Modeling and Design Analysis of Die Profile of Extrusion of Square Section from Round Billet through Non-linear Converging Dies

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Abstract: A non-linear converging die profile for extrusion of square section from round billet was designed using cosine function. MATLAB 7.1 was used to find out the co-ordinates of the cosine die profile. A solid model was generated using AutoCAD 2008 from the above generated points. The STL files of extrusion die generated in AutoCAD was used in DEFORM-3D for FEM simulation. Experimental investigation of extrusion of square section from round billet using non-linear converging dies were done for different percentage of reduction of cross section in dry and lubricated condition. All the experiments were done using FIE‟ Electronic Universal Testing machine (UTM), model UTS-100 with maximum capacity of 1000 kN with accuracy ±1.0% kN. Material properties of lead like flow stress and friction factor were determined using compression and ring compression tests under different boundary conditions. To study the flow pattern of the material during extrusion, experiments were done with split work piece in lubricated condition by making grid. The extrusion load with punch travel was compared with extrusion load with solid work piece under same experimental condition. It was found that extrusion load in splitted work piece is less than the work piece when solid. FEM simulation of extrusion of square section from round billet for pure Lead and aluminium-1100 as work material were done using DEFORM-3D 6.1 (sp1). For FEM simulation the linear converging and cosine (non-linear converging) die profiles are used. FEM simulation using leads were done for two frictional conditions 0.38 and 0.75 corresponding to dry and lubricated conditions. Extrusion was assumed to be isothermal. From present investigation it was found that the extrusion load in case of cosine (non-linear converging) die is less than the linear converging dies under same condition. The extrusion load increases with increase in reduction and friction factor.

Definition of Extrusion:

Extrusion can be defined as the process of subjecting a material to compression so that it is forced to flow through a confined space past a suitable opening called the die. The metal is forced through the die and the cross-section of the die determines the shape properties of the resulting product. Extrusion may be done either on cold metal or on heated metal. One of the analogies that can be offered to the process of extrusion is that of squeezing a tube of toothpaste. The metal billet is placed in the billet chamber and is forced by the ram through a die.

Hot extrusion is done to eliminate the cold working effects, reduce the force required, and reduce directional properties. However cold extrusion is also possible for many metals and has become an important commercial process. The reaction of the billet with the container and the die results in high compressive stresses that effectively reduce cracking of materials during primary breakdown from ingot. This is an important reason for increased commercial adoption of extrusion in the working of metals difficult to form such as stainless steel, nickel, nickel based alloys and other high temperature materials. Lead, brass, bronze, copper, aluminum, and some of the magnesium alloys are the most commonly extruded metals.

In general extrusion is used to produce cylindrical bars or hollow tubes. A large variety of irregular cross sections are also produced by this process using dies of complex shapes. The process has definite advantage over rolling for production of complicated section having re-entrant corners. In this process large reduction achieved even at high strain rates has made it one of the fastest growing metal working methods.

Classification of Extrusion

According to the flow direction of metal with respect to ram movement direction extrusion process can be classified under the following methods

- Direct extrusion
- Indirect extrusion
- Impact extrusion

Direct Extrusion

Direct extrusion is a process where the flow of material through the die is in the direction of the movement of the ram that is used to force the material. Here the billet is moved forward relative to the wall of the container, thereby giving rise to high resistance from friction.

Indirect Extrusion

Indirect extrusion is a process where the flow of the material through the die is in a direction opposite to the movement of the ram that is forcing the material to deform. Here there is no relative motion between the container and the billet and, therefore, the frictional force is minimal.

Impact Extrusion

Impact extrusion is a process where a single blow from the ram on the material causes the metal billet to be extruded between the die and the punch. This process is usually done cold and on low strength ductile materials such as lead, tin and aluminum. It is used to make collapsible tubes for toothpaste, shaving cream, and cans that are used to pack food.

Die Design Consideration

For die design of extrusion following factors are to be considered as given bellow.

