: Box Count in Green-Intensity Level

Box size(L)	Total no. of box (N _r)	Log-log plot of N _r	Reduction factor (1/r)	D
4	1343	7.2024	2.7722	2.5981
8	292	5.6769	2.0796	2.7298
16	54	3.9892	1.3865	2.8771
32	8	2.0792	0.6933	2.9989

Table-10: Box Count in Blue-Intensity Level

Box size(L)	Total no. of box (N _r)	Log-log plot of N _r	Reduction factor (1/r)	D
4	1460	7.2865	2.7728	2.6278
8	315	5.7531	2.0797	2.7663
16	56	4.0255	1.3865	2.9033
32	8	2.0792	0.6935	2.9981

Table-11: Average Box Count

Box size(L)	Total no. of box (N _r)	Log-log plot of N _r	Reduction factor (1/r)	D
4	1380.6667	7.2307	2.7729	2.6076
8	301.6667	5.7091	2.0789	2.7462
16	55.33	4.0137	1.3862	2.8954
32	8	2.0791	0.6937	2.9971

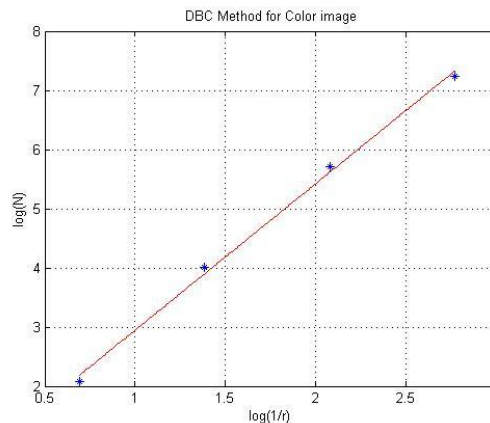


Fig-8: Best-fit line of various box size of color image using DBC method

The fractal dimension of color Baboon image is 2.2238. As per **figure 8** we found the best-fit line and mostly nearest point to the best-fit line are taken for calculation.

Fractal dimension as per graph

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{7.2308 - 5.7193}{2.7326 - 2.0784} = 2.1947$$

4. CONCLUSION

The fractal dimension of gray scale and color Baboon image are estimated using several principles. From cell counting method the fractal dimension gray scale Baboon image is estimated to be 2.2289. Using DBC method the fractal dimension of gray scale Baboon image is estimated to be 2.8298. For color Baboon image we have applied the above same method. For Differential Box Counting method the fractal dimension is found as 2.1947. Similarly for Reticular Cell Counting Method it is found to be as 2.2146.

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