Embedded Type-I and Type-II Censoring in Life-Testing Experiments to Determine the Reliability Sampling Plans for an Exponential Distribution

Ramya. D1, S. Devaarul2

1Dr. D. Ramya, Assistant Professor, Department of Statistics, P.S.G. College of Arts and Science, Coimbatore, Tamilnadu, India.
2Dr. S. Devaarul, Assistant Professor, Department of Statistics, Government Arts College, Coimbatore, Tamilnadu, India.

Abstract - Quality and Reliability Engineering has acquired its overwhelming application in industries as producer and consumer are aware of its vital importance in producing quality products. Reliability sampling is an algorithm driven tool of Statistical Quality Control which enables the Quality Control engineers to select appropriate sampling plans for testing the products and hence the decision of acceptance or rejection is made on the batches or lots. A new type of sampling plans called Mixed Censoring Reliability Sampling Plans are developed involving type I and type II censoring. In general, either type I censoring or type II censoring schemes are adopted in designing sampling plans. But in this study an attempt has been made to design sampling plans by blending the two censoring schemes. This pressurizes the producer to maintain the quality of the batches or lots. R-Language is used to determine the parameters of the reliability sampling plans. Necessary tables are constructed using the designing procedure and illustration is given for easy implementation in industries.

Key Words: Reliability sampling plans, Censoring, mixed censoring, exponential distribution, life-testing experiments.

1. INTRODUCTION

Censoring schemes are generally employed during the life test to make the inspection as a cost effective one. Time censoring (Type-I), Product censoring (Type-II) and hybrid censoring are some of the censoring schemes employed in the life test.

This paper proposes a general Bayesian framework for designing a variable acceptance sampling scheme with mixed censoring. A general loss function which includes the sampling cost, the time-consuming cost, the salvage value, and the decision loss is employed to determine the Bayes risk and the corresponding optimal sampling plan. An explicit expression of the Bayes risk is derived. The new model can easily be adapted to create life testing models for different distributions. Specifically, two commonly used distributions including the exponential distribution and the Weibull distribution are considered with a special decision loss function. We demonstrate that the proposed model is superior to models with Type I or Type II censoring.

This paper develops and validates a reliability sampling methodology using simulation, and re-sampling; and which incorporates unit-to-unit variation in the determination of significant sample sizes for analytically intractable reliability cases. This sample size determination is very important because the reliability of the sampled vehicles should represent the reliability of the entire fleet. Smaller-than-required sample sizes may lead to an incorrect representation of the reliability of the fleet, which may mislead the Army to make poor decisions, such as deploying a fleet that may not be reliable. These type II errors can be minimized by incorporating a more realistic sampling methodology, as developed in this research. Prior to using this methodology, analytical formulas were used to compute reliability sample sizes with unit-to-unit variation assumed to be constant. This new methodology confirms the analytically derived solutions for fixed usage & true failure rate, as well as for fixed usage & varying vehicle true failure rate. Existing reliability data shows that unit-to-unit variation does exist. Vehicle variation in true failure rate is modeled with a Gamma prior distribution. Recent reliability data are used to validate the hypothesis that this is an adequate tool for reliability sampling when unit-to-unit variation exists. Results of this validation accept the hypothesis, validating that the methodology is an adequate tool. The Army is currently using this methodology for fleet assessment.

The two censoring schemes have been studied by numerous authors who proposed a concept of grouping in which the experimenter might group the test units into several sets, each set as an assembly of test units, and then all the units are tested simultaneously until the first failures in each group are observed [11], [12], [10] and [8]. It was indicated that in a situation where the lifetime of a product is quite high and test facilities are scarce, but test material is relatively cheap, one can test ‘mn’ units by testing m sets, each containing n units [3]. The lifetest is then conducted by testing each of these sets of units separately until the occurrence of first failure in each set. Such a censoring
scheme is called first-failure censoring. The exact likelihood inference for the Exponential Distribution under Generalized Type-I and type-II hybrid censoring was discussed [4]. A reduction in testing effect and administrative convenience may be obtained by using intermittent inspection whereby items are inspected only at certain points of time using exponential distribution [9]. Already, mixed sampling plans by blending the process and product control measures was developed [6]. Also several mixed sampling plans have been proposed to suit the industrial requirements [16]. Mixed sampling product control plans for costly or destructive items were also developed [5].

In reliability studies the exponential distribution plays a role of importance analogous to that of the normal distribution in other areas of statistics. The desirability of the exponential distribution is due to its simplicity and its inherent association with the well-developed theory of Poisson process. Also, many times certain quantities computed from the exponential distribution serve as bounds for similar quantities that need to be computed from other, less tractable distributions. A reliability test plan for exponentiated log-logistic distribution was studied [14]. The Economic Reliability Group Acceptance Sampling Plans for Lifetimes using a Generalized Exponential Distribution was also developed [7]. A few authors proposed Group Acceptance Sampling Plans for Lifetimes using a Marshall-Olkin Extended Exponential Distribution [13]. An economic reliability test plan using the generalized exponential distribution was developed [1]. A new generalization of Lomax distribution called Exponential Lomax distribution was proposed [3].

