

# FEM Simulations in Convective Viscous Fluid Flow through Porous Vertical Channel

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## Abstract

*An attempt of Finite Element Method is made on the viscous fluid of free convection nature in a vertical porous channel. The porous medium in which the flow is assumed is bounded by impermeable parallel walls. A specific direction of axis of channel is considered. The viscous porous flow in the medium in the present investigation is an assumption with certain boundary conditions. Brinkman Model is implemented in the commutation of equations for heat and momentum concepts in developing conservation equations. The viscous and Darcy dissipation have been modeled in the energy equation for the heat flow phenomenon. In the simulation of the finite element method, the fluid and the porous matrix have been imagined to be in local thermal equilibrium and the flow is considered to be one direction of the buoyancy. The non-linear equations which govern the phenomenon of the flow, heat and mass transfer are clearly demonstrated in the simulation. Velocity profile and heat dissipation with respect to temperature, Sherwood number, concentration, Nusselt number are analyzed and their profile is studied in the paper.*

**Keywords:** *Nusselt number, Fluid Flow Sherwood Number, viscous fluid, Galerkin, Finite element method.*

## 1. Introduction

The Numerical simulation on various models convey the solutions to industrial systems where the heat and temperature effects involved[1]. It is observed that a chance of exponential increase of failure in the performance of a system with the increase in heat rate. The numerical attempt made in the paper lead to the solutions of enhancing the expected levels of performance of the working systems with the techniques of thermal managements through various techniques developed in simulation attempts. The efficiency in the performance can lead to reliability. The various cooling techniques is template.

## 2. Formulation and methodology

The model has been framed so as to trace the mathematical information and conclusion. The problem relating to convective phenomenon of Porous channel filled medium matrix that are bounded walls of impermeable nature has been

carefully modeled and fluid matrix is assumed to be with the direction of axis of flow channel. It has to be carefully seen to keep constant temperature at the surface parts of walls. The concept of Brinkman model is implemented in writing the equations of momentum conservation for analyzing flow. The fluid and the porous matrix are in local thermal equilibrium and the flow is unidirectional along the direction of the buoyancy. The possibility of dissipations due to various causes are considered and assumed as Darcy dissipation which needs to involve in the energy driving equations to describe the heat flow.

A reference frame of mathematical situation  $0(x, y, z)$  is considered in which the x-axis will be assumed to along normal direction against buoyancy. And it is to see that the vertical walls must be along  $(y, z)$  plane  $y = \pm b$ . Let  $(u, 0, 0)$  be the one indicating profile of velocity field only in one direction of flow and  $T$  the temperature can be an ambient temperature. The equations that govern the concepts of the flow and heat transfer will be as follows

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} = \mathbf{0}$$

$$(2.1)$$

$$\mu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{k} \right) + g\beta(T - T_0) + g\beta^*(C - C_0) = -v_0 \frac{\partial u}{\partial y} \quad (2.2)$$

under Boussinesq approximations (on dropping the asterisks) are

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - D^{-1}u + (\theta + NC) = -S \frac{\partial u}{\partial y} \quad (2.7)$$

$$\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\rho_0 \nu}{k_1} \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right] + \frac{\rho_0 \nu}{kk_1} u^2 - \frac{\partial}{\partial y} (r q_{dr}) = \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} + D^{-1}u^2 = -P_1 S \frac{\partial \theta}{\partial y} \quad (2.8)$$

$$D_1 \left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + K_{11} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) = 0 \quad (2.4)$$

$$\left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + \frac{S_0 S_c}{N} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) = 0$$