- Desired shape of the product
- Material
- Billet size
- Process capacity
- Extrusion ratio
- Number of die cavities
- Shrink factor
- Process tool
- Extrusion temperature
- Extrusion pressure
- Die material
- Heat treatment of die material

Advantages of Extrusion

Among the different metal forming processes, extrusion has definite advantages over others for the production of three dimensional section shapes. Now it is becoming essential to pay greater attention to the extrusion of section rod from round stock, as this operation offers the promises of an economic production route. The process is also attractive because press machines are readily available and the necessity to purchase expensive section stock corresponding to a multiplicity of required sections is eliminated. There are many advantage of extrusion as follows

- Uniform cross-sectional area over a long length.
- Low cost of dies making it economical to make small quantities of a shape.
- Good surface finish.
- Strain and hardness are increased due to strain hardening

Limitations of Extrusion

- Every process has some limitation; extrusion has also some limitation as given bellow
- Most materials require high temperature and pressure, which makes the equipment costly.
- Die material should be able to withstand the load, high temperature, and wear.
- In the case of steel, the equipment is costlier due to the magnitude of temperature to which the metal must be heated. (2300 F).
- Indirect extrusion complicates the handling of the extruded parts.
- Extrusion is limited to only a few metals and cannot be done on any metal chosen.

Applications of Extrusion

Extrusion is one of the most important methods of metal forming process with we can produce many product of high industrial applications with good quality. Some of the applications of the extrusion process are given bellow

- Helicopter blades
- Turbine blades
- Wingspans
- Columns used for creating structures
- Construction material

Upper Bound Solution

An upper bound analysis provides an overestimation of the required deformation force. It is more accurate because it will always result in an overestimation of the load that the press or the machine will be called upon to deliver. In this case factor of safety will be automatically built in. In this analysis, the deformation is assumed to take place by rigid body movement of triangular blocks in which all particles in a given element moves with the some velocity.

A kinematically admissible velocity should satisfy the

- Continuity equation
- Velocity boundary condition
- Volume constancy condition

The power of deformation calculated from this is higher than the actual one, called upper bound. When applying upper bound, the first step is to conceive of a velocity field for the deforming body.

- The field can be easily imagined and related to our visual experience.
- Velocity can be measured directly and is easily displayed in a physical manner.
- In this case factor of safety is automatically in built
- It is comparatively easy to analyze.
- There exists an infinite no stress field that satisfy the upper bound solution.

Finite Element Analysis

The finite element analysis method represents an extension of matrix method for the analysis of framed structures to the analysis of the continuum structure. The basic philosophy of this method is to replace the structure i.e. the continuum having an infinite or unlimited number of unknowns by a mathematical model, which has a limited or finite number of unknowns at certain chosen discrete points. This method is extremely powerful as it helps to accurately analyze structures with complex geometrical properties and loading condition.

In finite element method, a structure or a continuum is discredited and idealized by using a mathematical model, which is an assembly or subdivision or discrete elements. These elements known as finite elements are assumed to be interconnected only at the joints called nodes. Simple functions such as polynomials are chosen in terms of unknown displacements and or their derivative at the nodes to the approximate over the variation of the actual displacements over each finite element. The external loading is also transformed into equivalent forces applied at the nodes. The behavior of each element and later as an assembly of these elements is obtained by relating their response to that of the nodes in such way that the following basic conditions are satisfied at each node.

- The equation of equilibrium.
- The compatibility of displacements.
- The material constitutive relationship.

The equation, which are obtained using above condition and then these modified equations are solved to obtain displacements at the nodes, which are the basic unknowns of the finite element method. Finite element method involves extensive computations mostly repetitive in nature. Hence, this method is suitable for computer programming and solutions. Finite element computer programs have become widely available, easier to use and can display results with attractive graphics. Even an inept user can produce some kind of answer. It is hard to disbelieve finite element results because of the effort needed to get them and the polish of this presentation. However, smooth and colorful stress contours can be produced by any model, good or bad. It is possible that most finite element analyses are so flawed that they cannot be trusted. Even a poor mesh, improper element type, incorrect loads or improper supports may produce results that appear reasonable in causal inspection.

Experimental Investigation

The whole experimental investigation were done using „FIE" Electronic Universal Testing machine (UTM), model UTS-100 which can be used for conduction test in tension, compression and transverse test of metals and other material. Maximum capacity of the machine is of 1000 kN with measuring range between 0 to 1000 kN. The accuracy of measurement of the machine is ±1.0% kN Because load required for extrusion is of compressive type so, experiments were conducted using compression test.

