1. INTRODUCTION

Dry screw vacuum pumps work with two screw rotors rotating in opposite directions. This traps the medium to be pumped between the cylinder and the screw chambers and transports it to the gas discharge. The advanced screw design results in lower electric energy consumption than the standard screw designs. It also results in a lesser heat load of the compressed gas. Screw Vacuum pumps are such designed to ensure that the differential temperature remains in-tact and is optimized for operation. This internal compression is attained by reducing the thread pitch in the direction of the outlet. The gaps between the housing and the rotors, as well as between the rotors relative to one another, determine the ultimate pressure which a screw pump can achieve.

In this work the Lateral critical speeds of the shaft is evaluated by plotting Campbell diagrams using ANSYS® software. Later, Forced response analysis was conducted to know the peak amplitudes at the critical speeds. API 610 standards are followed in designing and analyzing the rotor.

2. PROBLEM DEFINITION

The main objective of this project is to evaluate the rotor dynamics of the drive shaft of Dry Screw Vacuum pump. To meet the higher target of power, vacuum pumps have to run at higher speeds. Because of this reason the pump shaft will be subjected to higher lateral and torsion vibrations due to gyroscopic effect and centrifugal force of the rotating elements mounted on the shaft. The resonance is produced when the rotor natural frequency coincides with operating frequency and the deflection of the rotor will be maximum. To avoid resonance condition, it is necessary to evaluate the stiffness and damping coefficients of the shaft. In this work the Lateral critical speeds of the shaft is evaluated by plotting Campbell diagrams using ANSYS® software. Later, Forced response analysis was conducted to know the peak amplitudes at the critical speeds. API 610 standards are followed in designing and analyzing the rotor.

The results obtained are evaluated on basis of the vacuum pump minimum required standards, then the design considerations are taken into account so as to optimize the shaft in order to minimize the various forces acting on the shaft so as to increase the life time and efficiency of the shaft. The modal analysis and harmonic analysis of the shaft with initial design considerations and optimized design considerations are compared and conclusions are made so that the errors can be eliminated.

3. METHODOLOGY

The methodology followed in this project is as follows:

1. Initially design of individual parts of the dry screw vacuum pump assembly is done by reverse engineering using NX-CAD software.

2. The parts are assembled in assembly module of NX-CAD to make final assembly of the dry screw vacuum pump.

3. The rotor shaft of the assembly is converted into parasolid format and imported into Ansys work bench to do finite element analysis.

4. Modal analysis is done on the rotor shaft. From the analysis the Campbell diagram is plotted. The Campbell diagram gives the relationship between damped natural frequencies and rotational speed.
5. From the modal analysis the critical speeds are identified and the stiffness and damping coefficients of the shaft is evaluated.

6. Harmonic analysis is carried out on the shaft near the critical speeds identified in the modal analysis. From the harmonic analysis deflections and stresses at the critical speeds, frequency responses of the bearings, point mass are calculated and documented.

7. Design modification is done on the shaft by adjusting the unbalanced mass to avoid higher lateral and torsion vibrations due to gyroscopic effect.

8. Modal analysis is done on the modified rotor shaft. From the analysis the Campbell diagram is plotted.

9. Harmonic analysis is carried out on the modifies shaft near the critical speeds identified in the modal analysis.

10. Conclusion is made by comparing the results from original and modified shaft model.

4. MODELING OF ROTOR SHAFT

The dimensional data for the dry screw vacuum pump is taken from the premier industries and designed in the NX-CAD. The model of the whole assembly created by taking the dimension from premier industry.

4.1 Shaft Assembly

The model of the shaft assembly developed in the NX-CAD. The motor is employed to power the driving shaft which intern driven the second shaft called driven shaft. A helical shape is mounted on the shaft to perform compression operations. Below figure shows the development of the helix. The specifications of the helix are

- Pitch is 55mm.
- Number of turns = 7
- Radius = 50.5mm

The use of sweep command creates the helical structure on the shaft with the given rectangular profile. The drafting of driving shaft with spiral spring is shown below. Projections, detailed views and dimensions are mentioned in the drafting.

4.2 Bearing Housing

Bearing house assembly mainly consists of three parts

1. Bearing housing
2. Bearing cover
3. Bearing house fitting
4. Cap

Bearing housing is used to provide support for rotating shafts.
4.3 Body Assembly

Body assembly consists of 3 parts

1. Bottom plate
2. Top plate
3. Body

Body acts as a support for whole assembly of vacuum pump. Top and bottom plates are modeled separately and then assembled to the body.