$$\rho = \rho_0 (1 - \beta(T - T_0) - \beta^*(C - C_0)) \quad (2.5)$$

Where

$$D = \frac{k}{b^2}$$

is the Darcy parameter

$$P = \frac{\mu C_p}{k_1}$$

is the Prandtl number

$$Ec = \frac{\beta g b}{C_p}$$

is the Eckert number

$$G = \frac{\beta g b^3 (T_1 - T_0)}{v^2}$$

is the Grashof number

$$S = \frac{v_0 b}{v}$$

is the suction Reynolds number

$$N = \frac{\beta^* (C_1 - C_0)}{\beta (T_1 - T_0)}$$

is the buoyancy ratio

$$S_c = \frac{v}{D_1}$$

is the Schmidt number

$$S_o = \frac{\beta^* K_{11}}{\beta v}$$

is the Soret number

$$N_1 = \frac{\infty \beta_R K_1}{4 \sigma T_e^3}$$

is the radiation parameter

$$N_2 = \frac{3N_1}{3N_1 + 4}$$

$$P_1 = PN_2 \quad \alpha_1 = \alpha N_2$$

The corresponding boundary conditions in the non-dimensional form are

$$u = 0, \theta = 1, C = 1 \text{ on } z = \pm 1$$

where  $\rho_0$  is the density at the ambient temperature  $T_0$  and concentration  $C_0$  and  $\beta, k, \nu$  are the coefficients of Kinematic viscosity, thermal conductivity and thermal expansion of the fluid respectively,  $\beta^*$  is the volumetric coefficient of expansion with mass fraction concentration,  $k$  is the permeability of the porous medium and  $C_p$  is the specific heat at constant pressure, this the molecular diffusivity and  $k_{11}$  is the cross diffusivity..

In view of the continuity equations, we take  $u = u(y, z)$

The boundary conditions are

$$u = 0 \text{ on } z = \pm b$$

$$T = \pm T_1, \quad C = \pm C_1 \quad (2.6)$$

$$\frac{\partial u}{\partial z} = 0, \quad \frac{\partial T}{\partial z} = 0 \quad \text{and} \quad \frac{\partial C}{\partial z} = 0 \quad \text{on } Z =$$

0 in view of the symmetry.

We introduce the following non-dimensional variables as follows.

$$z^* = \frac{z}{b}; \quad y^* = \frac{y}{b}; \quad \theta^* = \frac{T - T_0}{T_1 - T_0},$$

$$C^* = \frac{C - C_0}{C_1 - C_0},$$

$$u^* = \frac{\nu u}{\beta g b^2 (T_1 - T_0)}$$

Substituting these in the governing equations the corresponding dimensionless equations

$$\frac{\partial u}{\partial z} = 0, \quad \frac{\partial \theta}{\partial z} = 0 \quad \text{and} \quad \frac{\partial C}{\partial z} = 0 \quad \text{On} \\ Z = 0 \quad (2.10)$$

The observation of symmetrical mathematical flow in the present study is in view of mid plane of the fluid flow medium. It is investigated to observe the fluid flow pattern with respect to one half of the domain which was bounded by the walls of properties of impermeability. The finite element simulation is executed with functions having quadratic approximation as designed by eight noded rectangular elements in the normal cross sectional plane (y - z) that is completely bounded by the planes of z = 0 and 1.

### 3. FINITE ELEMENT ANALYSIS OF THE PROBLEM

If  $u^i$  and  $\theta^i$  are the approximations of  $u$  and  $\theta$  we define the errors (residual)  $E_1^i$  and  $E_2^i$  as

$$E_1^i = \frac{\partial^2 u^i}{\partial y^2} + \frac{\partial^2 u^i}{\partial z^2} - D^{-1}u^i + (\theta^i + NC^i) + S \frac{\partial u^i}{\partial y} \quad (3.1)$$

$$E_2^i = \frac{\partial^2 \theta^i}{\partial y^2} + N_2 \frac{\partial^2 \theta^i}{\partial z^2} + GP_1 Ec [(\frac{\partial u^i}{\partial y})^2 + (\frac{\partial u^i}{\partial z})^2] + D^{-1}(u^i)^2 + P_1 S \frac{\partial \theta^i}{\partial y} + \frac{\partial N_j^i}{\partial y} \frac{\partial u^i}{\partial z} - D^{-1}u^i N_j^i + (\theta^i + NC^i) N_j^i + S N_j^i \frac{\partial u^i}{\partial y} \quad (3.2)$$

