

Effect of Local Scour on Foundation of Hydraulic Structure

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Abstract - The best design of foundation of hydraulic structure requires accurate prediction of the maximum expected depths of scour of the stream. A study of the local scour at piles, groups for any bridge on river was experimentally investigated on laboratory faculty of engineering Al-Azhar University. The aim of the experimental is the investigation of the preferable separation distance between piles to reduce local scour around them to its minimum value, and it's noted that, the scour depth decreases as the spacing between piles increase. Also it noted that the scour depth for circular pile is decreasing than the scour depth for the square pile. I.e. the circular piles groups decrease the scour depth 29.5% than the square piles group. The study indicates the importance of arrangement piles in streamlined shape. Laboratory results were compared with the results obtained from FDOT(Florida department of transportation) program.

Key Words: Hydraulic structure, complex piles, local scour.

1. INTRODUCTION

Local scour is removal of sediment from a round bridge piers or abutments. Piers are the pillars supporting a bridge. Abutments are the supports at each end of a bridge. Water flowing past a pier or abutment may scoop out holes in the sediment these holes are known as scour. The main mechanisms of local scour are: (1) increased mean flow velocities and pressure gradients in the vicinity of the structure; (2) the creation of secondary flows in the form of vortices; and (3) the increased turbulence in the local flow field. Two kinds of vortices may occur: (1) wake vortices, downstream of the points of flow separation on the structure; and (2) horizontal vortices at the bed and free surface due to stagnation pressure variations along the face of the structure and flow separation at the edge of the scour hole. For geotechnical and economic reasons, multiple pile bridge piers have become more and more popular in bridge design. This type of piers can significantly reduce construction costs, compared to spread footer _gravity_ structures when sediment scour is a consideration. However, the scour mechanisms for pile groups are much more complex, and design of local scour depths more difficult to be predicted. While a substantial amount of knowledge has accumulated about scour around single piers over the past decade or so _Breusers and Raudkivi 1991; Melville and Coleman 2000, comparatively little is known about scour at pile groups _Hannah 1978; Salim and Jones 1998; Coleman 2005_.In the simplest case, pile groups are caped above the water surface and only the pile groups obstruct the flow field. Fig (1) shows the shape of scour on piles.



Fig (1) scour around complex piles

Hannah_1978_investigated scour around tow-pile, side-byside, and tandem arrangements, for different spacing's between the piles. Salim and Jones_1998_observed that the scour depth decreases as the spacing between the piles increases. Coleman_2005_presented a new methodology to predict local scour depth at a complex pier. The objectives of the present study are to investigate in a systematic manner the scour around pile groups. Experiments of local scour around pile groups are carried out under steady clear-water scour conditions. A variety of conditions including different pile group arrangements, spacing and streamlined are considered. Two wooden models circular and square shape of diameter and width of 2cm, were investigated in the present work. Where the total number of runs was ten runs.

2-Theoritical Approach

In the present work the different variables affecting the scour near, the groups of tested models of piles such as spacing between piles, shape of pile and arrangement of group piles. Dimensional analysis has been used by using π theory, the following π terms have included S/D, Hw/D, Ds/D, Q/Hw*V and by Modeling Similarity where:

$$\frac{D(\text{mod }el)}{D(\text{prototype})} = \frac{D_m}{D_p} = D_r(\text{scale} \cdot \text{ratio})$$

Where Dm, the diameter of model, Dp the diameter of prototype, Hw the height of water,S the spacing between piles, V the velocity of water and Ds the scour depth.

2-1 Methodology for Estimating Local Scour Depths at Complex Piers

This section presents a methodology for estimating equilibrium local scour depths at complex pier geometries under steady flow conditions. This methodology applies to structures that are composed of up to three components as shown in Figure (2).

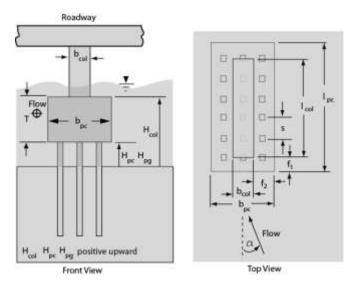


Figure (2) Complex pier configuration considered in this analysis

In this report these components are referred to as the 1) Column, 2) Pile Cap and 3) Pile Group.