4.4 Drive Cover Plate Assembly

Drive cover plate mainly consists of 5 parts.

1. Injector cap
2. Injector
3. Oil indicator
4. Drive cover plate
5. Drive side cover

The injector cap is used to inject the oil. Oil indicator indicates the level of oil. The drive cover acts as a cover for whole drive assembly.

5. MODAL ANALYSIS OF THE DRIVING SHAFT

In general modal analysis is performed to find the natural frequencies of the body. The frequency at which the body vibrates without the application of the external forces is called natural frequency. On performing modal analysis a desired set of natural frequencies will be obtained for the given conditions. A Campbell diagram plot illustrates a system's response spectrum as a function of its oscillation regime. In rotor dynamical systems, the Eigen frequencies often depend on the rotation rates due to the induced gyroscopic effects or variable hydrodynamic conditions in fluid bearings.

5.1 Geometric Model

The geometric model of the shaft was developed in the NX-CAD and converted into the parasolid file and then imported into the Ansys. In order to perform modal analysis of a model w.r.t rotor dynamics, the required model should be axisymmetric about the rotational axis. The shaft model used in this analysis is unsymmetrical about the axis because of the helical geometry.

Therefore the model is converted to axis symmetrical by

- Finding the properties like mass, center of mass and moment of inertia along the principal axis of the helix.
- Later, the helical volume is deleted and point mass is created along the center of mass of the helix with the properties of mass and moment of inertia.
- Now, this point mass is coupled with the remaining part of the structure there by converting it into an axis symmetrical model.
The final axisymmetric model after removing the helical volume with the point mass of the helical volume added at A as shown in the below figure.

![Final geometrical model used for modal analysis](image1)

**Figure -5.1:** Final geometrical model used for modal analysis

### 5.2 Material Properties

The shaft used for the analysis are assigned with the steel material properties with the following values:

- Young’s modulus: 200Gpa
- Poisson’s ratio: 0.3
- Density: 7850kg/m³
- Yield stress: 248MPa

### 5.3 Finite Element Model

All the entities of shaft is meshed with solid 186 element type. Structural mass 21 element type is used to apply the mass of the helical volume at the center of gravity calculated and spring 214 element type is used to model the bearing stiffness near the bearing locations. The element description and element shape of solid 180, mass 21 and spring 214 elements are explained below.

**Solid 186:** Solid186 is a higher order 3-D, 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element supports plasticity, hyper elasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyper elastic materials.

**MASS21** is a point element having up to six degrees of freedom: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes. A different mass and rotary inertia may be assigned to each coordinate direction.

**2D Spring Damper Bearing 214:** COMBI214 has longitudinal as well as cross-coupling capability in 2-D applications. It is a tension-compression element with up to two degrees of freedom at each node: translations in any two nodal directions (x, y, or z). COMBI214 has two nodes plus one optional orientation node. No bending or torsion is considered.

The spring-damper element has no mass. Masses can be added by using the appropriate mass element (MASS21). The spring or the damping capability may be removed from the element.

A total of 6014 elements and 29779 nodes have been created in the meshing. The meshed model figure is shown below.

![Finite element model of the driving shaft](image2)

**Figure -5.3:** Finite element model of the driving shaft

### 5.4 Boundary Conditions

As the maximum operating speed of the rotor shaft is 10,000 rpm, modal analysis has been carried out with load steps i.e 0 rpm and 20,000 rpm. The boundary conditions applied on the shaft for modal analysis are follows.

- Bearing elements are created at their respective positions on the shaft with bearing 214 elements and with the stiffness of 1e4 N/mm.

- As the mass is unsymmetrical along the axis, it is converted to a point mass located at the position of the center of mass. There are two point masses created i.e. at the location of the center of mass of the gear and at the location of the center of mass of the helical part of the shaft.

- In order to obtain the Campbell diagram, there should be multiple load steps. Thus rotational velocity is applied as two load steps i.e. as 0 rpm and 20,000 rpm. (0 rad/s and 2095 rad/s). The analysis is carried out for the rotational velocity of 0-20000 rpm.

### 5.5 Results of Modal Analysis

The modal analysis was carried out from 0 - 20000 rpm. From the analysis the natural frequencies and their corresponding mode shapes are calculated. Campbell diagram is also plotted to calculate the critical speeds of the
shaft. The natural frequencies obtained from the modal analysis of the shaft for different load steps is shown in the below table. As the natural frequencies depend on the geometry and the boundary conditions applied, it is found that the natural frequencies obtained for both the speeds are approximate to each other.