$$E_3^i = (\frac{\partial^2 C^i}{\partial y^2} + \frac{\partial^2 C^i}{\partial z^2}) + \frac{S_o S_c}{N} (\frac{\partial^2 \theta^i}{\partial y^2} + \frac{\partial^2 \theta^i}{\partial z^2}) \quad (3.3)$$

where

$$u^i = \sum_{k=1}^8 u_k^i N_k^i \quad (3.4)$$

$$\theta^i = \sum_{k=1}^8 \theta_k^i N_k^i \quad (3.5)$$

$$C^i = \sum_{k=1}^8 C_k^i N_k^i \quad (3.6)$$

These errors are orthogonal to the weight function over the domain of  $e^i$ . Under Galerkin we choose the approximation functions as the weight function. Multiply both sides of the equations (3.1) - (3.3) by the weight function i.e.,

each of the approximation function  $N_j^i$  and integrate over the surface  $\Omega_i$ , we obtain

$$\int_{\Omega_i} E_1^i N_j^i d\Omega_i = 0 \quad (j = 1, 2, \dots, 8) \quad (3.7)$$

$$\int_{\Omega_i} E_2^i N_j^i d\Omega_i = 0 \quad (j = 1, 2, \dots, 8) \quad (3.8)$$

$$\int_{\Omega_i} E_3^i N_j^i d\Omega_i = 0 \quad (j = 1, 2, \dots, 8) \quad (3.9)$$

$$\int_{\Omega_i} [\frac{\partial^2 u^i}{\partial y^2} + \frac{\partial^2 u^i}{\partial z^2} - D^{-1}u^i + (\theta^i + NC^i) + S \frac{\partial u^i}{\partial y}] N_j^i d\Omega_i = 0 \quad (3.10)$$

$$\int_{\Omega_i} [\frac{\partial^2 \theta^i}{\partial y^2} + N_2 \frac{\partial^2 \theta^i}{\partial z^2} + GP_1 Ec [(\frac{\partial u^i}{\partial y})^2 + (\frac{\partial u^i}{\partial z})^2] + D^{-1}(u^i)^2] + P_1 S \frac{\partial \theta^i}{\partial y} ] N_j^i d\Omega_i = 0 \quad (3.11)$$

$$\int_{\Omega_i} [(\frac{\partial^2 C^i}{\partial y^2} + \frac{\partial^2 C^i}{\partial z^2}) + \frac{S_o S_c}{N} (\frac{\partial^2 \theta^i}{\partial y^2} + \frac{\partial^2 \theta^i}{\partial z^2})] N_j^i d\Omega_i = 0 \quad (3.12)$$

$$= \int_{\Gamma} [N_j^i \frac{\partial u^i}{\partial y} n_y + N_j^i \frac{\partial u^i}{\partial z} n_z] d\Gamma_i \quad (3.13)$$

$$\int_{\Omega_i} [\frac{\partial N_j^i}{\partial y} \frac{\partial \theta^i}{\partial y} + N_2 \frac{\partial N_j^i}{\partial z} \frac{\partial \theta^i}{\partial z} + GP_1 Ec N_j^i [(\frac{\partial u^i}{\partial y})^2 + (\frac{\partial u^i}{\partial z})^2] + D^{-1}(u^i)^2 + P_1 S N_j^i \frac{\partial \theta^i}{\partial y}] d\Omega_i \\ = \int_{\Gamma} [N_j^i \frac{\partial \theta^i}{\partial y} n_y + N_j^i \frac{\partial \theta^i}{\partial z} n_z] d\Gamma_i \quad (3.14)$$

$$\int_{\Omega_i} [N \frac{\partial N_j^i}{\partial y} (N \frac{\partial C^i}{\partial y} + S_c S_o \frac{\partial \theta^i}{\partial y}) + \frac{\partial N_j^i}{\partial z} (N \frac{\partial C^i}{\partial z})] d\Omega_i \\ = \int_{\Gamma} [N_j^i (N \frac{\partial C^i}{\partial y} + S_c S_o \frac{\partial \theta^i}{\partial y}) n_y + N_j^i (N \frac{\partial C^i}{\partial z} + S_c S_o \frac{\partial \theta^i}{\partial z}) n_z] d\Gamma_i \quad (3.15)$$