The methodology for predicting local scour depths at complex pier geometries begins with the following three assumptions:

1- The structure can be divided into no more than three components as shown in Figure (1).

2- The equilibrium local scour depth for the total structure can be estimated by summing the scour produced by each of the structure components.

3- For scour calculation purposes, each component can be replaced by a single, surface penetrating, circular pile with an effective diameter, D*, that depends on the shape, size and location of the component and its orientation relative to the flow.

2-2 Local Scour Depth Calculation Procedure

The procedure involves decomposing the structure into its (up to three) components, computing the scour depth produced by each component (starting with the upper most component and working down) and summing these depths. Adjustments to both the bed elevation and the depthaveraged velocity are made after each component scour is computed. As stated above, the local scour depth at the composite pier is the sum of the scour depths for the individual components. The step-by-step procedure for computing the equilibrium scour depth for each component is presented below.

1. Divide the pier into its components

Divide the complex pier into its components. Note that y0 is the initial water depth before aggradations/degradation and contraction scour.

2. Sum the aggradations/degradation and contraction scour depths

Sum the computed scour depths produced by aggradations/degradation and contraction scour and adjust the bed elevation accordingly. That is, if there is a net lowering of the bed due to these mechanisms then the bed elevation must be lowered by this amount and increase the initial, un-scoured water depth, y0, to y1.

y1 = y0 + (degradation + contraction scour)

3. Compute the Critical Depth Averaged Velocity

The Critical Depth Averaged Velocity, Vc, (i.e. the velocity that will initiate sediment motion on a flat bed for the given upstream water depth and sediment properties). The critical velocity for a given situation can be determined from the equations or plots given in Appendix B of this report. These equations/plots are based on Shield's curve. Note that this value of critical velocity is used for all of the following calculations

4. Compute the local scour depth due to the Pile Group

a. Lower the bed elevation (and increase the water depth) by one-half the scour depth produced by the Pile Cap, ys (pc),

$$y_3 = y_2 + \frac{y_{s(pc)}}{2}$$
.

b. Adjust the velocity to account for the increased crosssectional area using the following equation:

$$\mathbf{V}_3 = \mathbf{V}_2 \frac{\mathbf{y}_2}{\mathbf{y}_3} \,.$$

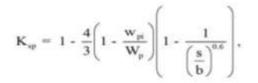
c. Determine the value of the coefficient, Km, which accounts for the number of piles in the direction of the unskewed flow. Note that this coefficient also depends on the pile centerline spacing, s, normalized by the single pile diameter/width, b (i.e. Km depends on s/b as well as the number of piles in the unskewed flow direction, m).



$$\begin{split} \mathbf{K}_{\mathrm{IIII}} = & \begin{cases} f_1(\mathbf{m}) \ f_2\left(\frac{\mathbf{s}}{\mathbf{b}}\right) & \text{for flow skew angle, } \alpha \ \leq \ 3 \text{ degrees} \\ & \mathbf{f}_1 = \begin{pmatrix} 0.875 + 0.125 \mathbf{m} & 1 \le \mathbf{m} \le 5 \\ 1.5 & 5 < \mathbf{m} & , \end{cases} \\ & f_2 = & \begin{cases} \mathbf{a}_0 + \mathbf{a}_1\left(\frac{\mathbf{s}}{\mathbf{b}}\right) & 1 \le \left(\frac{\mathbf{s}}{\mathbf{b}}\right) \le 3 \\ \mathbf{a}_2 + \mathbf{a}_3\left(\frac{\mathbf{s}}{\mathbf{b}}\right) & 3 \le \left(\frac{\mathbf{s}}{\mathbf{b}}\right) \le 10 \\ \mathbf{a}_2 + \mathbf{a}_3(10) & 10 < \left(\frac{\mathbf{s}}{\mathbf{b}}\right) \end{cases} \end{split}$$
where $& \mathbf{a}_0 = -0.5 + 1.5 \left(\frac{1}{f_1}\right), \\ & \mathbf{a}_1 = 0.5 - 0.5 \left(\frac{1}{f_1}\right), \\ & \mathbf{a}_2 = 1.429 - 0.429 \left(\frac{1}{f_1}\right) \text{ and} \\ & \mathbf{a}_3 = -0.143 + 0.143 \left(\frac{1}{f_2}\right). \end{split}$

d. Determine the "projected width" of the piles, Wp. This is the sum of the non -overlapping projections of the piles in the first two rows and the first column onto a vertical plane normal to the flow (Figure 3). The first column is the upstream column for piers skewed to the flow.