'FIE' Electronic Universal Testing machine (UTM)

The UTM consists of three major parts

- Machine frame or Loading unit
- Hydraulic system
- Electronic control panel

Machine frame or Loading unit

Machine frame and loading unit consist of two cross heads and one lower table. Center cross head are adjustable by means of geared motor. Compression test is carried out between center and lower table while tension test is carried out between center table and upper cross head. Load is sensed by means of precision pressure transducer of strain gauge type

Hydraulic System unit

Hydraulic system unit consists of motor pump unit with cylinder and piston. Safety valve is provided for additional safety.

Electronic control unit

Electronic control unit control the process by controlling the input parameter like load rate, strain rate, maximum load etc.

Selection of the work-piece

The selection of material depends upon the properties of die material and load required for deformation. Because load required for extrusion is very high so we had to select softer materials, so we select lead as workpiece material of experimental investigation. Lead is a highly dense, corrosion resistive and very soft ductile material. Properties of lead in detail are given in table.

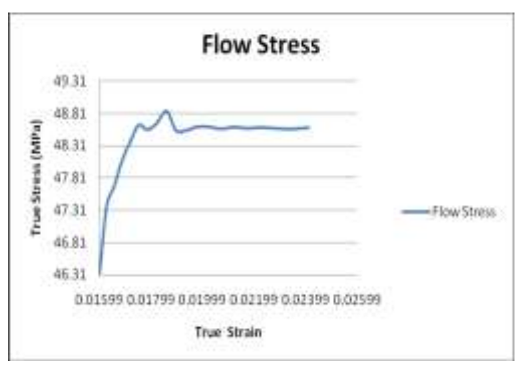
Mechanical properties of lead
Thermal Properties

Mechanical properties		condition
Density	11300 kg/m³	298.15 K
Young's Modulus of Elasticity	16000MPa	
Poisson Ratio	0.44	
Thermal Expansion Coefficient	29e-6	298.15 K

Thermal Properties		condition
Melting point	600.61 K	
Boiling point	2022.15 K	
Critical temperature	5500 K	
Heat capacity	130 J/kg-K	298.15 K
Thermal conductivity	35.3 W/m-K	300 K

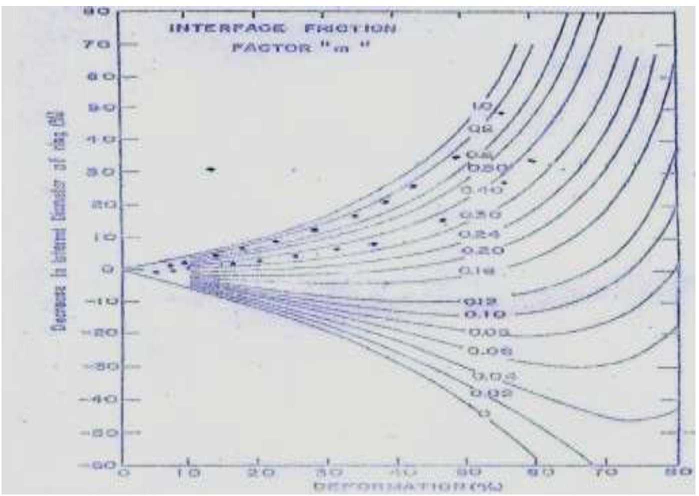
Compression test

Pure lead cylinders with a 50mm × 30 mm (H× D) were taken to obtain the stress-strain curve by a compression test using „FIE" Electronic Universal Testing machine (UTM), model UTS-100 at room temperature. The compression rate is maintained same as that adapted for the experiments. The specimen has oil grooves on both the ends to entrap lubricant during the compression test. The compression load is recorded at every 0.5 mm of punch ravel. After compressing the specimen to about 10 mm it is taken out of the press. Compressed material re-machined to cylindrical shape with original diameter, and tested in compression till the specimen is reduced to another 10 mm. The stress-strain diagram was drawn. The average flow stress of the pure lead is found to be 48.85 N/mm2



Ring Compression Test

A ring compression test was carried out at dry condition and commercially available grease lubrication condition. The rings were compressed up to the 4 mm inner diameter, at each 0.5 mm of punch travel inner diameter and height was recorded. The friction factors were found to be 0.75 for dry condition and 0.38 for the lubricated condition by comparing our result with the calibration curve of Male and Cockcroft as shown in the fig.



