DESIGN OF GIRDER USING THIN BEAM WALLED THEORY WITH INFLUENCE OF SHEAR

Mr Ajay Prakash Yadav[1], Mr. G.V. Joshi[2]

[1]*M.Tech Final Year Student, GHRCEM, Wagholi, Pune*
[2]*Professor, Dept. of Civil Engineering, GHRCEM, Wagholi, Pune*

Abstract - *A railway U-girder bridge is a kind of structure which is composed of a plate and two side girders. Compared with a concrete box-girder bridge and T-beam bridge, the U-shaped Bridge has many advantages. This has been widely used in railway and urban rail transit bridges U-shaped prestressed concrete bridge decks, simply supported, are now being increasingly used in railways and highways. Simplified methods of analysis are commonly used in design practice.*

It is especially suitable when a new or modified alignment structure for railway/highway use requires an increase in the vertical clearance beneath the bridge; this also reduces the earthwork in the approach embankments. This concept can be used for overpasses, under-crossings, viaducts and so on (Gibbens and Smith, 2004). The two webs are integrally connected to and positioned above and on either side of the deck slab, providing a U-shaped girder cross-section. The resulting requirement for the depth of the girder section below the passageway level is significantly lower, compared to the convectional beam-and-slab type deck; this is its main functional advantage.

Key Words: U-Girder, Bridge, Thin Beam Walled Theory, Railway, Metro

1. INTRODUCTION

The Euler-Bernoulli beam theory as well as the Vlasov thin-walled beam theory does not take into account shear deformations due to shear forces. The shear effect, as well as Poisson's effect, can be included by methods of theory of elasticity, but in that case the problem is no longer one-dimensional.

Thus, approximate methods to include the shear effect are developed; particularly in the analysis of displacements, by deriving an adequate stiffness matrix. The concept of shear factors, first introduced by Timoshenko was used as the ratio of the maximum shear stress to the average shear stress over a cross-section. Recent approaches to the problem are based on geometric assumptions or shear energy relations. Numerical examples comparing results obtained by different approaches can be found. Approximate analytical solutions for stresses along the beam cross-section contour as well as for stresses and displacements along the beam length are given

1.1 Assumptions

- Beams with cross-sections with one and two axes of symmetry are considered.
- Poisson's effect is ignored. (Its influence on both the stresses and the displacements in the case of common open cross-sections is small, even for extremely low ratios of beam length to cross section contour dimensions)
- The warping effect, defined by the "non-uniform warping bending theory", is also ignored. (This effect remains much localized close to the clamped ends, where by the non-uniform warping theories warping due to shear is restricted.)

2. STRAINS AND DISPLACEMENTS

The displacement of an arbitrary point S (x,s) at the middle line in the case of bending of thin-walled beams of open sections with one axis of symmetry can be expressed as

$$u_s = -\frac{dw}{dx} z + u + \int_0^s \gamma_{x\xi}\, ds \qquad \text{---- (1)}$$

where w=w (x) is the displacement in the z-direction, i.e. the displacement of the cross-section middle line as a rigid line in the plane of symmetry, z = z (s) is the rectangular coordinate, u= u (x) is the displacement of the cross-section middle line as a rigid line in the x-direction, $\gamma_{x\xi} = \gamma_{x\xi}$ (x, s) is the shear strain in the middle surface, s is the curvilinear coordinate of the middle line, ξ is the tangential axis on the curvilinear coordinate s; Oxyz is the orthogonal coordinate system, where the z-axis is the axis of symmetry (Fig. 1).