![Figure 5.5: Natural frequencies for different rotation velocity](image)

<table>
<thead>
<tr>
<th>Mode No</th>
<th>Frequencies at 0 rpm</th>
<th>Frequencies at 20000 rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>122.85</td>
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<td>123.14</td>
<td>123.42</td>
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<td>261.44</td>
<td>256.54</td>
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<td>458.89</td>
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<td>637.9</td>
<td>624.17</td>
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<td>638.14</td>
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</tr>
<tr>
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</tr>
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</tr>
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<td>17</td>
<td>2224.8</td>
<td>2266.6</td>
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<td>18</td>
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<td>19</td>
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<td>2969.4</td>
</tr>
<tr>
<td>20</td>
<td>3032.6</td>
<td>3032.6</td>
</tr>
</tbody>
</table>

Table -5.5: Natural frequencies for different rotation velocity

5.6 Campbell Diagram

After completion of modal analysis with several rotational velocity load steps, Campbell diagram analysis is performed. The analysis gives the following:

- Visualize the evolution of the frequencies with the rotational velocity
- Check the stability and whirl of each mode
- Determine the critical speeds.
- Determine the stability threshold

5.7 Mode Shapes

5.7.1 Mode shapes for 0 rpm rotation velocity:

The mode shapes of vibrations obtained for the above mentioned critical speeds at 0 rpm are shown below. The mode shape of vibration for the critical speeds of 7381.4 rpm at 123.12 Hz is shown in below figure.
5.7.1 Mode shapes for 0 rpm rotation velocity:
The mode shapes of vibrations obtained for the above mentioned critical speeds at 0 rpm are shown below. The mode shape of vibration for the critical speeds of 7381.4 rpm at 122.85 Hz is shown in below figure

Figure 5.7.1.2: Mode shape at critical speed of 7395 rpm at 123.14 Hz for 0 rpm

Figure 5.7.1.3: Mode shape at critical speed of 15459 rpm at 261.44 Hz for 0 rpm

Figure 5.7.1.4: Mode shape at critical speed of 15922 rpm at 261.48 Hz for 0 rpm

5.7.2 Mode shapes for 20000 rpm rotation velocity:
The mode shapes of vibrations obtained for the above mentioned critical speeds at 20000 rpm are shown below. The mode shape of vibration for the critical speeds of 7381.4 rpm at 122.85 Hz is shown in below figure

Figure 5.7.2.1: Mode shape at critical speed of 7381 rpm at 122.85 Hz for 20000 rpm

Figure 5.7.2.2: Mode shape at critical speed of 7395 rpm at 123.42 Hz for 20000 rpm

Figure 5.7.2.3: Mode shape at critical speed of 15459 rpm at 256.54 Hz for 20000 rpm

Figure 5.7.2.4: Mode shape at critical speed of 15922 rpm at 266.37 Hz for 20000 rpm
From the mode shapes of vibration, it is found that there only two critical speeds 7381 and 7394 rpm occurring in the operation range of 10000 rpm. The critical speeds are occurring at 123.12 and 123.14 Hz for rotation velocity of 0 rpm and 122.85 and 123.42 Hz for rotation velocity of 20000 rpm. These vibration modes indicate that mass unbalance is present in the appreciable range and has to be attended which otherwise will cause the damage of the rotor shaft and bearing operating at higher speeds. Harmonic response analysis is carried out to check the effect of mass imbalance at the critical speeds. Deflections and stresses are plotted from the harmonic analysis. The summary of the vibration modes at critical speeds is given in the below table.

Table 5.7: Summary of vibration modes at critical speeds

<table>
<thead>
<tr>
<th>S.No</th>
<th>Frequency</th>
<th>Whirl Direction</th>
<th>Mode Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>123.12 Hz</td>
<td>Backward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>2</td>
<td>123.14 Hz</td>
<td>Forward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>3</td>
<td>261.44 Hz</td>
<td>Backward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>4</td>
<td>261.48 Hz</td>
<td>Forward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>5</td>
<td>122.85 Hz</td>
<td>Backward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>6</td>
<td>123.42 Hz</td>
<td>Forward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>7</td>
<td>256.54 Hz</td>
<td>Backward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>8</td>
<td>266.37 Hz</td>
<td>Forward Whirl</td>
<td>Stable</td>
</tr>
</tbody>
</table>

6.1 Boundary Conditions

The boundary conditions and loading used for doing harmonic analysis are as follows:

- Bearing elements are created at their respective positions on the shaft with bearing 214 elements and with the stiffness of 14 N/m.
- As the mass is unsymmetrical along the axis, it is converted to a point mass located at the position of the center of mass. There are two point masses created i.e. at the location of the center of mass of the gear and at the location of the center of mass of the helical part of the shaft.
- An unbalanced mass of 10.272 Kg i.e. mass of the eccentric is applied in the form of the rotating force along the axis of the shaft.

