Stability Analysis of DC-DC Boost Converter using Sliding Mode Controller

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***Abstract*— This paper presents sub-optimal design of fixed-frequency pulse width-modulation-based sliding-mode controllers for dc–dc Boost converters operating in the continuous conduction mode. Controller are designed to control and fixed the error in output along with giving fixed switching frequency to the converter so to maintain stability in output voltage. The switching frequency control system is modelled and design criteria for the control parameters are derived for guaranteeing closed-loop stability, under different approaches and taking into account the most expectable working scenarios. In the present work, the boost converter is modeled and PWM based sliding mode controller is designed and simulated using MATLAB/SIMULINK.

***Keywords*— sliding mode (SM) control, fixed switching frequency, nonlinear controller, continuous conduction mode (CCM), Boost converter.**

1. INTRODUCTION

The DC-DC converters family that interfaces the energy sources with the power grids and loads. Applications of DC-DC converters are hybrid electric vehicles, regenerative braking of dc motors, trolley cars, forklift trucks, Uninterruptable Power Supply (UPS). In order to design the DC-DC converter with sufficient relative stability, good tracking performance, and constant output voltage, a control circuit is required. Due to parameters variations like huge variations in Load or input voltage these controllers are not robust and linear control methods like PI or PID control are not suitable for DC-DC converters. Hence nonlinear controllers are appropriate for controlling DC-DC converters as it displaying non minimum phase characteristics while examined to variations of small signal.

Sliding Mode (SM) controller is a kind of nonlinear controller are well known for their robustness and stability. Ideally, SM controllers operate at infinite, varying, and self-oscillating switching frequency such that the controlled variables can track a certain reference path to achieve the desired steady-state operation [1]. However, due to the varying and high switching frequency, the feasibility of applying SM controllers in power converters is challenged. According to the structure control law the physical structures of variable structure control system are varied regarding to time However, due to the varying and high switching frequency, the feasibility of applying SM controllers in power converters is challenged. First, extreme high-speed switching operation in power converters results in excessive switching losses, inductor and transformer core losses, and electromagnetic interference (EMI) issues. Second, variable switching frequency complicates the design of the input and output filters [3]-[5].

In earlier works in this field, the typical choice for the controlling function s is a linear combination of the error of the variable to be controlled and its r^{th} time derivative [6]-[7]. Alternatively, modulation of infinite varying frequency of sliding mode controller limit to finite fixed frequency by using pulse width modulation (PWM) which remove variable frequency problem, further it can also called as duty cycle control. This idea can be rooted back to one of the earliest papers on SM controlled power converters [11], which suggested that under SM control operation, the control signal of an *equivalent control approach* u_{eq} in SM control is equivalent to the *duty cycle control signal* D of a PWM controller. It has been shown that as the switching frequency tends to infinity, the averaged dynamics of a SM controlled system is equivalent to the averaged dynamics of a PWM controlled system, thus establishing the relationship $u_{eq} = d$. On the other hand, the same correlation has been derived in Martinez*etal* [14], where the nonlinear PWM continuous control was compared with an equivalent control. So Pulse-width modulation based sliding mode controller can overcome deficiency present in conventional PWM controllers by giving a good regulation at any operating condition. Systematic step by step method for the design of proposed methodology is shown in addition control equations obtained are implemented on boost converter and simulated using MATLAB/SIMULINK.

2. MODELLING OF CONVERTER

Fig. 1 and Fig. 2 demonstrates switch ON and OFF of the boost converter where L, C, R_L , V_i and $V_o$be the inductor, capacitor, resistive load, input and output voltage respectively. Here, i_r , i_l, and i_c resembles load, inductor and capacitor currents. From switch ON and switch OFF condition of boost converter the equations are obtained.