1.1 Formulation of the plan

In this Mixed Censoring Reliability Sampling Plans there are two stages, first stage being type I censoring and the second stage is dealt with type II censoring, where it is assumed that the random variable X follows an exponential distribution. The probability density function (pdf) is defined as:

\[ f(x, \theta) = \theta e^{-\theta x}, \theta > 0 \]  

(1)

The cumulative distribution function (cdf) of exponential distribution is

\[ F(x, \theta) = 1 - e^{-\theta x}, \theta > 0 \]  

(2)

where \( \theta \) is the parameter of the distribution.

1.2. Characteristics of Exponential Distribution:

Mean of the Exponential distribution

\[ \bar{x}_E = \frac{1}{\theta} \]

Variance of the Exponential distribution

\[ \sigma^2 = \frac{1}{\theta^2} \]

Skewness of the Exponential distribution

\[ \beta_1 = \frac{3}{\sqrt{\theta}} \]

Kurtosis of the Exponential distribution

\[ \beta_2 = \frac{6}{\theta^2} \]

2. ALGORITHM FOR SENTENCING A LOT

Let the two stages be independent.

**Step 1:** Draw a random sample of size \( n_1 \) from the lot and put them into life test using type II censoring. Let it be \( x_1, x_2, ..., x_k \).

**Step 2:** Determine the skewness and kurtosis from the sample observations and hence the parameter \( \theta \) is determined. Now determine the mean \( \bar{x}_E \).

**Step 3:** If \( \bar{x}_E \geq L \) accept the lot.

**Step 4:** If \( \bar{x}_E < L \) take a second sample of size \( n_2 \) and put them into life test using type I censoring.

**Step 5:** In the second stage of life test, inspect and find the number of failures (d).

**Step 6:** Accept the batch or lot if \( d \leq c \), otherwise reject it.

3. MEASURES OF THE MIXED CENSORING RELIABILITY SAMPLING PLANS

Let \( x_1, x_2, ..., x_n \) be a random sample which follows an exponential distribution. Also let \( d_1, d_2, ..., d_n \) be the probability of failures. Hence the different measures are as follows

Operating Characteristic function:

By the addition theorem on probability, we get

\[ P_a(p) = P_{Type-II,n_1}[\bar{x} \geq L] + P_{Type-I,n_2}[\bar{x} < L]P[d \leq c] \]  

(3)

Average Sample Number (ASN):

\[ ASN = n_1P[\bar{x} \geq L] + n_2P[\bar{x} < L].P[d \leq c] \]  

(4)

Average Outgoing Quality (AOQ):

\[ AOQ \approx p.P_a(p) \]  

(5)
4. DESIGNING THE MIXED CENSORING RELIABILITY SAMPLING PLANS INDEXED THROUGH AQL AND LQL

**Step 1:** Assume that the two stages are independent. Let \( \beta_1 = 0.95 \) and \( \beta_2 = 0.10 \)

**Step 2:** The first stage probability of acceptance is separated. Let it be \( \beta'_1 \) and \( \beta'_2 \) respectively. Also \( \beta_1 \geq \beta'_1 \) and \( \beta_2 \geq \beta'_2 \)

**Step 3:** Using the procedure developed for non-normal distributions, the values of \( n_1 \) and \( k \) are determined using the following formula [17]:

\[
 n_1 = c_L \left( \frac{K_1 + K_2}{K_2 - p_2} \right)^2 
\]

(6)

\[
 k = -\frac{K_2 K_1 - p_2 + K_3 K_1 - p_3}{K_2 + K_3} 
\]

(7)

**Step 4:** The probability of acceptance associated with \( p_1 \) and \( p_2 \) is determined. Let them be denoted by \( \beta'_1 \) and \( \beta'_2 \)

Where

\[
 \beta'_1 = \frac{\beta - \beta'_1}{\beta_1 - \beta'_1} 
\]

(8)

\[
 \beta'_2 = \frac{\beta - \beta'_2}{\beta_2 - \beta'_2} 
\]

(9)

**Step 5:** The values of \( n_2 \) and the corresponding acceptance number \( c \) are determined, such that

\[
 \sum_{x=0}^{\infty} e^{-\theta x} \frac{(\theta x)^{n_2}}{x!} \beta'_1 \]

and

\[
 \sum_{x=0}^{\infty} e^{-\theta x} \frac{(\theta x)^{n_2}}{x!} \beta'_2 \]

(10)

5. ILLUSTRATION

Let \( p_1 = 0.01 \) and \( p_2 = 0.1 \), the corresponding probability of acceptance be 0.95 and 0.10 respectively. Obtain the parameters of Mixed Censoring Reliability Sampling Plans using exponential distribution for the acceptance failure constant \( c = 3 \).