6.2 Results of Harmonic analysis

As the critical speeds are occurring at 123.12 and 123.14 Hz for rotation velocity of 0 rpm and 122.85 and 123.42 Hz for rotation velocity of 20000 rpm, harmonic analysis is carried out in the frequency range of 120 Hz to 124 Hz and the substeps are defined to exactly extract the results at the critical frequencies. The boundary conditions applied on the shaft for harmonic analysis are follows.

- Deflection at 123.12 Hz at 0 rpm rotational velocity: A maximum deflection of 12.014 mm is obtained for the shaft at 123.12 Hz operating at a rotational velocity of 0 rpm and occurring at the critical speed of 7381.4 rpm.
- Deflection at 123.14 Hz at 0 rpm rotational velocity: A maximum deflection of 12.65 mm is obtained for the shaft at 123.14 Hz operating at a rotational velocity of 0 rpm and occurring at the critical speed of 7394 rpm.
- Deflection at 122.85 Hz at 20000 rpm rotational velocity: A maximum deflection of 3.42 mm is obtained for the shaft at 122.85 Hz operating at a rotational velocity of 20000 rpm and occurring at the critical speed of 7381.4 rpm.
- Deflection at 123.42 Hz at 20000 rpm rotational velocity: A maximum deflection of 21.56 mm is obtained for the shaft at 123.42 Hz operating at a rotational velocity of 20000 rpm and occurring at the critical speed of 7394 rpm.
The material property of steel used for the shaft has a yield strength of 248 Mpa. By plotting the VonMises stress we can determine the factor of safety of the shaft at different critical speeds. From the analysis VonMises stress at 123.12 and 123.14 Hz for rotation velocity of 0 rpm and 122.85 and 123.42 Hz for rotation velocity of 20000 rpm where critical speeds are occurring are:

- **VonMises stress at 123.12 Hz at 0 rpm rotational velocity:** A VonMises stress of 1303 Mpa is observed on the edges of the shaft at 123.12 Hz operating at a rotational velocity of 0 rpm and occurring at the critical speed of 7381.4 rpm. The VonMises stress obtained is more than the yield strength of the material (248 Mpa). The FOS for the shaft is 248/1302=0.19 which is less than 1. So it can be concluded that the shaft is not safe for the operating at a speed of 7381.4 rpm for a rotational speed of 0 rpm.

- **VonMises stress at 123.14 Hz at 0 rpm rotational velocity:** A VonMises stress of 1372 Mpa is observed on the edges of the shaft at 123.14 Hz operating at a rotational velocity of 0 rpm and occurring at the critical speed of 7394 rpm. The VonMises stress obtained is more than the yield strength of the material (248 Mpa). The FOS for the shaft is 248/1372=0.18 which is less than 1. So it can be concluded that the shaft is not safe for the operating at a speed of 7394 rpm for a rotational speed of 0 rpm.

- **VonMises stress at 122.85 Hz at 20000 rpm rotational velocity:** A VonMises stress of 361 Mpa is observed on the edges of the shaft at 122.85 Hz operating at a rotational velocity of 20000 rpm and occurring at the critical speed of 7381.4 rpm. The VonMises stress obtained is more than the yield strength of the material (248 Mpa). The FOS for the shaft is 248/361=0.68 which is less than 1. So it can be concluded that the shaft is not safe for the operating at a speed of 7381.4 rpm for a rotational speed of 20000 rpm.

- **VonMises stress at 123.42 Hz at 20000 rpm rotational velocity:** A VonMises stress of 2348 Mpa is observed on the edges of the shaft at 123.42 Hz operating at a rotational velocity of 20000 rpm and occurring at the critical speed of 7394 rpm. The VonMises stress obtained is more than the yield strength of the material (248 Mpa). The FOS for the shaft is 248/2348=0.10 which is less than 1. So it can be concluded that the shaft is not safe for the operating at a speed of 7394 rpm for a rotational speed of 20000 rpm.