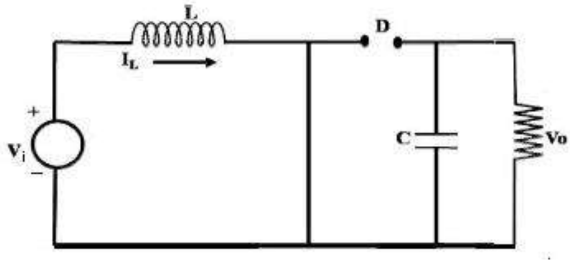

Fig. 1 Switch ON condition

$$i_L = \frac{1}{L}\int V_i u dt \quad (1)$$

$$\frac{dV_o}{dt} = -\frac{V_o}{RC} \quad (2)$$

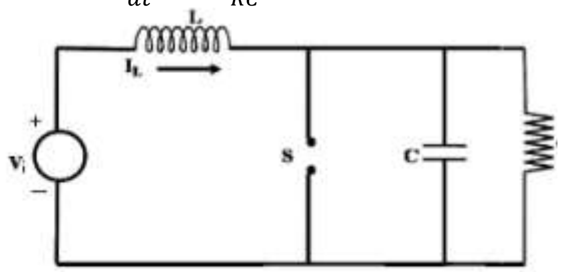

Fig. 2 Switch OFF condition

$$L\frac{di_L}{dt} = V_i - V_o \quad (3)$$

$$i_L = \frac{1}{L}\int (V_i - V_o)\bar{u} dt \quad (4)$$

$$V_o = \frac{1}{C}\int (i_L - i_R) dt \quad (5)$$

Combine equation (1) and (4)

$$i_l = \frac{1}{L}\int (V_i u + (V_i - V_o)\bar{u}) dt \quad (6)$$

u indicates the state of a switch S and $\bar{u}$ is inverse logic of *u*

3. DESIGN PROCEDURE

Modelling of the system and design procedure for PWM based SM controller for boost converter in continuous conduction mode are explained.

A. System Modelling

converter model in terms of the desired control variables (i.e., voltage and/or current etc.). Our focus in this paper is the application of SM control to converters operating in CCM. State space description can developed for the control variables like voltage and current in the system. The state variables are portrayed as,

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} V_{ref} - \beta V_0 \\ \frac{d(V_{ref} - \beta V_0)}{dt} \\ \int (V_{ref} - \beta V_0) dt \end{bmatrix} \quad (7)$$

Where X_1, X_2 *and* X_3 are the voltage error ,the voltage error dynamics (or the rate of change of voltage error), and the integral of voltage error. ; V_{Ref} and , V_o are instantaneous input and instantaneous output voltages with **β** denotes the feedback network ratio.

Substitution of the converters' behavioral models under CCM into (7) produces the following control variable descriptions as

$$X_{boost} = \begin{bmatrix} X_1 = V_{ref} - \beta V_0 \\ X_2 = \frac{\beta V_0}{r_L C} + \int \frac{(V_{ref} - \beta V_0)}{LC} dt \\ X_3 = \int X_1 \end{bmatrix} \quad (8)$$

State space descriptions can obtain from equation (8) by differentiating with respect to time which is essential for control design of boost converter.

$$\dot{x}_1 = \frac{d}{dt}\left(V_{ref} - \beta V_0\right) = x_2 \quad (9)$$

$$\dot{x}_1 = -\beta\frac{dV_o}{dt} = x_2 \quad (10)$$

$$\dot{x}_2 = \frac{\beta}{r_L C}\frac{dV_o}{dt} + \frac{\beta}{LC}(V_0 - V_i)\bar{u} \quad (11)$$

$$\dot{x}_2 = \frac{-x_2}{r_L C} + \left(\frac{\beta V_0}{LC} + \frac{\beta V_i}{LC}\right)\bar{u} \quad (12)$$

$$\dot{x}_3 = V_{ref} - \beta V_0 = x_1 \quad (13)$$

Obtaining equations (10), (12), (13) in matrix form,

$$\dot{x}_{boost} = Ax_{boost} + B\bar{u}$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{r_L C} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\beta V_0}{LC} - \frac{\beta V_i}{LC} \\ 0 \end{bmatrix} \bar{u} \quad (14)$$

Where $\bar{u}$=1-*u* is the inverse logic of *u,* used particularly for modeling the boost topologies. Where *v =u or* $\bar{u}$ (Depending on topology)

B. Controller Design

Having obtained the state-space descriptions, the next stage is the design of the controller. For these systems, it is appropriate to have a general SM control law that adopts a switching function such as,

$$u = \begin{cases} 1, & when\ S > 0 \\ 0, & \text{When S} < 0 \end{cases} \quad (15)$$

where is S the instantaneous state variable's trajectory, and is described as.

$$S = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 = \boldsymbol{J^T X} \quad (16)$$

$\boldsymbol{J^T}$= $[\alpha_1\ \alpha_2\ \alpha_3]$ and α_1, α_2, and α_3 representing the control parameter termed as sliding coefficients. By selecting the accurate values for sliding coefficients the sliding mode control can be satisfied from the three conditions i.e. hitting, existence and stability condition. At any operating load and input parameters the controller must fulfill these requirements.