where  $\Omega_i$  is the serendipity element bounded by  $\Gamma_i$ ,  $n_y$ ,  $n_z$  are the direction cosines normal to  $\Gamma_i$ . Substituting (3.4), (3.5) & (3.6) in L.H.S of (3.13), (3.14) & (3.15) we get

$$\int_{\Omega_i} \sum_{k=1}^8 u_k^i \left[ \frac{\partial N_j^i}{\partial y} \frac{\partial N_k^i}{\partial y} + \frac{\partial N_j^i}{\partial z} \frac{\partial N_k^i}{\partial z} - D^{-1} N_k^i N_j^i + S N_j^i \frac{\partial N_k^i}{\partial y} \right] d\Omega_i + \int_{\Omega_i} \sum_{k=1}^8 (\theta_k^i + N C_k^i) N_j^i N_k^i d\Omega_i = Q_j^i \quad (3.16)$$

$$\int_{\Omega_i} \sum_{k=1}^8 \theta_k^i \left[ \frac{\partial N_j^i}{\partial y} \frac{\partial N_k^i}{\partial y} + N_2 \frac{\partial N_j^i}{\partial z} \frac{\partial N_k^i}{\partial z} + P_1 S N_j^i \frac{\partial N_k^i}{\partial y} \right] d\Omega_i + \int_{\Omega_i} \sum_{k=1}^8 u_k^i G P_1 E c \left[ \left( \frac{\partial N_k^i}{\partial y} \right)^2 + \left( \frac{\partial N_k^i}{\partial z} \right)^2 + D^{-1} (N_k^i)^2 \right] d\Omega_i = (Q^T)_j^i \quad (3.17)$$

$$\int_{\Omega_i} \left[ \sum_{k=1}^8 C_k^i \left\{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} \right\} d\Omega_i - \int_{\Omega_i} N_2 S c \sum_{k=1}^8 u_k^i (N_j^i N_k^i) d\Omega_i - S c S_o \int_{\Omega_i} \sum_{k=1}^8 \theta_k^i \left\{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} \right\} d\Omega_i \right] = (Q^C)_j^i \quad (3.18)$$

where

$$Q_j^i = \int_{\Gamma_i} (N_j^i) \left( \frac{\partial u^i}{\partial y} n_y + N_j^i \frac{\partial u^i}{\partial z} n_z \right) d\Gamma_i$$

$$(Q^T)_j^i = \int_{\Gamma_i} (N_j^i) \left( \frac{\partial T^i}{\partial y} n_y + N_j^i \frac{\partial T^i}{\partial z} n_z \right) d\Gamma_i \quad (j = 1, 2, \dots, 8)$$

$$(Q^C)_j^i = \int_{\Gamma_i} \left[ N_j^i \left\{ N \frac{\partial C^i}{\partial y} + S_o S c \frac{\partial \theta^i}{\partial y} \right\} n_y + N_j^i \left\{ N \frac{\partial C^i}{\partial z} + S_o S c \frac{\partial \theta^i}{\partial z} \right\} n_z \right] d\Gamma_i \quad j=1,2,\dots,8.$$

Choosing different  $N_k^i$ 's corresponding to each element  $e_i$  (3.11) results in sixteen equations for two sets of unknown  $(u_k^i)$  and  $(\theta_k^i)$  viz

$$(a_{kj}^i)(u_k^i) = Q_j^i \quad (3.19)$$

$$(b_{kj}^i)(\theta_k^i) + (c_{kj}^i)u_k^i = (Q^T)_j^i \quad (j = 1, 2, \dots, 8) \quad (3.20)$$

$$(m_{kj}^i)(C_k^i) + (l_{kj}^i)(u_k^i) = (n_{kj}^i)(\theta_k^i) + (Q_j^C)^i \quad (j,k=1,2,\dots,8) \quad (3.21)$$

where  $(a_{kj}^i)$ ,  $(b_{kj}^i)$ ,  $(c_{kj}^i)$ ,  $(m_{kj}^i)$ ,  $(n_{kj}^i)$  and  $(l_{kj}^i)$