From the harmonic analysis deflections and VonMises stress at 123.12 and 123.14 Hz for rotation velocity of 0 rpm and 122.85 and 123.42 Hz for rotation velocity of 20000 rpm where critical speeds are occurring are calculated and found that the FOS at all the critical speeds is less than 1. Therefore it is concluded that the mass unbalance present in the shaft is causing a high damage when it is operated at 7381 and 7391 rpm. Thus there comes the need for the modification and optimization of the shaft. Different iterations were made on the shaft by changing the helix geometry by shifting the mass moment of inertia towards centre of gravity of shaft, thereby reduces the gyroscopic effect. Thus the unsymmetrical part of the shaft is modified as shown in the below figure and the rotor dynamic analysis is performed on the shaft to validate the design and its results. The optimized model of the shaft is shown in the following figure.

**Figure - 6.2:** 3D model of modified shaft in isometric view

### 7. MODAL ANALYSIS OF THE MODIFIED DRIVING SHAFT

#### 7.1 Geometric Model

The geometric model of the modified shaft was developed in the NX-CAD and converted into the parasolid file and then imported into the Ansys. In order to perform modal analysis of a model w.r.t rotor dynamics, the required model should be axisymmetric about the rotational axis. The modified shaft model used in this analysis is unsymmetrical about the axis because of the helical geometry present in the model. Therefore the model is converted to axis symmetrical by:

- Finding the properties like mass, center of mass and moment of inertia along the principal axis of the helix.
- Later, the helical volume is deleted and point mass is created along the center of mass of the helix with the properties of mass and moment of inertia.
- Now, this point mass is coupled with the remaining part of the structure there by converting it into an axis symmetrical model.

The final axisymmetric model after removing the helical volume with the point mass of the helical volume added at A as shown in the below figure.
7.2 - Material Properties:

The shaft used for the analysis is assigned with the steel material properties with the following values:

- Young’s modulus: 200Gpa
- Poisson’s ratio: 0.3
- Density: 7850kg/m³
- Yield stress: 248MPa

7.3 - Finite Element Model:

All the entities of modified shaft is meshed with solid 186 element type. Structural mass 21 element type is used to apply the mass of the helical volume at the center of gravity calculated and spring 214 element type is used to model the bearing stiffness near the bearing locations. A total of 6524 elements and 30874 nodes have been created in the meshing.

7.4 - Boundary Conditions:

As the maximum operating speed of the rotor shaft is 10,000 rpm, modal analysis has been carried out with load steps i.e. 0 rpm and 20,000 rpm. The boundary conditions applied on the shaft for modal analysis are follows.

- Bearing elements are created at their respective positions on the shaft with bearing 214 elements and with the stiffness of 1.4 N/mm.
- As the mass is unsymmetrical along the axis, it is converted to a point mass located at the position of the center of mass. There are two point masses created i.e. at the location of the center of mass of the gear and at the location of the center of mass of the helical part of the shaft.
- In order to obtain the Campbell diagram, there should be multiple load steps. Thus rotational velocity is applied as two load steps i.e. as 0 rpm and 20,000 rpm (0 rad/s and 2095 rad/s). The analysis is carried out for the rotational velocity of 0-20000 rpm.

7.5 - Results of Modal Analysis

The modal analysis was carried out from 0 - 20000 rpm. From the analysis the natural frequencies and their corresponding mode shapes are calculated. Campbell diagram is also plotted to calculate the critical speeds of the shaft. The natural frequencies obtained from the modal analysis of the shaft for different load steps is shown in the below table. As the natural frequencies depend on the geometry and the boundary conditions applied, it is found that the natural frequencies obtained for both the speeds are approximate to each other.

<table>
<thead>
<tr>
<th>Mode No</th>
<th>Frequencies at 0 rpm</th>
<th>Frequencies at 20000 rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5534e-003</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>2.8659e-003</td>
</tr>
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<td>4</td>
<td>8.1175e-003</td>
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</tr>
<tr>
<td>7</td>
<td>291.12</td>
<td>283.06</td>
</tr>
<tr>
<td>8</td>
<td>291.25</td>
<td>299.4</td>
</tr>
<tr>
<td>9</td>
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<td>20</td>
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<td>3010.8</td>
</tr>
</tbody>
</table>

Table -7.5: Natural frequencies for different rotation velocity for modified shaft
7.6 · Campbell Diagram

After completion of modal analysis with several rotational velocity load steps, Campbell diagram analysis is performed. The analysis gives the following:

- Visualize the evolution of the frequencies with the rotational velocity
- Check the stability and whirl of each mode
- Determine the critical speeds.
- Determine the stability threshold

![Campbell Diagram](image)