e. Determine the coefficient that accounts for the pile spacing, Ksp. Knowing the centerline spacing between the piles, s, and the pile width, b, Ksp can be obtained from the following equation:



Where

wpi \equiv projected width of an unobstructed single pile in the group.

 $Wp \equiv$ the projected width.

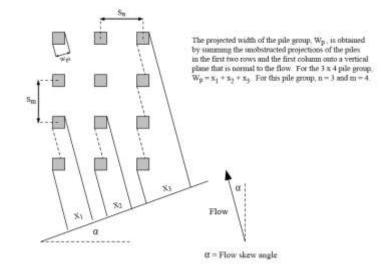


Figure (3) Diagram illustrating the projected width of a pile group, Wp.

f. Estimate the effective diameter/width of the pile group using the following equation:

$$D_{pg}^* \approx \sqrt{\frac{W_p K_m H_{pg}}{5}}$$
,

Where

1

H pg \equiv Distance from the adjusted bed to the top of the piles

g. Compute the value of the parameter, y3(max), using the following equation (this places a limit on the magnitude of the water depth for the pile group scour calculations):

$$y_{3(\text{max})} = \begin{cases} y_3 & \text{for } y_3 \le 2.5 \text{D}_{\text{pg}}^* \\ 2.5 \text{D}_{\text{pg}}^* & \text{for } y_3 > 2.5 \text{D}_{\text{pg}}^* \end{cases}.$$

h. If the tops of the piles in the group are at or above the water surface set Kh = 1. If the tops of the piles are submerged (i.e. if Hpg is less than y3(max)) this will influence the scour depth produced by the group.

The coefficient that accounts for the height of the pile group above the adjusted bed, Kh , can be obtained from the following equation:

$$K_{y} = 3.08 \left(\frac{H_{py}}{y_{3(max)}}\right) - 5.23 \left(\frac{H_{py}}{y_{3(max)}}\right)^{2} + 5.25 \left(\frac{H_{py}}{y_{3(max)}}\right)^{4} - 2.10 \left(\frac{H_{py}}{y_{3(max)}}\right)^{4}.$$

NOTE: If H pg /y3max is greater than 1, set it equal to 1.

i. Knowing Wp, Ksp, Km, and Kh the effective diameter of the pile group can be determined from the following equation:



$$D_{pg}^* = K_m K_{sp} K_h W_p$$

Where

 $D^*pg \equiv$ effective diameter/width of the pile group (i.e. the diameter/width of a water surface penetrating single circular pile that will experience the same local scour depth as the pile group).

k. Compute the Shape Factor, Ks, for the pile group (which is a function of the skew angle, $\boldsymbol{\alpha}$) using the following equation:

$$\mathbf{K}_{s} = \begin{cases} 1.0 & \text{for circular individual piles} \\ 0.9 + 0.66 & \left| \alpha - \frac{\pi}{4} \right|^{4} & \text{for square piles} \end{cases}$$

l. Knowing D*pg, D50, y3, V3, Vc, and Ks, the scour depth created by the pile group, ys (pg), can be estimated using single pile, equilibrium scour depth equations.

2-3 Bridges scour evolution Program.

GCI Incorporated has been involved in the Florida Department of Transportation (FDOT) Bridge Scour Evaluation and Remediation (Maintenance) program throughout the State of Florida since 1993. The purpose of this program has been to evaluate bridges over tidal and non-tidal waterways with scour able beds to determine the risk of failure from scour. A multidiscipline team of engineers was assembled in each District of the Florida Department of Transportation (FDOT) to ensure proper evaluation. Experts in bridge hydraulics/hydrology, structures and geotechnical worked together to come to a consensus on potential bridge scour-related problems and possible corrective actions. Scour is a complicated phenomenon and one can safely say that predicting scour cannot be established by testing and calculations only. Therefore, GCI has led the way in advancing an approach based on "Sound Engineering Judgment" in addressing scour impact on existing and new bridges over waterways. GCI advocated early in the program that the assessment of reasonableness of the predicted scour depth should be established jointly by the hydraulic, geotechnical and structural engineers (consultants and FDOT staff) working on the project.

