# Order Reduction Technique using Improved Padé Approximation and Routh Array Method

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**Abstract**—*In this paper, a mixed-method is used to reduce the higher order of the frequency domain system to lower-order frequency domain system by using improved Padé approximations and Routh array method. The denominator of the proposed method is computed by the Routh array method whereas the numerator is computed by improved Padé approximation. The proposed method helps generate 'r' number of $r^{th}$ reduced-order models from the original high order system. With the help of a numerical example taken from literature, we can observe the superiority of the proposed method by comparing the proposed method with a well-known existing order reduction technique.*

***Keywords***: Linear Dynamic System, *Routh Array Method, Order reduction technique, Padé approximation, Improved Padé approximation.*

## 1. INTRODUCTION

The order reduction technique is a smart idea that reduces the complexity of a higher-order system. The order reduction technique reduces an original high order system into a low order system, yet still, the reduced system retains the main properties of the original system in an optimum manner. Therefore, by converting the original model to a reduced model, the higher-order original system can be analyzed easily. The order reduction technique is a very useful tool in the various field of engineering design such as control theory, power system, fluid dynamics, etc. In the proposedmethod two different order reduction techniques (improved Pad approximation and Routh array method) are mixed to form a new technique for order reduction.in this paper, the Routh array method is used for computation of denominator and improved Pad approximation is used for computation of numerator polynomial. Krishnamurthy [2], had introduced the Routh stability criterion and derived the methodology for the computation of the reduced-order technique in 1978. Routh array method is used for the reduction of both numerator and denominator polynomial but in the proposed method only denominator has been reduced by using the Routh array method while numerator polynomial is reduced by using improved Padé approximation. Sambariya and Prasad, have presented the application of four different methods for finding a decreased order model of a linear time-variant system [3, 5, 6]. Four methods of finding reduced order for the linear dynamic system have been presented by the Sambariya and Prashad [3,5,6], which are applicable in the field of the power system and also used to reduce the model of the single infinite bus power system (SMIB) [7,8].A mixed approach was introduced by Satakshi[10] for order reduction of a linear time-invariant system. Padé approximation method [13] is used for matching the forced response because it retains even small initial time moments.The inability to retain the stability of the reduced-order system was removed by improved Padé approximations method [13]. Both methods i.e. Routh array method and improved Padé approximations [13] are mixed to form a better technique for order reduction of the linear dynamic systems. Computational simplicity, robustness and stability retention, etc. are some major advantages of the proposed method. Improved page approximation is also able to generate an 'r' number of $r^{th}$ order reduction.

## 2. PROBLEM FORMULATION

Let G(s) be the transfer function of Higher-order i.e.

$$\mathrm{G}(s) = \frac{N_n(s)}{D_n(s)} = \frac{c_1 s^{n-1} + c_2 s^{n-2} + c_3 s^{n-3} + \cdots + c_n s^0}{d_1 s^n + d_2 s^{n-1} + d_3 s^{n-2} + \cdots + d_n s^0} \quad (1)$$

Where $(c_1, c_2, c_3 \cdots c_n)$ and $(d, d_2, d_3 \cdots d_n)$ are unknown constants.

And $G_r(s)$ is the reduced transfer function of 'r' order i.e.

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{e_1 s^{r-1} + e_2 s^{r-2} + e_3 s^{r-3} + \cdots + e_r s^0}{f_1 s^r + f_2 s^{r-1} + f_3 s^{r-2} + \cdots + f_r s^0} \quad (2)$$

Where $(c_1, c_2, c_3 \cdots c_n)$ and $(d, d_2, d_3 \cdots d_n)$ are unknown constants.
Here the problem is to derive a method to find the reduced-order system(2) from the original $n^{th}$-order system i.e. (1) so that the obtained system preserves all its specifications as much as possible.

## 3. DESCRIPTION OF THE METHOD

The method of calculating the order reduction is described stepwise below:

### 3.1 Reduction methodology of the numerator polynomial

- Routh Array Method

Let the transfer function for the higher system be

$$G(s) = \frac{c_1 s^{n-1} + c_2 s^{n-2} + c_3 s^{n-3} + \cdots + c_n s^0}{d_1 s^n + d_2 s^{n-1} + d_3 s^{n-2} + \cdots + d_n s^0} \quad (4)$$

Where n = 1,2, 3…….
The Routh stability array for the numerator and denominator polynomials of (4) are shown below in Tables 1 and 2, respectively. It is well known that the first two rows of Tables 1 and 2 are constructed from the coefficients of the two polynomials, respectively.