Experimental procedure

Before starting the test the die sets, die holder and inside face of extrusion chamber were cleaned. The two halves of the die set were then push fitted into die holder and total assembly were secured by screwing four bolts. The full assembly was then placed in between the base plate and center table of „FIE" Electronic Universal Testing machine (UTM), model UTS-100 in upside down position. It was done so that extrude product would get enough clearance, when it comes out from the die. For carrying out an extrusion test the pure lead specimen was placed inside the extrusion chamber or container. The punch was then inserted into its position. After centering the apparatus under machine lower table, Machine was started and extrusion process was continued. Punch load was recorded at every 1 mm movement of punch travel, which was read from computer fitted to the UTM. The application of load was continued till it reaches the steady state and up to certain length of product comes out side. At this position Machine was stopped and test was terminated the die holder was then separated from extrusion chamber and finally die halves with extruded product were pushed out from die holder and extrusion chamber. Experiments were conducted for three different reductions 50%, 70% and 90% to get square section product at two different experimental conditions (dry and lubricated) from round cross-section billet.

Split Test

Lead material round billet of 30mm diameter and 50mm length is taken as work material, the work material is spited in to two parts along it diameter. A grid of 5mm x 5mm is made in to inner surface of the slitted part. In fig 5 grid pattern is shown. Rubber band is put in to the grid pattern to prevent material mixing during extrusion, because lead is a very soft ductile metal. Now, both part are again join using araldite (gum) and clamp it for 48 hour so that both part joined properly like single piece.

Extrusion is carried out in lubricated condition of split work material. The flow pattern is studied for different percentage of reduction. Work material after extrusion with 50% reduction is shown in fig 6. In fig 7 flow pattern of material with 70% reduction in area is shown. Fig 8 shows the flow pattern of material with 90% reduction.

Finite element analysis

Finite element analysis modeling is done using DEFORM-3D Version 6.1(sp1). DEFORM-3D is a Finite Element Method (FEM) based process simulation system designed to analyze various forming and heat treatment processes. By simulating manufacturing processes on a computer, this advanced tool allows designers and engineers to:

- Reduce the need for costly shop floor trials and redesign of tooling and processes
- Improve tool and die design to reduce production and material costs
- Shorten lead time in bringing a new product to market.

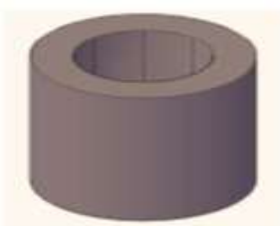
Unlike general purpose FEM codes, DEFORM is tailored for deformation modelling. A user friendly graphical user interface provides easy data preparation and analysis so engineers can focus on forming, not on learning a cumbersome computer system.

The DEFORM-3D consists of three major components:

- Pre-processor: used for creating assembly or modifying the data required to analyze the simulation, generating mesh and for generating the required database file.
- Simulation engine: Used for performing the numerical calculations required to analyze the process, and writing the results to the database file. The simulation engine reads the database file, performs the actual solution calculation, and appends the appropriate solution data to the database file.
- Post-processor: Used for reading the database file from the simulation engine and displaying the results graphically and for extracting numerical data.

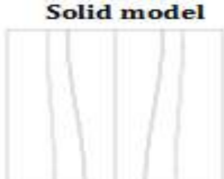
Solid Modelling

MATLAB 7.1 was used to generate the no fo points using above generated die profile equation. The generated points were used to create solid model using AutoCAD 2008. Solid model generated from AutoCAD 2008 were shown in the following figures



Solid model

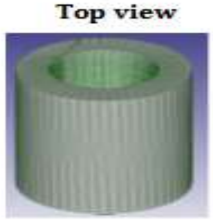
wire-frame model



wire-frame longitudinal view

Top view

Bottom view



STL file

Dual stream function :

General equation of any point on the cross-section using cosine function will be given by the equation (1) and (2)

$$F(z_1) = x = \left[\frac{R*\sqrt{(1-\frac{P}{N})}+A\left(1-\frac{P}{N}\right)}{2}\right] + \left[\frac{R*\sqrt{(1-\frac{P}{N})}-A\left(1-\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right) \quad (1)$$

$$F(z_2) = y = \left[\frac{R*\sqrt{(P/N)}+A\left(\frac{P}{N}\right)}{2}\right] + \left[\frac{R*\sqrt{(\frac{P}{N})}-A\left(\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right) \quad (2)$$