Eq. (1) may be expressed as

$$u_s = \beta z + u + \int_0^s \gamma_{x\xi}\, ds \qquad \text{---- (2)}$$

Where $\beta = -dw/dx$ is the angular displacement of the middle line as rigid line with respect to the y-axis, orthogonal to the z-axis. It is assumed that the middle line rotates with respect to the y-axis as rigid line, expressed by the first member of Eq. (2), as in the case of the ordinary theory of bending; In addition, it is assumed that the middle line is displaced due to shear, expressed by the second and third members of Eq. (2)

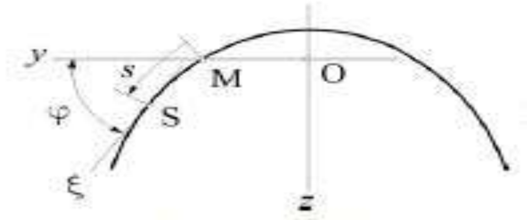

Fig -1: Cross-section middle-line

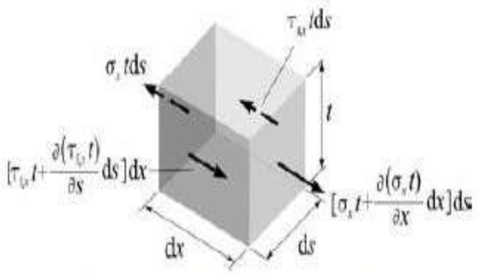

Fig -2: The equilibrium of the element of the wall

The displacements can be separated as follows:

$w = w_b + w_a$, $u = u_a$ ----- (3)

where wb = wb (x) is the displacement of the cross-sections as plane sections in the z-direction, as in the case of the ordinary theory of bending, wa = wa (x) is additional displacement due to shear in the z-direction, ua = ua (x) is the additional displacement due to shear in the x-direction.

$\beta = \beta_b + \beta_a$, $\beta_b = -\,dw_b/dx$ $\beta_a = -\,dw_a/dx$ ------- (4)

The strain in the beam longitudinal direction may then be expressed as

$$\varepsilon_x = \frac{\partial u}{\partial x} = -\frac{d^2w}{dx^2}z - \frac{du}{dx} + \int_0^s \frac{\partial \gamma_{x\xi}}{\partial x} ds \qquad \text{---------- (5)}$$

3. STRESSES AND DISPLACEMENT

Hooke's law may be simplified as

$$\sigma_x = E\varepsilon_x \qquad \tau_{x\xi} = G\gamma_{x\xi} \qquad \text{-------(6)}$$

Where E is the modulus of elasticity and G is the shear modulus.

Thus,

$$\sigma_x = -E\frac{d^2w}{dx^2}z + E\frac{du}{dx} + \frac{E}{G}\int_0^s \frac{\partial \tau_{x\xi}}{\partial x} ds \qquad \text{---------(7)}$$

From the equilibrium of a differential portion of the beam wall (Fig. 2), it may be written

$$\tau_{x\xi} = \frac{1}{t}\left[-\int_0^s \frac{\partial(\sigma_x t)}{\partial x} ds + f(x)\right] \quad . \quad f = t(M).\tau_{x\xi}(x, M) = T_M(x) \quad \text{-----(8)}$$

Where $t = t(s)$ is the wall thickness and M is the starting point of the curvilinear coordinate s.

If $\partial \tau_{x\xi}/\partial x = Constant$. Referring to (7), one has

$$\tau_{x\xi} = \frac{1}{t}\left[T_m + E\left(-\frac{d^3w}{dx^3}S_y(s) + \frac{d^2u}{dx^2}A(s)\right)\right], \qquad S_z(s) = \int_0^s y dA, \quad A(s) = \int_0^s dA,$$

$$dA = tds$$

------------------- (9)

Eq. (9) may be rewritten as

$$\tau_{x\xi} = \frac{E}{T}\left(-\frac{d^3w}{dx^3}S_y^* + \frac{d^2u}{dx^2}A^*\right), S_y^* = \int_{s^*} zdA^* \;,$$

$$= \int_{s^*} dA^* \;, dA^* = tds^*, ds^* = -ds$$

------ (10)

where $S_y^* = S_y^*(s)$ is the moment of the cut-off portion of area with respect to the y-axis, $A^* = A^*(s)$ is the cut-off portion of the beam wall area with respect to the y-axis, s^* is the Curvilinear coordinate of the cut-off portion of the beam wall area, from the free edge, i.e. where $\tau_{x\xi} = 0$. It is assumed that the normal stress given by Eq. (7) and the shear stress given by Eqs. (9) and (10) are constant across the wall thickness.