**Figure - 7.6: Campbell diagram for modified shaft**

From the above Campbell diagram it can be observed that there are 2 critical speeds at 17055 and 17913 respectively at frequencies of 291.12 and 299.4 Hz respectively. The critical speeds obtained for the shaft for different rotation velocities is plotted in the below table.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Whirl Direction</th>
<th>Mode Stability</th>
<th>Critical Speed in rpm</th>
<th>Freq at 0 rpm</th>
<th>Freq at 20000 rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BW</td>
<td>Stable</td>
<td>17055</td>
<td>291.12 Hz</td>
<td>283.06 Hz</td>
</tr>
<tr>
<td>2</td>
<td>FW</td>
<td>Stable</td>
<td>17913</td>
<td>291.25 Hz</td>
<td>299.4 Hz</td>
</tr>
</tbody>
</table>

**Table - 7.6: Critical speeds for different rotation velocity for modified shaft**

7.7 · Mode Shapes

**Mode shapes for 0 rpm rotation velocity:**

The mode shapes of vibrations obtained for the above mentioned critical speeds at 0 rpm are shown below. The mode shape of vibration for the critical speeds of 17055 rpm at 291.12 Hz is shown in below figure.

![Mode shape at critical speed of 17055 rpm at 291.12 Hz for 0 rpm](image)

**Figure 7.7.1: Mode shape at critical speed of 17055 rpm at 291.12 Hz for 0 rpm**

![Mode shape at critical speed of 17913 rpm at 291.25 Hz for 0 rpm](image)

**Figure 7.7.2: Mode shape at critical speed of 17913 rpm at 291.25 Hz for 0 rpm**

**Mode shapes for 20000 rpm rotation velocity:**

The mode shapes of vibrations obtained for the above mentioned critical speeds at 20000 rpm are shown below. The mode shape of vibration for the critical speeds of 17055 rpm at 283.06 Hz is shown in below figure.

![Mode shape at critical speed of 17055 rpm at 283.06 Hz for 20000 rpm](image)

**Figure 7.7.3: Mode shape at critical speed of 17055 rpm at 283.06 Hz for 20000 rpm**

![Mode shape at critical speed of 17913 rpm at 299.4 Hz for 20000 rpm](image)

**Figure 7.7.4: Mode shape at critical speed of 17913 rpm at 299.4 Hz for 20000 rpm**
From the mode shapes of vibration, it is found that there no critical speeds occurring in the operation range of 10000 rpm. The critical speeds are occurring at 291.12 and 291.25 Hz for rotation velocity of 0 rpm and 283.09 and 299.4 Hz for rotation velocity of 20000 rpm. These critical speeds are occurring at 17055 rpm and 17913 rpm which are not in the operational range. However Harmonic response analysis is carried out to check the effect of mass imbalance at the critical speeds. Deflections and stresses are plotted from the harmonic analysis. The summary of the vibration modes at critical speeds is given in the below table.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Frequency</th>
<th>Whirl Direction</th>
<th>Mode Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>283.09</td>
<td>Backward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>2</td>
<td>291.12 Hz</td>
<td>Backward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>3</td>
<td>291.25 Hz</td>
<td>Forward Whirl</td>
<td>Stable</td>
</tr>
<tr>
<td>4</td>
<td>299.4 Hz</td>
<td>Forward Whirl</td>
<td>Stable</td>
</tr>
</tbody>
</table>

Table - 7.7: Summary of vibration modes at critical speeds for modified shaft

8. HARMONIC ANALYSIS OF THE MODIFIED DRIVING SHAFT

Harmonic analysis was carried on the modified drive shaft after obtaining the critical speeds from the modal analysis. Harmonic analysis is performed to plot the frequency responses of the bearings, point mass and also to plot the deflections and stresses of the shaft for the critical speed obtained in the modal analysis. To perform a harmonic analysis of an unbalanced excitation, the effect of the unbalanced mass is represented by forces in the two directions perpendicular to the spinning axis. The forces are applied on a node located on the axis of rotation. The SYNCHRO command is used to specify that the frequency of excitation is synchronous with the rotational velocity.

8.1 - Boundary Conditions

The boundary conditions and loading used for doing harmonic analysis are as follows:

- Bearing elements are created at their respective positions on the shaft with bearing 214 elements and with the stiffness of 1e4 N/mm.
- As the mass is unsymmetrical along the axis, it is converted to a point mass located at the position of the center of mass. There are two point masses created i.e. at the location of the center of mass of the gear and at the location of the center of mass of the helical part of the shaft.
- An unbalanced mass of 10.272 Kgs i.e. mass of the eccentric is applied in the form of the rotating force along the axis of the shaft.