Let the stream function are

Now V_x, V_y, V_z are velocity function in x, y &z direction respectively. Then velocity field in x, y and Z direction can be given as follows.

$$\varphi 1 = \frac{-x}{F(z_1)} \quad (3)$$

$$\varphi 1 = \frac{\pi R^2 v_b y}{F(z_2)} \quad (4)$$

$$v_x = \left\{\frac{\partial\varphi_2}{\partial y} * \frac{\partial\varphi_1}{\partial z}\right\} - \left\{\frac{\partial\varphi_1}{\partial y} * \frac{\partial\varphi_2}{\partial z}\right\} \quad (5)$$

$$v_y = \left\{\frac{\partial\varphi_2}{\partial z} * \frac{\partial\varphi_1}{\partial x}\right\} - \left\{\frac{\partial\varphi_1}{\partial z} * \frac{\partial\varphi_2}{\partial x}\right\} \quad (6)$$

$$v_z = \left\{\frac{\partial\varphi_2}{\partial x} * \frac{\partial\varphi_1}{\partial y}\right\} - \left\{\frac{\partial\varphi_1}{\partial x} * \frac{\partial\varphi_2}{\partial y}\right\} \quad (7)$$

Derivation of kinematically admissible velocity field

Using equation (1) & (2) velocity function can be written as follow

$$v_x = \left\{\frac{\partial\varphi_2}{\partial y} * \frac{\partial\varphi_1}{\partial z}\right\} - \left\{\frac{\partial\varphi_1}{\partial y} * \frac{\partial\varphi_2}{\partial z}\right\}$$

$$v_x = \frac{\pi R^2 v_b y}{F(z_2)} * \frac{x}{F(z_1)^2} \quad (8)$$

$$v_y = \left\{\frac{\partial\varphi_2}{\partial z} * \frac{\partial\varphi_1}{\partial x}\right\} - \left\{\frac{\partial\varphi_1}{\partial z} * \frac{\partial\varphi_2}{\partial x}\right\}$$

$$v_y = \frac{\pi R^2 v_b y}{F(z_2)^2} * \frac{1}{F(z_1)} \quad (9)$$

$$v_z = \left\{\frac{\partial\varphi_2}{\partial x} * \frac{\partial\varphi_1}{\partial y}\right\} - \left\{\frac{\partial\varphi_1}{\partial x} * \frac{\partial\varphi_2}{\partial y}\right\}$$

$$v_y = \frac{\pi R^2 v_b}{F(z_2)} * \frac{1}{F(z_1)} \quad (10)$$

Derivation of strain function

Now ε_{xx} ,ε_{yy}, ε_{zz} strain function in x, y & z direction respectively.

$$\varepsilon_{xx} = \frac{\partial v_x}{\partial x} \tag{11}$$

$$\varepsilon_{yy} = \frac{\partial v_y}{\partial y} \tag{12}$$

$$\varepsilon_{zz} = \frac{\partial v_z}{\partial z} \tag{13}$$

Using equation (1) & (2) strain function can be written as follow

$$\varepsilon_{xx} = \frac{\partial v_x}{\partial x}$$

$$\varepsilon_{xx} = \frac{\pi R^2 v_b}{F(z_2)} * \frac{1}{F(z_1)^2} \tag{14}$$

ε_{xx} =

$$\frac{\pi R^2 v_b}{\left[\frac{R*\sqrt{(P/N)}+A\left(\frac{P}{N}\right)}{2}\right]+\left[\frac{R*\sqrt{\left(\frac{P}{N}\right)}-A\left(\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right)*\left\{\left[\frac{R*\sqrt{\left(1-\frac{P}{N}\right)}+A\left(1-\frac{P}{N}\right)}{2}\right]+\left[\frac{R*\sqrt{\left(1-\frac{P}{N}\right)}-A\left(1-\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right)\right\}^2}$$

$$\varepsilon_{yy} = \frac{\partial v_y}{\partial y}$$

$$\varepsilon_{yy} = \frac{\pi R^2 v_b}{F(z_2)} * \frac{1}{F(z_1)} \tag{15}$$

ε_{yy}

$$= \frac{\pi R^2 v_b}{\left[\frac{R*\sqrt{(P/N)}+A\left(\frac{P}{N}\right)}{2}\right]+\left[\frac{R*\sqrt{\left(\frac{P}{N}\right)}-A\left(\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right)*\left\{\left[\frac{R*\sqrt{\left(1-\frac{P}{N}\right)}+A\left(1-\frac{P}{N}\right)}{2}\right]+\left[\frac{R*\sqrt{\left(1-\frac{P}{N}\right)}-A\left(1-\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right)\right\}}$$

$$\varepsilon_{zz} = \frac{\partial v_z}{\partial z}$$

$$\varepsilon_{zz} = -\pi R^2 v_b\left\{\frac{1}{F(z_1)*F(z_2)^2}+\frac{1}{F(z_2)*F(z_1)^2}\right\} \tag{16}$$