**TABLE-1:** ROUTH ARRAY OF STABILITY FOR NUMERATOR

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{14}$ | $C_{15}$ | $C_{16}$ | | ------------------------------------------------------------------ |
| $C_{21}$ | $C_{22}$ | $C_{23}$ | $C_{24}$ | $C_{25}$ | $C_{26}$ | | ------------------------------------------------------------------ |
| $C_{31}$ | $C_{32}$ | $C_{33}$ | $C_{34}$ | $C_{35}$ | - | - | |
| - | - | - | | | | | |
| - | - | - | | | | | |
| $C_{n1}$ | | | | | | | |
| $C_{n+1,1}$ | | | | | | | |

**TABLE-2:** ROUTH ARRAY OF STABILITY FOR DENOMINATOR

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $d_{11}$ | $d_{12}$ | $d_{13}$ | $d_{14}$ | $d_{15}$ | $d_{16}$ | | ------------------------------------------------------------------ |
| $d_{21}$ | $d_{22}$ | $d_{23}$ | $d_{24}$ | $d_{25}$ | $d_{26}$ | | ------------------------------------------------------------------ |
| $d_{31}$ | $d_{32}$ | $d_{33}$ | $d_{34}$ | $d_{35}$ | - | - | |
| - | - | - | | | | | |
| - | - | - | | | | | |
| $d_{n1}$ | | | | | | | |

$d_{n+1,1}$

In both the table (table-1 &table-2) the first row consists of odd coefficients and the second row consists of even coefficients. The first two rows are directly generated from the given transfer function and other remaining rows are calculated by below Eq.5.

$$X_{ij} = X_{i-2,j+2} - \frac{X_{i-2,1} \times X_{i-1,j+1}}{X_{i+1,1}} \quad (5)$$

For I $\geq$ 3 and In$\leq$ $\left[\left\{\frac{n-1+3}{2}\right\}\right]$,
Here:
n-order of polynomial,
Starting two rows are directly obtained from given polynomial and other rows are constructed with the help of a conventional method i.e. eq.(5).

### Reduction methodology of the denominator polynomial

- Improved padé approximation

By using binomial theorem original high order system can be written as

$G(s) = \sum_{i=0}^{\infty} M_i s^{-i-1}$ (about s=∞)

$= \sum_{i=0}^{\infty} T_i s^{i}$ (about s=0)

Here$T_i$-$i^{th}$ Time moment and
$M_i$-$i^{th}$Markov parameter of G(s).
Reduced $r^{th}$order system is considered as
$H_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{\sum_{i=0}^{r-1} e_i s^i}{\sum_{i=0}^{r} f_i s^i}$

The coefficient of$N_r(s)$can be obtained with the help of the following equations.

$e_0 = f_0 T_0$
$e_1 = f_0 T_1 + f_1 T_0$
$e_2 = f_0 T_1 + f_1 T_1 + f_2 T_0$
…
…
$e_{\alpha-1} = f_0 T_{\alpha-1} + f_1 T_{\alpha-2} + \cdots + f_{\alpha-2} T_1 + f_{\alpha-1} T_0$
$e_{r-\beta} = f_r M_{\beta-1} + f_{r-1} M_{\beta-2} + \cdots + f_{r-\beta+2} M_1 + f_{r-\beta+1} M_0$
$e_{r-\beta+1} = f_r M_{\beta-2} + f_{r-1} M_{\beta-3} + \cdots + f_{r-\beta+3} M_1 + f_{r-\beta+2} M_0$
…
…
$e_{r-2} = f_r M_1 + f_{r-1} M_0$
$e_{r-1} = f_r M_0$

Finally, $r^{th}$order numerator$N_r(s)$ can be obtained solving 'r' linear equation,
$N_r(s) = e_0 + e_1 s + e_2 s^2 + \cdots + e_r s^{r-1}$

## 4. METHOD OF COMPARISON

Accuracy of the proposed method can be checked by the integral square error [ISE] and the integral of the square of error [IAE].[ISE] and[IAE] is calculated by the given formula defined as follows:

ISE = $\int_0^\infty [y(t) - y_k(t)]^2 dt$

IAE = $\int_0^\infty |y(t) - y_k(t)| dt$

Where

y(t) – unit response of the original system,

$y_k(t)$ - unit step responses of reduced order systems and

$y(\infty)$- the steady-state value of the original high order system.

## 5. NUMERICAL EXAMPLE

Example 1- Consider a system of six order taken from [5]

$$G(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1}$$

Routh stability array for denominator:

**Table -3:** Denominator Table

| | | | | |
|---|---|---|---|---|
| $s^6$ | 2 | 155.94 | 102.42 | 1 |
| $s^5$ | 33.6 | 209.46 | 18.3 | |
| $s^4$ | 143.472 | 101.3307 | 1 | |
| $s^3$ | 185.7291 | 18.065 | | |
| $s^2$ | 87.3466 | 1 | | |
| $s^1$ | 15.9386 | | | |
| $s^0$ | 1 | | | |

Obtained reduced-order denominator derived from the table is:
$D_r(s) = 87.3466s^2 + 15.9386\, s + 1$

Using improved Padè approximation technique in numerator polynomial:
For the calculation of reduced numerator, some required Time moments and Markov parameter are as
Time moments: $T_1 = 1, T_2 = -8.3, T_3 = 69.47$
Markov parameters: $M_1 = 1, M_2 = -15.3, M_3 = 187.07$
Since $\alpha + \beta = k$
Where $\alpha$ -Number of Time moments
$\beta$ - Number of Markov parameters
$k$ - Order of reduced model