3-Estimate pile groups scour by FDOT Program.

By study some factors affecting the scour around pile groups using FDOT program, which include:

1 – The effect of ratio S/D

Where:

S= is the pile spacing (from center line to center line of pile)

D = is the diameter or width of a single pile.

2- The shape of the pile (rectangular or circular).

The results getting from (FDOT) program were comparison with laboratory results.

3-1The effect of pile shape (rectangular or circular)

Installed all the variables and the experience of the program so that the shape of the pile rectangular. Erosion was recalculated with the same variables, but the form of the pile a circular. It was noted that depth of erosion is greater in the case of the pile rectangular as shown in table (1).

Table (1) Scour depth in case of rectangular pile and circular pile

Rectangular pi	le	Circular pile			
Effective	ctive Scour		Scour		
			500 ai		
diameter (m)	depth(m)	diameter (m)	depth(m)		
4.35	3.75	5.10	4.62		
Conjulie Plan .	The local occur is	\$75 oro			
Longer and the set of	The structures D* in	4 31 (m)			
Mate: •					
Figure and Section and	Column Data	Complex Pier data Pile Cap Data	Rie Group Dep		
Fiber and Settiment	825 B _{at} (m)	Pier Cup Deta	Mill Group Data		
Sediment Dentity (ngrow)	(BO) An ini	3_(m)			
Mater Temp. (C*)	225 H _{att} (m)	17.010	b (ro) 1		
Salinity (pp)	4 8.00	H _{ph} (m)	(s, (rt) 2		
Stever Angle	1 1,00	Shape	5		
5. (H)	3 944	Mi	Pie Cicalar		
Vanio	1		W _a (m) 2 (w _a (m) 5		
	R No Column	St No File Day	T' No Pile Group		
Reset Debulm	1	Calculate Case 1 Scour			
1200101010000					
		Cave 2 Cave 3			
Typical Saletty Values	Case 5 Pile Cap Above Bett P		, , h ,		
The average scean raining to 35 ppt 1 Particle were latered about 32 and 3					
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Rematlen cause the senatory. The end					
the Black Seals to dialectly rise tur seemon saleity is only 10.001	No.				
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Figure (4) Calculation scour depth using FDOT program in case of circular pile



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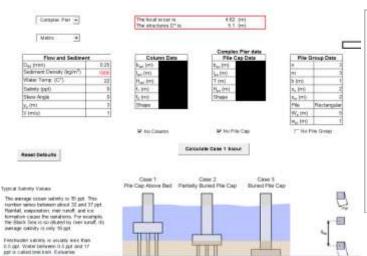


Figure (5) Calculation scour depth using FDOT program in case of rectangular pile

As shown in figures (3) & (4) and table (1) the scour depth for square pile is (1.25) times the scour depth for the circular pile.

3-2 Study the effect of ratio S/D

Choosing the pile spacing depends on several factors, the most costs of foundation, nature of the soil and the behavior of piles in the group. Should be the optimal distance between the intra-Piles be from 2.5 to 3.5 times diameter pile. Egyptian code of practice specifics the minimum distance between any two adjacent piles by three times pile diameter (3D) in the case of the use of friction piles, while not less than two and half diameter of the pile (2.5D) in the case of using of bearing piles. In some special cases this distance was taken twice the diameter of the pile (2D). The largest distance between the piles, it does not specify in the codes. Preferably be from eight to ten times the pile diameter (8:10D).