1. Hitting Condition

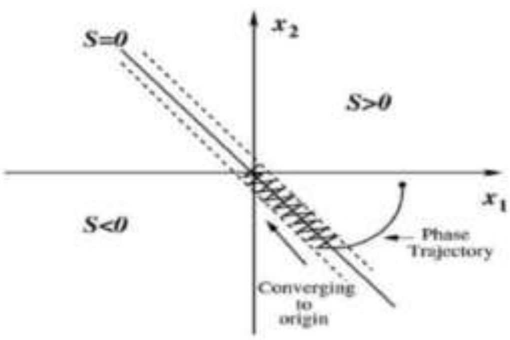

Fig. 3 Trajectory approach and reach to the sliding manifold

The aim of the hitting condition is make sure that irrespective of initial conditions of locations, the trajectory will moved within a vicinity δ, of the sliding manifold as seen in the Fig. 3.

At initial state
Vector $x_i = x(t = 0)$, trajectory $S_t = S(t = 0)$
Distance away from sliding manifold $\delta = 0$.
This condition can be satisfied by resulting control $u_t = u(t > 0)$ having state variable vector $x_t(t > 0)$ also controlled trajectory S (t > 0). This required condition accepts the given expression:
$S\frac{dS}{dt} < 0\ \ (for\ (t > 0) and\ that\ |S| \geq \delta)$

2. Existence Condition

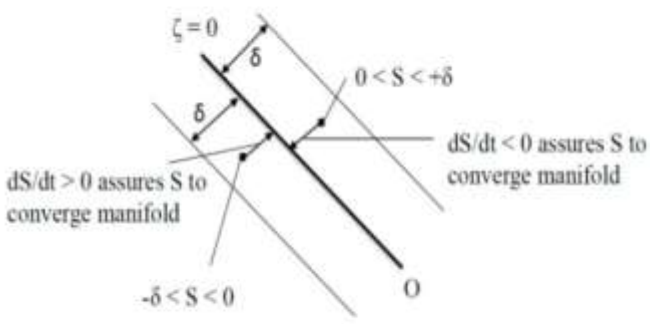

Fig. 4 Trajectory converging to the sliding manifold

Examine the existence condition is essential in the system which leads to make sure that the sliding manifold must be within $0 < |\delta| < \delta$ at the locations of trajectory, as seen in the Fig.4. The existence of sliding mode operation is satisfied by making the condition of local reachability,

$$\lim_{s\to 0} S\frac{ds}{dt} < 0 \qquad (17)$$

Equation (17) determined by,

$$\left.\begin{aligned}\dot{S}_{s\to 0^+} &= \boldsymbol{J^T Ax} + \boldsymbol{J^T B} v_{s\to 0^+} + \boldsymbol{J^T D} < 0\\ \dot{S}_{s\to 0^-} &= \boldsymbol{J^T Ax} + \boldsymbol{J^T B} v_{s\to 0^-} + \boldsymbol{J^T D} > 0\end{aligned}\right\} \qquad (18)$$

There will be two cases arise as shown below:

Case1: $S \to 0^+$, $\dot{S} < 0$
Substitution of $v_{s\to 0^+} = \bar{u} = 0$ and the matrices Eq (14) into (18) gives,

$$-\alpha_1\frac{\beta i_C}{C} + -\alpha_2\frac{\beta i_C}{r_L C^2} + -\alpha_3(V_{ref} - \beta V_0) < 0 \qquad (19)$$

Case 2: $S \to 0^-$,$\dot{S} > 0$
Substitution of $v_{s\to 0^-} = \bar{u} = 0$ and the matrices Eq. (3) into (18) gives,

$$-\alpha_1\frac{\beta i_C}{C} + -\alpha_2\frac{\beta i_C}{r_L C^2} + -\alpha_3\left(V_{ref} - \beta V_0\right) - \alpha_2\frac{\beta v_i}{LC} + \alpha_3\frac{\beta v_o}{LC} > 0 \qquad (20)$$

Finally, the combination of (19) and (20) gives the simplified existence condition.