8.2 Results of Harmonic analysis

From the analysis deflections at 291.12 and 291.25 Hz for rotation velocity of 0 rpm and 283.06 and 299.4 Hz for rotation velocity of 20000 rpm where critical speeds are occurring are plotted.

Deflection at 291.12 Hz at 0 rpm rotational velocity: A maximum deflection of 2.75 mm is obtained for the modified shaft at 291.12 Hz operating at a rotational velocity of 0 rpm and occurring at the critical speed of 17055 rpm.

Deflection at 291.25 Hz at 0 rpm rotational velocity: A maximum deflection of 1.62 mm is obtained for the shaft at 291.25 Hz operating at a rotational velocity of 0 rpm and occurring at the critical speed of 17913 rpm.
Deflection at 283.06 Hz at 20000 rpm rotational velocity: A maximum deflection of 6.58 mm is obtained for the shaft at 283.06 Hz operating at a rotational velocity of 2000 rpm and occurring at the critical speed of 17055 rpm.

Deflection at 299.4 Hz at 20000 rpm rotational velocity: A maximum deflection of 78 mm is obtained for the shaft at 299.4 Hz operating at a rotational velocity of 2000 rpm and occurring at the critical speed of 17913 rpm.

The material property of steel used for the shaft has a yield strength of 248 Mpa. By plotting the VonMises stress we can determine the factor of safety of the shaft at different critical speeds. From the analysis VonMises stress at 291.12 Hz and 291.25 Hz for rotation velocity of 0 rpm and 283.06 and 299 Hz for rotation velocity of 20000 rpm where critical speeds are occurring are plotted.

VonMises stress at 291.12 Hz at 0 rpm rotational velocity: A VonMises stress of 331.6 Mpa is observed on the edges of the modified shaft at 291.12 Hz operating at a rotational velocity of 0 rpm and occurring at the critical speed of 17055 rpm. The VonMises stress obtained is more than the yield strength of the material (248 Mpa). The FOS for the shaft is 248/331.6 = 0.75 which is less than 1. So it can be concluded that the shaft is not safe for the operating at a speed of 17055 rpm for a rotational speed of 0 rpm.

VonMises stress at 291.25 Hz at 0 rpm rotational velocity: A VonMises stress of 169 Mpa is observed on the edges of the modified shaft at 291.25 Hz operating at a rotational velocity of 0 rpm and occurring at the critical speed of 17913 rpm. The VonMises stress obtained is less than the yield strength of the material (248 Mpa). The FOS for the shaft is 248/169 = 1.46 which is less than 1. So it can be concluded that the shaft is safe for the operating at a speed of 17913 rpm for a rotational speed of 0 rpm.

VonMises stress at 283.06 Hz at 20000 rpm rotational velocity: A VonMises stress of 899 Mpa is observed on the edges of the modified shaft at 283.06 Hz operating at a rotational velocity of 20000 rpm and occurring at the critical speed of 17055 rpm. The VonMises stress obtained is more than the yield strength of the material (248 Mpa). The FOS for the shaft is 248/899 = 0.27 which is less than 1. So it can be concluded that the shaft is not safe for the operating at a speed of 17055 rpm for a rotational speed of 20000 rpm.

VonMises stress at 283.06 Hz at 20000 rpm rotational velocity: A VonMises stress of 899 Mpa is observed on the edges of the modified shaft at 283.06 Hz operating at a rotational velocity of 20000 rpm and occurring at the critical speed of 17055 rpm. The VonMises stress obtained is more than the yield strength of the material (248 Mpa). The FOS for the shaft is 248/899 = 0.27 which is less than 1. So it can be concluded that the shaft is not safe for the operating at a speed of 17055 rpm for a rotational speed of 20000 rpm.
VonMises stress at 299.4 Hz at 20000 rpm rotational velocity: A VonMises stress of 10012 Mpa is observed on the edges of the modified shaft at 299.4 Hz operating at a rotational velocity of 20000 rpm and occurring at the critical speed of 17913 rpm. The VonMises stress obtained is more than the yield strength of the material (248 Mpa). The FOS for the shaft is \( \frac{248}{10012} = 0.024 \) which is less than 1. So it can be concluded that the shaft is not safe for operating at a speed of 17913 rpm for a rotation speed of 20000 rpm.

From the harmonic analysis the frequency responses (Frequency Vs Amplitude) at 3 bearing locations and point mass where the unsymmetrical load was applied are plotted below.