For circular pile groups, (S/D) were gradually changed considering S/D = 2, 3..., 6, where the FDOT program does not support ratio (S/D) less than 2 and not more than 6. Where:

S= is the pile spacing (from center line to center line of pile)

D = is the diameter or width of a single pile.

h= the water depth in up stream

Table (2) effect of ratio (S/D) on scour depth

S/D	Circular pile					
	Scour depth(m)	Effective diameter (m)				
2	0.95	0.79				
3	0.91	0.74				
4	0.73	0.56				
5	0.73	0.56				
6	0.73	0.56				

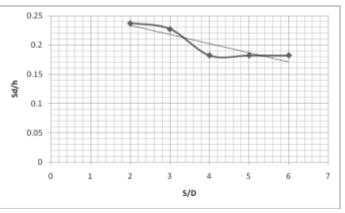


Figure (5) Effect of ratio (S/D) on scour depth

As shown in table (2) and figure (5 it was noted that, the high of the ratio S / D decreases the scour depth.

Where:

Sd= Scour depth,

h= the water depth in upstream.

4-Experimental work

4-1-The flume

The experimental work has been carried out using a model of wooden piles placed on glass-side rectangular flume in faculty of engineering, Al Azher University. The flume is 6 m long, 0.30 m wide and 0.30 m deep. The flume had a working section in the form of a 0.08 m depth recess below its bed that was filled with movable sands. Details of the flume and experimental setup are shown in Figure (6).



Figure (6) Experimental set-up



The flume is proved with high power and two pumps to re circulate the water throw pipes from the front tank to the rear tank to feed the flume with the required flow discharge. Figure (7) shows the pipes, the valves and the pumps.





Figure (7) the pipes, the valves and the pumps.

4-2Procedure of work

The following steps are made in conducting the tests:

1-The first the wooden piles are fixed (according to case study) carefully in the flume.

2-The top level of sand bed is smoothed and measured and the sand bed level is recorded.

3-The pump is operated to fill the front tank of the flume.

4-The flume pump are operated to feed the flume with required flow discharge.

5-After 6 hours, the flume pump is stopped.

6-The maximum scour depth and dimension of scour hole are measured.

7-The sand bed level is recorded(using a plate glass perforated every 5 cm in two directions and laser) every 5 cm cross direction and 5cm in longitudinal direction till the end of scour effects, cut and fill, to have a complete mesh.

8-The previous steps are repeated with every case study.

9- The number of runs are ten runs

4-2-1- Study the effect the ratio S/D

A study of the local scour at piles groups was experimentally investigated. The case of six piles having the same diameter aligned with the flow direction in two rows altering the separation distance between the centerline of the three piles was established. The aim of the experimental is the investigation of the preferable separation distance between three piles to reduce local scour around them to its minimum value. (S/D) was gradually changed considering S/D = 1, 2, 3 and 4Where S is the pile Spacing, D is the diameter or width of a single pile, as shown in figures (8)&(9). The test conditions together with the experimental results are listed in Table (3). In this table, D is pile diameter, h is upstream water depth, d50 is the mean particle diameter, S/D is pile spacing, and Sd is maximum scour depth.

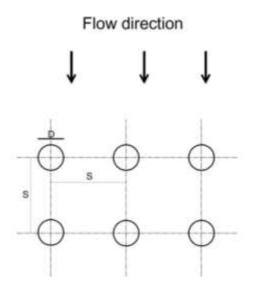


Figure (8) the effect the ratio S/D for circular piles



Figure (9) The wooden piles in the flume

Table (3): Summary of test conditions and experimental results

Exp. No.	D	d50 (mm)	h	S/D	Sd(cm)
No.	(cm)		(cm)		
1	2	0.25	15	1	1.5
2	2	0.25	15	2	1.0
3	2	0.25	15	3	0.75
4	2	0.25	15	4	0.50



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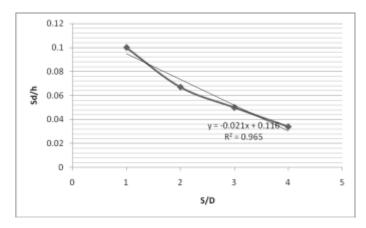


Figure (10) the relation between S/D and scour depth resulting from experimental work

As shown in table (3) and figure (10) it was noted that, the higher the ratio S / D decreased the scour depth.