$$0 < \beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right)i_C - LC\frac{\alpha_2}{\alpha_3}\left(V_{ref} - \beta V_0\right) < \beta(V_o - V_i) \qquad (21)$$

4. Stability Condition

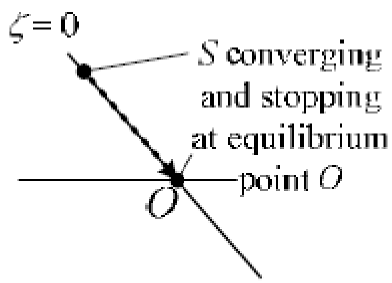

Fig. 5 Stable converging region

Trajectory is directed by sliding manifold near a stable equilibrium point. Sliding mode system will have to design so that it will always converge and stop at equilibrium point to maintain stability of the system.

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A. Selection of Sliding Coefficients

Depending on dynamic properties the sliding coefficients are selected. System also becomes stable based on the selection of coefficient. Depending on the value of damping ratio and settling time, the sliding coefficients are obtained. The desired settling time $T_s = 5\tau s$ (1% criteria), where τ indicates natural time constant , can be set by tuning using (a_1/a_2)

$$\frac{\alpha_1}{\alpha_2} = \frac{10}{T_s} \qquad (22)$$

and the desired damping ratio can be set using

$$\frac{\alpha_3}{\alpha_2} = \frac{25}{\zeta^2 {T_s}^2} \qquad (23)$$

Thus, the design of the sliding coefficients is now dependent on the desired settling time of the response and the type and amount of damping required, in conjunction with the existence condition.

B. Derivation of Control Equations for PWM Based Controller

Ramp signal and control signal are compared to get output switching signal which is having frequency identical to the frequency of ramp signal. By fixing ramp signal frequency so that output switching signal frequency will remains constant. Therefore by using PWM technique in controller design, fixed frequency for the proposed method is possessed. From first step, equivalent control signal, u_{eq} is originated using invariance condition and second step is to translate u_{eq} to duty ratio d of PWM is carried during the derivation process.

u_{eq} is obtained from the equation $\dot{S} = \boldsymbol{J^T A x} + \boldsymbol{J^T B \overline{U}_{eq}} = 0$

$$\bar{u}_{eq} = -[J^T B]^{-1} J^T A x$$

$$\bar{u}_{eq} = \frac{\beta L}{\beta(v_0 - v_i)}\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) i_C - \frac{LC\alpha_3}{\alpha_2 \beta(v_0 - v_i)}\left(V_{ref} - \beta v_0\right)$$
(24)

Where $\overline{U}_{eq}$ is continuous and $0 < \bar{u}_{eq} < 1$. Substitution into the inequality gives .($u_{eq} = 1 - \bar{u}_{eq}$)

$$0 < u_{eq} = 1 - \left(\frac{\beta L}{\beta(v_0 - v_i)}\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) i_C + \mathrm{LC}\frac{\alpha_3}{\alpha_2 \beta(v_0 - v_i)}\left(V_{ref} - \beta v_0\right)\right) < 1 \qquad (25)$$

Multiplication by $\beta(v_o - v_i)$ to eq (25) gives,

$$0 < {u_{eq}}^* = -\beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) i_C + LC\frac{\alpha_3}{\alpha_2}\left(V_{ref} - \beta v_0\right) + \beta(v_0 - v_i) < \beta(v_0 - v_i) \qquad (26)$$

Finally, the mapping of the equivalent control function (26) onto the duty ratio control d.
Where,

$$0 < d = \frac{v_C}{\hat{v}_{ramp}} < 1 \qquad (27)$$

gives the following relationships for the control signal v_C and $\hat{v}_{ramp}$ ramp signal for the practical implementation of the PWM based SM controller.

$$0 < \frac{-\beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) i_C + LC\frac{\alpha_3}{\alpha_2}(V_{ref} - \beta v_0) + \beta(v_0 - v_i)}{\beta(v_0 - v_i)} < 1 \qquad (28)$$