are  $8 \times 8$  stiffness matrices and  $Q_j^i$ ,  $(Q^T)_j^i$  and  $(Q_j^C)^i$  are  $8 \times 1$  dimensional matrices. The process of simulation at each point of element mn has been periodically made with the consideration of boundary conditions and then an integration of elements in the matrix was done. The unknown parameters of  $u$ ,  $\theta$  and  $C$  in the process of computation is attempted with respect to global nodes which ultimately determine them on solving the matrix equation and it may lead to solution of matrix solution.

In order to compute, It is to consider a serendipity element having the vertices of (0,0), (0,1) (1,0) and (1,1). The eight nodes of the selected and computed element are clearly given in Fig.(a) and the operations of functions for the quadratic interpolation at these nodes have been as under

$$\begin{aligned} N_1 &= -2(y-1)(z-1)(z+y-\frac{1}{2}) & ; & & N_2 &= -4z(z-1)(y-1) \\ N_3 &= -2z(y-1)(z-y+\frac{1}{2}) & ; & & N_4 &= -4yz(y-1) \\ N_5 &= 2yz(z+y-\frac{3}{2}) & ; & & N_6 &= -4yz(z-1) \\ N_7 &= 2y(z-1)(z-y+\frac{1}{2}) & ; & & N_8 &= -4y(z-1)(y-1) \end{aligned}$$

Substituting these shape functions in (3.19) and integrating over the element domain the matrix for the global nodes of  $u$  viz.  $u_i$  ( $i = 1,2,\dots$ ) reduces to a  $8 \times 8$  matrix equations.

This  $8 \times 8$  matrix equations can be partitioned in the form

$$\begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix} \begin{bmatrix} \Delta_U^1 \\ \Delta_U^2 \end{bmatrix} = \begin{bmatrix} F_U^1 \\ F_U^2 \end{bmatrix} \quad (3.22)$$

Where  $\Delta_U^1, \Delta_U^2, F_U^1, F_U^2$  are column matrices given by

$$\Delta_U^1 = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}, \quad \Delta_U^2 = \begin{bmatrix} U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix}$$

Equation (3.20) yields the following two equations in terms of the partitioned matrices.

$$[S^{11}] [\Delta_U^1] + [S^{12}] [\Delta_U^2] = [F_U^1]$$

$$[S^{21}] [\Delta_U^1] + [S^{22}] [\Delta_U^2] = [F_U^2] \tag{3.23}$$

Similarly the  $8 \times 8$  matrix equations for  $\theta_j, C_j$  ( $j = 1, 2, \dots, 8$ ) in the partitioned form are

$$\begin{bmatrix} B^{11} & B^{12} \\ B^{21} & B^{22} \end{bmatrix} \begin{bmatrix} \Delta_\theta^1 \\ \Delta_\theta^2 \end{bmatrix} = \begin{bmatrix} F_\theta^1 \\ F_\theta^2 \end{bmatrix} \tag{3.24}$$

$$\begin{bmatrix} L^{11} & L^{12} \\ L^{21} & L^{22} \end{bmatrix} \begin{bmatrix} \Delta_C^1 \\ \Delta_C^2 \end{bmatrix} = \begin{bmatrix} F_C^1 \\ F_C^2 \end{bmatrix} \tag{3.25}$$

Where  $\Delta_\theta^1, \Delta_\theta^2, F_\theta^1, F_\theta^2, \Delta_C^1, \Delta_C^2, F_C^1, F_C^2$  are column matrices given by

$$\Delta_\theta^1 = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}; \quad \Delta_\theta^2 = \begin{bmatrix} \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \end{bmatrix};$$

$$\Delta_C^1 = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}; \quad \Delta_C^2 = \begin{bmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix}$$