4-2-2- Study the effect of pile shape

A comparison between the two groups of piles (six piles) consist of two rows and three columns in flow direction (three piles in row), once square shape and other a circular shape as shown in figure (11)

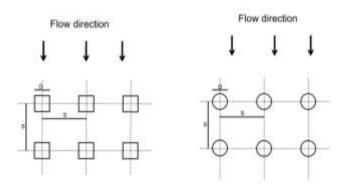


Figure (11) the effect of pile shape

The test conditions together with the experimental results are listed in Table (3).

Table (4): Summary of test conditions and experimental results

Exp. No.	Туре	Pile shape	D (cm)	d50 (mm)	h (cm)	S/D	Sd(cm)
1	3*2	circular	2	0.25	15	2	1.0
1	3*2	square	2	0.25	15	2	1.4



Figure (12) the bed profile around square piles after scour test



Figure (13) the bed profile around square piles before scour test

As shown in Figures (12) & (13) and table (4), the scour depth for square pile is (1.40) times the scour depth for the circular pile. The maximum scour occurs around front piles and the diameter of scour holes nearly equal to 3 times the pile diameter.

4-2-3 Study of the effect of the arrangement

Acomparsion between the two groups of piles, the first group consist of two rows (three piles in row). The second group consist of the same number of piles and one pile in front and other in rear to take the streamlined shape . As shown in figures (14) & (15).

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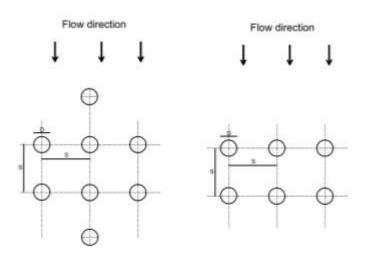


Figure (14) the effect of streamlined shape for piles

Table (5): Summary of test conditions and experimental
results

Exp.	Туре	Pile	D	d50	h	S/D	Sd(cm)
No.		shape	(cm)	(mm)	(cm)		
1	3*2	circular	2	0.25	15	2	1.25
1	3*2+1(in front)+1 (in rear)	circular	2	0.25	15	2	0.75



Figure (15) the effect of streamlined shape for piles

According to laboratory results it was noted that:

1- The results of pile group arrangement indicate that $(3 \times 2+1(in \text{ front}) +1 (in \text{ rear}))$ is better than 3x2 arrangement with the same spacing.

2- The maximum scour depth occurs at the pile front side.

3-In case of streamlined shape, the scour depth reduced a third.

5- A comparison between experimental results and FDOT program results

1-As shown in figures (16) and table (6), it can be seen that the scour depth decreases as the spacing between the pile increases. A slight increase in scour depth was noted as S/D increased from one (piles touching) to two; otherwise the scour depth gradually decreased as the spacing increased. The results are consistent with the findings fromFDOTprogram.

Table (6): The scour depth resulting from experimental
work and FDOT program results

	Туре	D	h	S/D	Sd(cm) from	Sd(cm)
Exp.		(cm)	(cm)		experimental	FDOT
No.					work	program
1	3*2	2	15	1	1.5	2.50
2	3*2	2	15	2	1.10	2
3	3*2	2	15	3	0.70	2
4	3*2	2	15	4	0.50	1

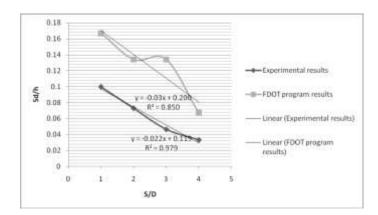


Figure (16) the relation between S/D and scour depth resulting from experimental work and FDOT program results

2-The FDOT program results showed that, the scour depth for square pile is (1.25) times the scour depth for the circular pile, where experimental results shown, the scour depth for square pile is (1.40) times the scour depth for the circular pile.

3-In experimental work, the maximum scour occurs around front piles, where in FDOT program the places of maximum scour not mentioned.

4- In experimental work, as the piles closer the sides, the scour depth increases. Where in FDOT program the effect of the piles proximity from river side unclear

5- FDOT program supports only arrangement piles in rows and columns.

5-CONCLUSIONS

1-For pile groups arranged in multiple columns and rows, the scour depth decreases as the spacing between piles increase.

2-The scour depth for circular piles is less than the scour depth for the square piles by 29.5% nearly, so a square pile must be avoided.

3- In case of streamlined shape, the scour depth reduced a third.

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