Compare equations (27) and (28),

$$v_C = {u_{eq}}^* = -\beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) i_C + LC\frac{\alpha_3}{\alpha_2}\left(V_{ref} - \beta v_0\right) + \beta(v_0 - v_i)$$

$$v_C = -K_{p1} i_C + K_{p2}\left(V_{ref} - \beta v_0\right) + \beta(v_0 - v_i) \qquad (29)$$

$$\hat{v}_{ramp} = \beta(v_0 - v_i) \qquad (30)$$

Where,

$$K_{p1} = \beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) \qquad (31)$$

$$K_{p2} = \frac{\alpha_3}{\alpha_2} LC \qquad (32)$$

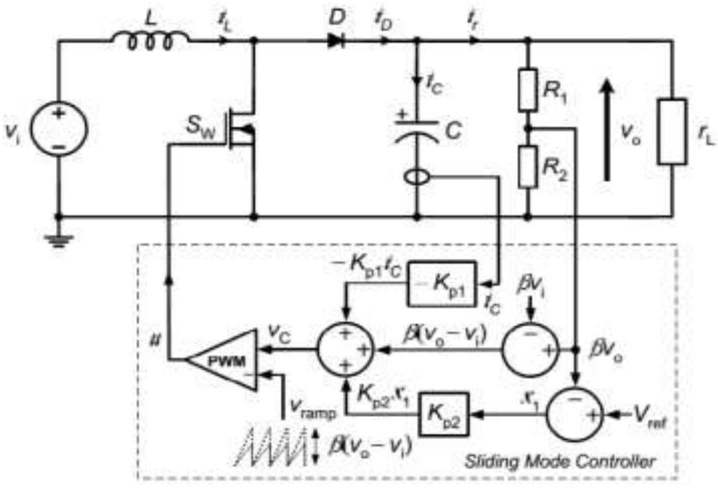

Fig. 6 Control structure of sliding mode boost converter

Fig. 6 represents the proposed method which is simulated in MATLAB/SIMULINK. Control and ramp voltage equations obtained are used to design a controller which is implemented on boost converter. the controlled gate pulse can obtained by modulating control signal over ramp signal. Constant gain parameters K_{p1} and K_{p2} for the feedback signals i_c and $V_{ref} - \beta v_0$. When there is a giant disturbance, the integral term of control variable i.e x_3 has the ability to control in the system. Differential term of control variable i.e. x_2 lower the settling time and peak overshoot and has the ability to enhance transient part not by altering steady state. The controller structure which is similar to the PWM Proportional-Derivative (PD) control in addition having extra parameters like instantaneous output voltage βv_o and instantaneous input voltage βv_i so to reduce the steady-state dc error of the practical SM controlled system. . To achieve robust to parameter like load and line regulation at various conditions these components helps the controller.

6. DESIGN AND CALCULATION OF SM CONTROLLER FOR BOOST CONVERTER

The below table I shows the specification for 100W boost converter. The Sliding Mode converter is design with a bandwidth of $\omega_n = 1.25Krad/s$, i.e., $\tau = 0.8ms$, and $T_s = 4\ ms$, and with damping coefficient $\zeta = 0.2$. From (22) and (23) the sliding coefficient are determined as,

TABLE I BOOST CONVERTER SPECIFICATION

Description	Parameters	Nominal values
Input voltage	v_i	24V
Inductance	L	100mH
Inductor resistance	l_r	0.14 Ω
Capacitance	C	2100 uF
Capacitor ESR	c_t	40mΩ
Minimum load resistance	r_{min}	100Ω
Maximum Load resistance	r_{max}	400 Ω
Desired Output Voltage	v_o	48V

$$\frac{\alpha_1}{\alpha_2} = \frac{10}{T_s} = \frac{10}{4*10^{-3}} = 2500$$

$$\frac{\alpha_3}{\alpha_2} = \frac{25}{\zeta^2 {T_s}^2} = \frac{25}{(0.2^2)*((4*10^{-3})^2)} = 39130435$$

$$\beta = \frac{V_{ref}}{V_O} = \frac{1}{6}$$

$$K_{p1} = \beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) = 0.0416$$

$$K_{p2} = \frac{\alpha_3}{\alpha_2} LC = 0.3913$$

7. SIMULATION RESULTS

Simulations are done by using MATLAB/SIMULINK for boost converter using SM controller with variable load resistance.