The boundary conditions (essential) on the primary variables are

$$u_3 = u_4 = u_5 = 0;$$

$$\theta_3 = \theta_4 = \theta_5 = 1 \text{ and}$$

$$C_3 = C_4 = C_5 = 1 \quad \text{on} \quad y = 1 \tag{3.26}$$

In view of the symmetry conditions we obtain

$$Q_1 = Q_2 = Q_6 = Q_7 = Q_8 = 0$$

$$Q_1^T = Q_2^T = Q_6^T = Q_7^T = Q_8^T = 0 \tag{3.27}$$

$$Q_1^C = Q_2^C = Q_3^C = Q_4^C = Q_8^C = 0$$

Solving the ultimate  $8 \times 8$  matrices we determine the unknown global nodal values of  $u_i, \theta_i$

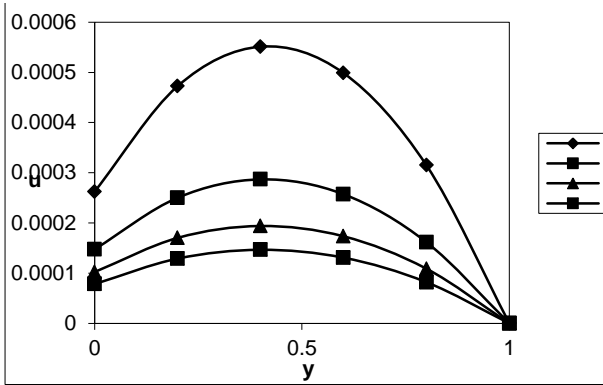
( $i = 1, 2, \dots, 8$ ).

The solution for  $u, \theta$  may now be represented as

$$u = \sum_{k=1}^8 u_k N_k, \quad \theta = \sum_{j=1}^8 \theta_j N_j \text{ and } C = \sum_{i=1}^8 C_i N_i$$

#### 4. DISCUSSION OF NUMERICAL RESULTS

It is observed in the results of simulations that the behaviour of fluid flow in the direction of planes  $y=0 \& 1$  and  $Z=0 \& Z=1/2$  can be accompanied with movement in the direction from  $y=0$  to  $1$  in vertical manner. A higher value is observed to at  $y=0.4$  while at  $Z=1/2$  level it can be maximum at the position of  $y=1$ . It is clearly noticed that the velocity is decreases with increase in  $D^{-1}$ . Permeability of the porous medium smaller and therefore the velocity in the fluid region can also be very low. The variation of  $u$  along the normal planes  $y=0 \& 1$  shows that  $u$  decreases with  $D^{-1}$  (Figs 1, 2). This situation in the stud gives the fact that lesser the permeability of the porous medium smaller the magnitude of  $u$ . The values of  $u$  at  $y=0$  are higher than those at  $y=1$  level for all variations in parameters. Figures 3 and 4 show the variation of non-dimensional temperature at the horizontal levels  $y=0 \& 1/2$  with respect to  $\alpha$ , and  $N_1$ . A marginal depreciation in  $\theta$  is observed to be with increase in the heat source parameter  $\alpha$ . The variation of  $\theta$  with radiation parameter  $N_1$  reveals that the enhancement in  $N_1$  may cause an increase in the actual temperature at  $y=0 \& 1$  levels. The variation of  $\theta$  with  $\alpha$ , and  $N$ , at the vertical levels  $Z=0 \& 1/2$  is shown in figures. From figures 7&8 it can concluded that the actual temperature experiences depreciation with increase in the heat source parameter  $\alpha$ . This indicates that the presence of the heat generating source in the fluid region leads to a reduction in the actual temperature at both the vertical levels.

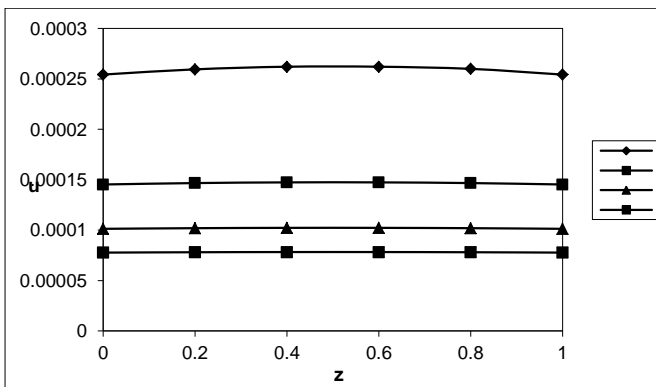


**Fig. 1**

**Variation of u with  $D^{-1}$  at  $y=0.5$  level**

$M=5; G=200; N_1=0.5; S=0.8; k=0.5; P=0.71; D^{-1}=2000; z=0.5$

	I	II	III	IV
$D^{-1}$	$2 \times 10^3$	$4 \times 10^3$	$6 \times 10^3$	$8 \times 10^3$