**Frequency Vs Amplitude at bearing1:**

Frequency Vs Amplitude at bearing1 was obtained over the frequency range as shown in the below graph. From the graph the maximum amplitude of 2.58 mm is obtained at frequency of 1527 Hz for bearing1.

**Frequency Vs Amplitude at bearing2:**

Frequency Vs Amplitude at bearing2 was obtained over the frequency range as shown in the below graph. From the graph the maximum amplitude of 1.01 mm is obtained at frequency of 1527 Hz for bearing2.

**Frequency Vs Amplitude at bearing3:**

Frequency Vs Amplitude at bearing3 was obtained over the frequency range as shown in the below graph. From the graph the maximum amplitude of 0.06 mm is obtained at frequency of 2079 Hz for bearing3.

**Frequency Vs Amplitude at point mass 1:**

Frequency Vs Amplitude at unsymmetrical point mass was obtained over the frequency range as shown in the below graph. From the graph the maximum amplitude of 0.5 mm is obtained at frequency of 1527 Hz for point mass.

**Frequency Vs Amplitude at point mass 2:**

Frequency Vs Amplitude at unsymmetrical point mass was obtained over the frequency range as shown in the below graph. From the graph the maximum amplitude of 0.17 mm is obtained at frequency of 1527 Hz for point mass.
Harmonic analysis of the modified driving shaft of dry screw vacuum pump shows that there is no mass imbalance present in the operating range of 0-10,000 rpm.

10. CONCLUSIONS

- In this project, the rotor dynamic analysis of the driving shaft of the dry screw vacuum pump is performed to find the natural frequencies and critical speeds.
- As the operational speed of the shafts is 4500 rpm and the maximum speed of the shaft is 10000 rpm, the analysis is done up to 10000 rpm to find the critical speeds.
- From the analysis VonMises stress at 123.12 and 123.14 Hz for rotation velocity of 0 rpm and 122.85 and 123.42 Hz for rotation velocity of 20000 rpm where critical speeds are occurring are calculated for original shaft and found that the FOS at all the critical speeds is less than 1. Therefore it is concluded that the mass unbalance present in the original shaft is causing a high damage when it is operated at 7381 and 7391 rpm.
- Therefore the shaft is modified to shift the mass moment of inertia towards the center of gravity of shaft, thereby reducing the gyroscopic effect.
- From the analysis VonMises stress at 291.12 and 291.25 Hz for rotation velocity of 0 rpm and 283.06 and 299.4 Hz for rotation velocity of 20000 rpm where critical speeds are occurring are calculated for modified shaft and found that the FOS at all the critical speeds is less than 1. Therefore it is concluded that the mass unbalance present in the original shaft is causing a high damage when it is operated at 7381 and 7391 rpm. Therefore the shaft is modified to shift the mass moment of inertia towards the center of gravity of shaft, thereby reducing the gyroscopic effect.

9. DISCUSSION OF RESULTS

Model analysis of the driving shaft of dry screw vacuum pump shows that there only two critical speeds 7381 and 7394 rpm occurring in the operation range of 10000 rpm. The critical speeds are occurring at 123.12 Hz and 123.14 Hz for rotation velocity of 0 rpm and 122.85 and 123.42 Hz for rotation velocity of 20000 rpm. These vibration modes indicate that mass unbalance is present in the appreciable range and has to be attended which otherwise will cause the damage of the rotor shaft and bearing operating at higher speeds.

Harmonic analysis of the driving shaft of dry screw vacuum pump shows that VonMises stress of 1303 Mpa, 1372 Mpa is observed on the edges of the shaft at 123.12 Hz, 123.14 Hz occurring at the critical speed of 7381.4 rpm, 7394 rpm respectively at 0 rpm. A VonMises stress of 361 Mpa, 2348 Mpa is observed on the edges of the shaft at 122.85 Hz, 123.42 Hz occurring at the critical speed of 7381.4 rpm, 7394 rpm respectively operating at a rotational velocity of 20000 rpm. The VonMises stress obtained is more than the yield strength of the material thus the factor of safety (fos) is less than 1. So it can say that shaft is not safe for all these operating speeds.

Model analysis of the modified driving shaft of dry screw vacuum pump shows that there are no critical speeds occurring in the operation range of 10000 rpm. The critical speeds are occurring at 291.12 and 291.25 Hz for rotation velocity of 0 rpm and 283.09 and 299.4 Hz for rotation velocity of 20000 rpm. These critical speeds are occurring at 17055 rpm and 17913 rpm which are not in the operational range.
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