4. Equilibrium equations

It is assumed that the beam loads are reduced to loads $q_z = q_z(x)$ in the beam plane of Symmetry.

$$q_z = \int p_z ds \qquad \text{---------------- (11)}$$

where $p_z = p_z\,(x, s)$ are the surface loads with respect to the z-axis and L is the cross-section middle line length.

For a portion of the beam wall, the following equilibrium equations can be written

$$\sum F_x = \int \frac{\partial(\sigma_x t)}{\partial x} dxds = 0, \quad \sum F_z = \int \frac{\partial(\tau_{x\xi} t)}{\partial x} sin\emptyset \; dxds + q_z dx = 0 \qquad \text{------------- (12)}$$

Eqs. (12) Can be rewritten as

$$\sum F_x = \int \frac{\partial(\sigma_x t)}{\partial x} dxds = 0,$$

$\sum F_z = \int \frac{\partial(\tau_{x\xi}t)}{\partial x} sin\Phi \, dxds + q_z dx = 0$ ---------- (13)

By integrating by parts one has

$\int \frac{\partial(\sigma_x t)}{\partial x} dxds = 0, \quad \frac{\partial(\tau_{x\xi}t)}{\partial x} z|_{e_1}^{e_2} - \int z \frac{\partial}{ds}\left[\frac{\partial(\tau_{x\xi}t)}{\partial x}\right] ds + q_z = 0$
----------------- (14)

Where $e1$ and $e2$ are the boundaries, where $\tau_{x\xi} = 0$

Thus,

$\int \frac{\partial \sigma_x}{\partial x} dA = 0, \; \int z \frac{\partial}{dx}\left[\frac{\partial(\tau_{x\xi}t)}{\partial s}\right] ds + q_z = 0$ ---------- (15)

By substituting Eqs. (7) and (9) one has

$-ES_y \frac{d^3y}{dx^3} + EA\frac{d^2u}{dx^2} = 0, \; EI_y \frac{d^4w}{dx^4} - ES_y \frac{d^3u}{dx^3} = q_z$ ---- (16)

Where

$A = \int dA, \; S_y = \int zdA, \; I_y = \int z^2 dA$ -------- (17)

If y is the centroid coordinate, when Sy = 0, Eqs. (16) take the following simple form

$\frac{d^2u}{dx^2} = 0, \; EI_y \frac{d^4w}{dx^4} = q_z$ --------- (18)

3. CONCLUSIONS

A theory of bending of thin-walled beams with the influence of shear for sections with one and two axes of symmetry is developed. The theory is based on the classical Timoshenko bending theory. The shear factor with respect to the bending in the beam plane of symmetry is given in an analytical form. It is proved that the beam with a single symmetrical section, loaded in the plane of symmetry, is subjected also to tension/compression due to shear. Thus, a new factor of shear is given, with respect to tension/compression due to shear.

In the case of a double symmetrical section this factor vanishes: the beam is subjected to bending with the influence of shear only.

For various types of cross-sections with one and two axes of symmetry, the shear factors are given in the parametric forms. Stresses can be obtained in the analytical form both along the cross-section middle line and the beam length. Various boundary conditions and loadings are considered.

Several examples are analyzed in comparison with the finite element method. Excellent agreements of the results for displacements are obtained, as well as for stresses. Some discrepancies for normal stresses are noticed at beam ends in the case of clamped ends, as a result of different boundary conditions, both in the presented theory and the finite element method. Corresponding cross-section functions are given in the appendix.

REFERENCES

1. Ladeveze, P., Simmonds, J.G.: New concepts for linear beam theory with arbitrary geometry and loading, Comptes Rendus Acad. Sci. Paris, Vol. 332, Ser.IIb, 455-462, 1998.

2. Senjanović, I., Fan, Y.: The bending and shear coefficient of thin-walled girders, Thin-Walled Structures,

BIOGRAPHIES

Mr.Ajay Prakash Yadav
Structural Engineering Post Graduate 2nd Year Student

Prof. G. V. Joshi
Assistant Professor at G.H.Raisoni College of Engineering and Management, Wagholi, Pune