(1)

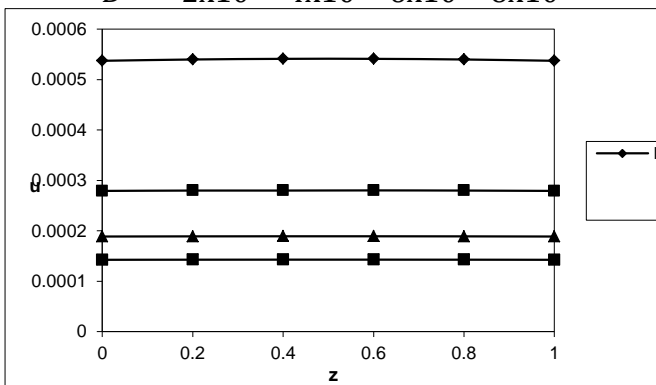
**Fig. 2**

**Variation of u with  $D^{-1}$  at  $z=0$  level**

$M=5; G=200; N_1=0.5; S=0.8; k=0.5; P=0.71; \beta=2;$

$D^{-1}=2000; z=0.5$

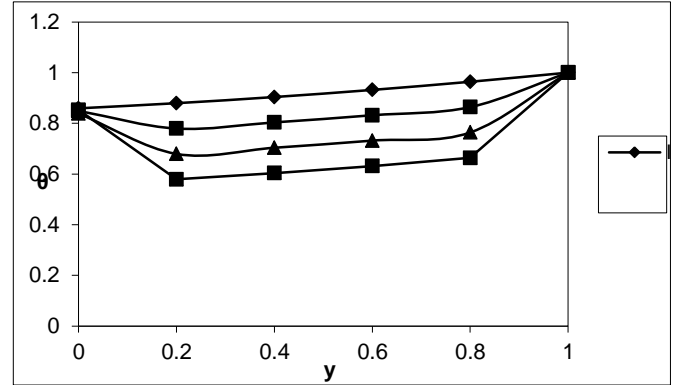
	I	II	III	IV
$D^{-1}$	$2 \times 10^3$	$4 \times 10^3$	$6 \times 10^3$	$8 \times 10^3$



**Fig. 3**

**Variation of u with  $D^{-1}$  at  $z=0.5$  level**  
 $M=5; G=200; cn=0.5; S=0.8; k=0.5; P=0.71; D^{-1}=2000; z=0.5$

	I	II	III	IV
$D^{-1}$	$2 \times 10^3$	$4 \times 10^3$	$6 \times 10^3$	$8 \times 10^3$