For the calculation of coefficients of numerator following table is provided

| Calculation of numerator coefficients | |
|---|---|
| Numerator coefficients $(c_0, c_{1..})$ taking $\alpha = 1, \beta = 1, r = 2$ | Numerator coefficients $(c_0, c_{1..})$ taking $\alpha = 2, \beta = 0, r = 2$ |
| $c_0 = d_0 T_0 = 1$<br><br>$c_{r-1} = d_r M_0$<br>$c_1 = d_1 M_0 = 15.9386 \times 1 = 15.9386$ | $c_0 = d_0 T_0 = 1$<br><br>$c_1 = d_0 T_1 + d_1 T_0$<br>$c_1 = (1 \times -8.3) + (15.9386 \times 1) = 7.6386$ |

Thus, $2^{nd}$ order reduced transfer function is obtained as

$R(s) = \frac{7.638\ s+1}{87.3466\ s^2+15.9386\ s+1}$ where($\alpha = 2$ , $\beta = 0, r = 2$)

$R(s) = \frac{15.9386\ s+1}{87.3466\ s^2+15.9386\ s+1}$ where($\alpha = 1$ , $\beta = 1, r = 2$)

**Table -4:** Comparison of proposed method with other order reduction methods

| Reduction methods | Reduced order system | ISE | IAE |
|---|---|---|---|
| Proposed Model | $R(s) = \frac{7.638\ s + 1}{87.3466\ s^2 + 15.9386\ s + 1}$ | 0.001895 | 0.1117 |
| Padé approx., Soloklo,2013[9] | $R(s) = \frac{6.87815\ s + 1.09228}{89.54625\ s^2 + 12.96860\ s + 0.99732}$ | 0.01208 | 0.2642 |
| Routh-Padéapprox. using harmony search,Soloklo[9] | $R(s) = \frac{7.80016\ s + 0.81849}{87.58712\ s^2 + 12.43314\ s + 0.81857}$ | 0.0066 | 0.1855 |
| Model order reduction by differentiation equation method using with Routh array Method [15] | $R(s) = \frac{2535.69\ s + 357.14}{2458.08\ s^2 + 2196\ s + 360}$ | 6.376 | 4.812 |

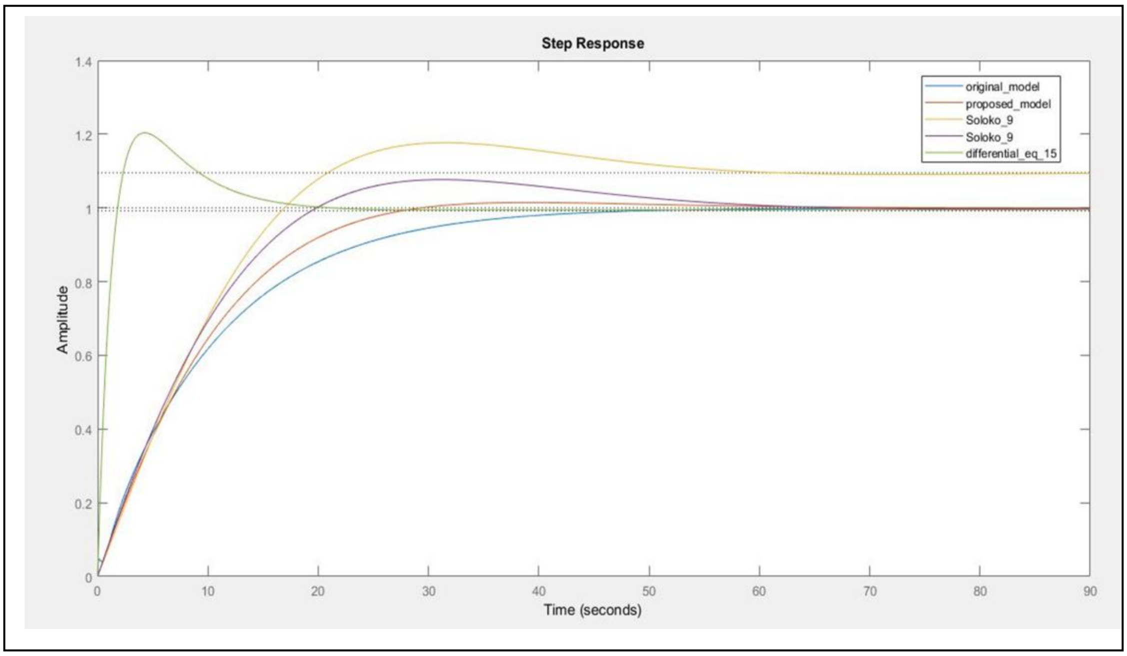


**Chart -1**: Step response comparison of example-1 with other order reduction method.

## 6. CONCLUSION

Here a higher-order frequency domain system is reduced and analyzed by using the order reduction technique using improved pade approximation and routh array method.in proposed we can see that the denominator term of the given system is solved by routh array approximation and numerator term is solved by improved pade approximation. Calculated IAE and ISE of the proposed method are better than the other model order reduction technique which is shown in the above table.Both order reduction method preserves the stability of the original system after reduction, which is very important.

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