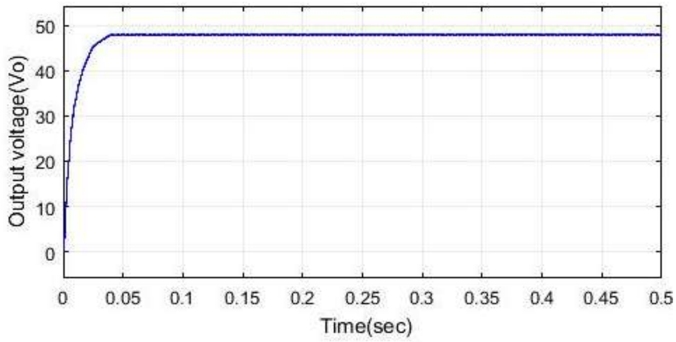

Fig 7 Output Voltage of Boost Converter for variable load(100 Ω – 1000 Ω)

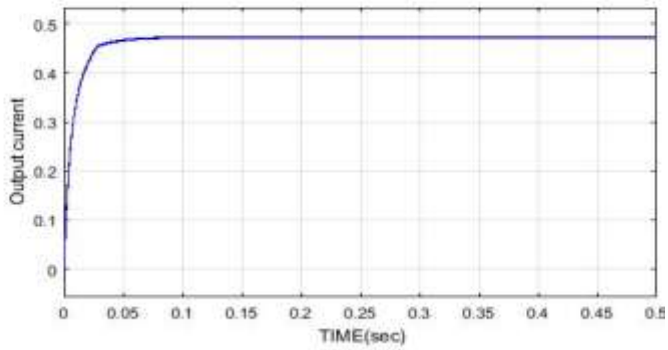

Fig. 8 Output Current of Boost Converter for 100 Ω Load.

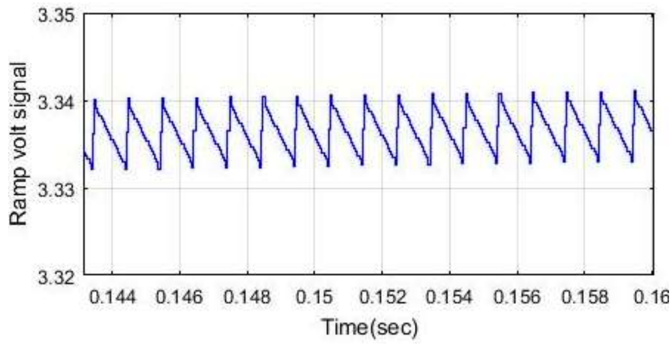

Fig. 9 Ramp signal $\beta(v_o - v_i)$ waveform

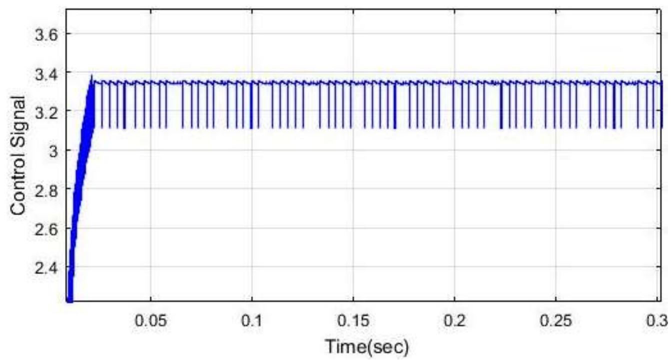

Fig. 10 Control signal $v_c$waveform

Fig 7 show obtained output voltage of boost converter for variable load with stable output of 48V with deviation of 0.2 volt in output. The output is stable for huge range of load with fewer disturbances in output voltage as shown in fig.7. the output current waveform is also having improve in transient response than open loop with stability in current for huge range of load. Fig 9 and 10 show ramp $\beta(v_o - v_i)$ and control signal v_c which modulated with pulse-width modulation as shown in fig .6 so to form controlled gate signal for the Boost converter. The resistive load of sliding mode boost converter is varied for different values from 100Ω to 1000Ω, same performance is seen when there is a load variations. Hence this sliding mode control is robust to load variations and provide consistent performance.

8. CONCLUSION

Boost converter is widely used in energy management of electronic devices. In this research sliding mode control is put into practice for Dc-Dc Boost converter. This paper enlightens step by step procedure for the design of PWM based SM controller for boost converter is derived and control equations obtained are implemented on boost converter and simulated using MATLAB/SIMULINK. Output results tells that sliding mode controller clearly refuses the load disturbances by maintaining stable transient response and from the steady state output it is observed that output voltage is stable for huge range of load résistance. Hence SM controller is suitable for common Dc-Dc boost converter applications.

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