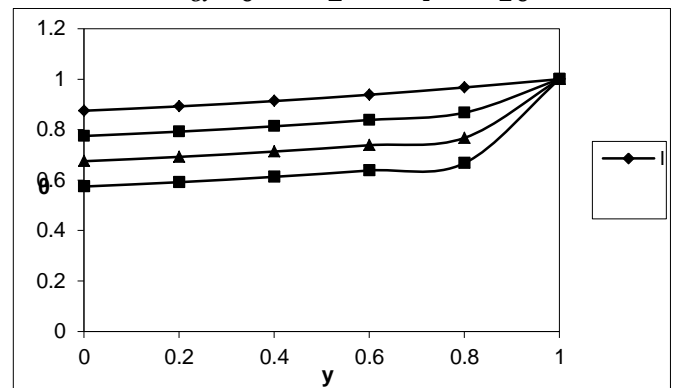
**Fig. 4**

**Variation of  $\theta$  with  $\alpha$  at  $y=0$  level**

$M=5; G=200; N_1=0.5; S=0.8; k=0.5; P=0.71; \beta=2;$

$D^{-1}=2000; z=0.5$

	I	II	III	IV
$\alpha$	0	2	4	10

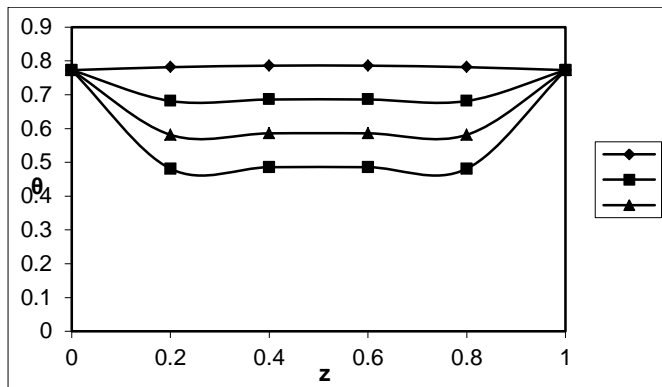


**Fig. 5**

**Variation of  $\theta$  with  $\alpha$  at  $y=0.5$  level**

$M=5; G=200; N_1=0.5; S=0.8; k=0.5; P=0.71; D^{-1}=2000; z=0.5$

	I	II	III	IV
$\alpha$	0	2	4	10

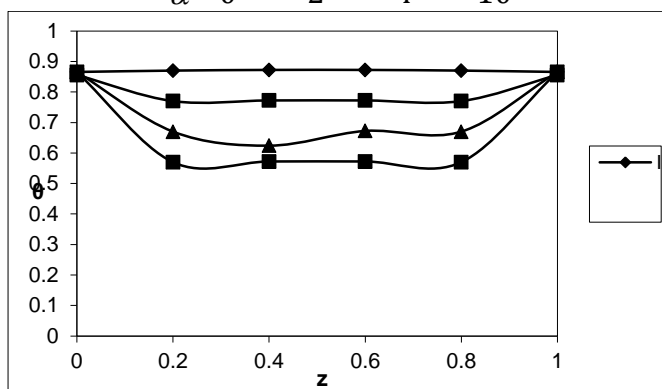


**Fig. 6**

**Variation of  $\theta$  with  $\alpha$  at  $z=0$  level**

$M=5; G=200; N_1=0.5; S=0.8; k=0.5; P=0.71; \beta=2;$   
 $D^{-1}=2000; z=0.5$

	I	II	III	IV
$\alpha$	0	2	4	10

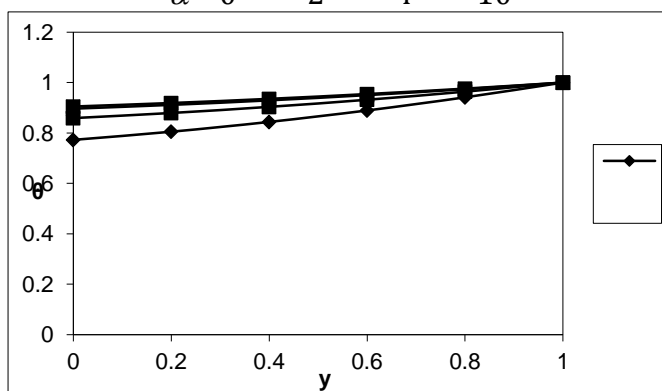


**Fig. 7**

**Variation of  $\theta$  with  $\alpha$  at  $z=0.5$  level**

$M=5; G=200; N_1=0.5; S=0.8; k=0.5; P=0.71; D^{-1}=2000; z=0.5$

	I	II	III	IV
$\alpha$	0	2	4	10



**Fig. 8**

**Variation of  $\theta$  with  $N_1$  at  $y=0$  level**

$M=5; G=200; S=0.8; k=0.5; P=0.71; \beta=2; D^{-1}=2000; z=0.5$

	I	II	III	IV
$N_1$	0.5	1.5	5	10

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