Solution:

It is given that the probability of acceptance is 0.95 and 0.10 at AQL and LQL respectively

By assuming the first stage probability of acceptance \( \beta'_1 = 0.65 \) and \( \beta'_2 = 0.05 \)

from the Table 1,

We get \( n_1 = 9; k = 4.7555; n_2 = 78 \) for \( C = 3 \).

Algorithm for sentencing a lot:

**Step 1:** Draw a sample of size 9.

**Step 2:** Find the skewness and kurtosis using the sample observations and hence determine the parameter \( \theta \).

**Step 3:** Find the mean \( \bar{X} \)

**Step 4:** If \( \bar{X} \geq L \) accept the lot

**Step 5:** If \( \bar{X} < L \) take a second sample of size 78 and put them into life test using type I censoring

**Step 6:** In the second stage of life test, inspect and find the number of failures (d).

**Step 7:** Accept the lot if \( d \leq 3 \), otherwise reject it.

Table 1: Mixed Censoring Reliability Sampling plans indexed through AQL and LQL based on Exponential distribution (\( \beta_1 = 0.95; \beta_2 = 0.10; \beta'_1 = 0.65; \beta'_2 = 0.05; \beta_1 = 0.86; \beta_2 = 0.05 \))
Table 2: Probability of Acceptance values and Average Sample Number values for Mixed Censoring Reliability Sampling Plans using Exponential distribution

<table>
<thead>
<tr>
<th>p</th>
<th>P a1</th>
<th>P a2</th>
<th>P a3</th>
<th>ASN 1</th>
<th>ASN 2</th>
<th>ASN 3</th>
</tr>
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<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
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<td>10</td>
</tr>
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<tr>
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<td>0.8982</td>
<td>0.9333</td>
<td>10</td>
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</table>

5.1 Construction of Tables

Table 1:

i) The values of p 1 and p 2 are assumed to known.

ii) The probabilities of acceptance values such as β 1 = 0.95, β 2 = 0.10, β 1' = 0.65 and β 2' = 0.05 are assumed.

iii) For the second stage, the probabilities of acceptance are obtained using equations (8 and 9).

iv) The sample size n 1 and the corresponding values of k of the first stage for the given values of p 1 and p 2 are obtained.

v) Here, variable sampling plans using exponential distribution is used for obtaining the first stage parameters. And hence n 1 and k are determined using equations (6 and 7).

vi) The operating ratio \( Z = \frac{p_2}{p_1} \) is obtained for this sampling plan using chi-square variates.

vii) In the second stage c is fixed and hence n 2 is obtained using equation (10). The values of n 2 are obtained through AQL and LQL.

viii) The values of n 1 and n 2 so obtained are the minimum sample sizes and they are presented in Table 1.

Table 2:

i) The values of p are assumed to be known.

ii) The corresponding probabilities of acceptance namely p a1, p a2 and p a3 for varying values of n 1, n 2, k and C are obtained using equation (3).

iii) Then the ASN values are obtained using equation (4) and are presented.

iv) Using the Table 2, the OC and ASN curves are constructed and presented in Figure 1 and Figure 2.
5.3 Construction of ASN Curves

The ASN curves for the Mixed Censoring Reliability Sampling Plans are constructed and presented in Figure 2. The Average Sample Number (ASN) values are obtained using equation (4) for \( n_1=10, n_2=10 \) and \( C=2, 3, 4 \) and are presented in Table 2. From the Figure 2, ASN1 curve is obtained for \( n_1=10, n_2=10 \) and \( C=2 \), ASN2 curve is obtained for \( n_1=10, n_2=10 \) and \( C=3 \) and ASN3 curve is obtained for \( n_1=10, n_2=10 \) and \( C=4 \).

6. CONCLUSION

In this paper, the Mixed Censoring Reliability Sampling Plans are developed for one parameter exponential distribution. The operating procedure for these plans is very simple and can be used effectively in the quality control section. It is found that the new reliability sampling plans give more pressure on the producer to maintain the quality of the lots or batches. It is observed from the tables that the first stage requires minimum sample sizes when compared to the second stage. Also as the acceptance constant increases the sample size increases. And it is found that the sample size increases in case of type I censoring compared to type II censoring. Whenever the probabilities of acceptance are known, various parameters like \( n_1, n_2, k \) and \( c \) can be easily obtained. Tables are given for easy selection and implementation in industries.

References