ε_{zz} =

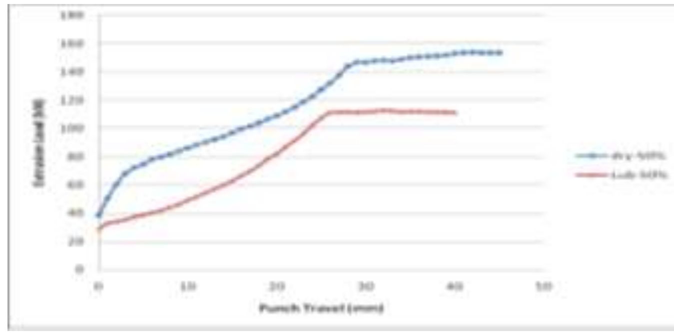
$$= -\pi R^2 v_b\left\{\frac{1}{\left\{\left[\frac{R*\sqrt{(P/N)}+A\left(\frac{P}{N}\right)}{2}\right]+\left[\frac{R*\sqrt{\left(\frac{P}{N}\right)}-A\left(\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right)\right\}^2*\left\{\left[\frac{R*\sqrt{\left(1-\frac{P}{N}\right)}+A\left(1-\frac{P}{N}\right)}{2}\right]+\left[\frac{R*\sqrt{\left(1-\frac{P}{N}\right)}-A\left(1-\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right)\right\}} + \frac{1}{\left[\frac{R*\sqrt{(P/N)}+A\left(\frac{P}{N}\right)}{2}\right]+\left[\frac{R*\sqrt{\left(\frac{P}{N}\right)}-A\left(\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right)*\left\{\left[\frac{R*\sqrt{\left(1-\frac{P}{N}\right)}+A\left(1-\frac{P}{N}\right)}{2}\right]+\left[\frac{R*\sqrt{\left(1-\frac{P}{N}\right)}-A\left(1-\frac{P}{N}\right)}{2}\right]\cos\left(\frac{\pi z}{L}\right)\right\}^2}\right\}$$

Adding equation (1), (2) & (3) we get

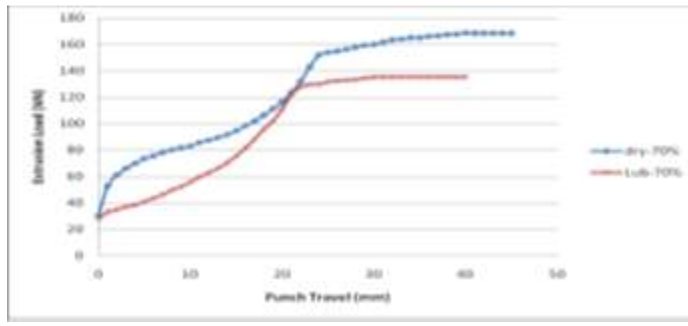
$$\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0$$

Results and Discussions:

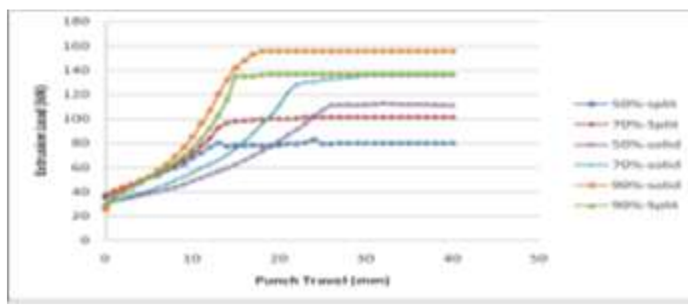
Experimental Investigation



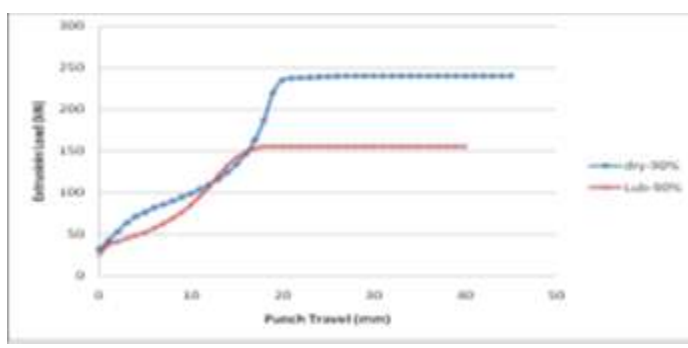
Comparison of extrusion load with punch travel in dry and lubricated condition for 50% reduction



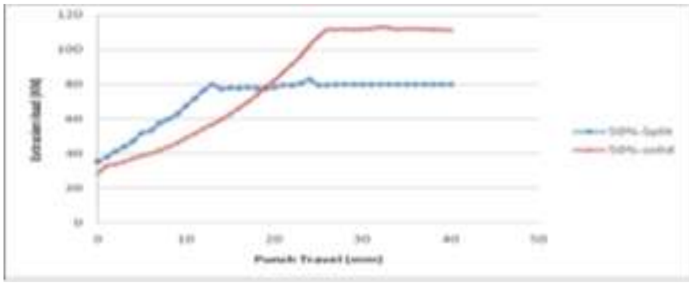
Comparison of extrusion load with punch travel in dry and lubricated condition for 70% reduction



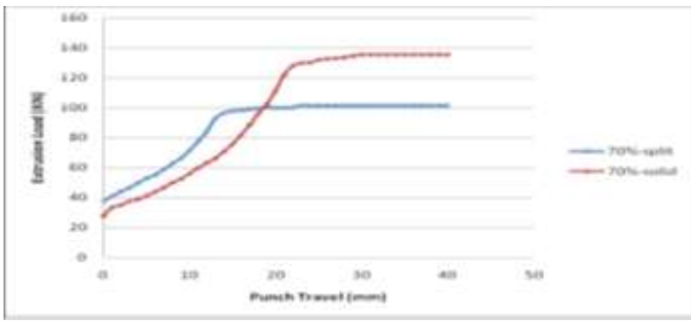
Comparison of extrusion load with punch travel in dry and lubricated condition for 90% reduction



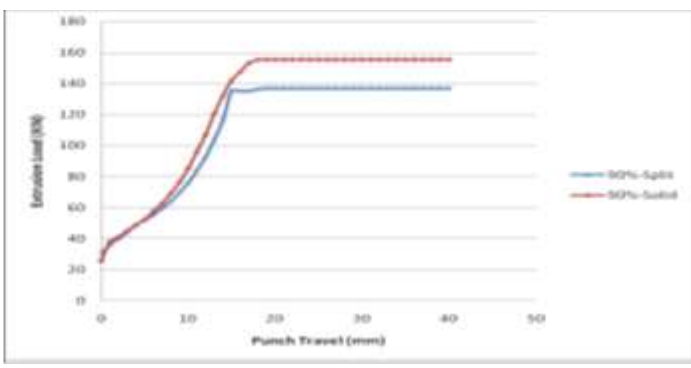
Comparison of extrusion load with punch travel in dry condition for different reduction



Comparison of extrusion load with punch travel in lubricated condition for solid and split die with 50% reduction



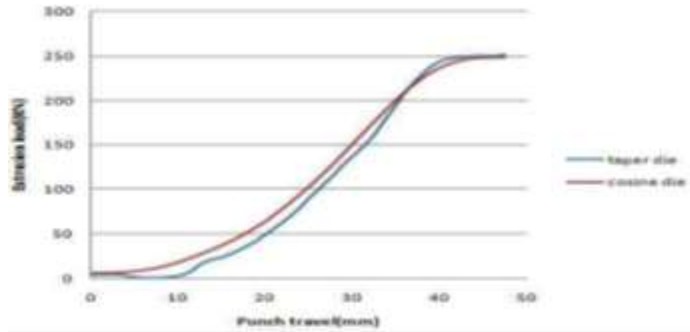
Comparison of extrusion load with punch travel in lubricated condition for solid and split die with 70% reduction



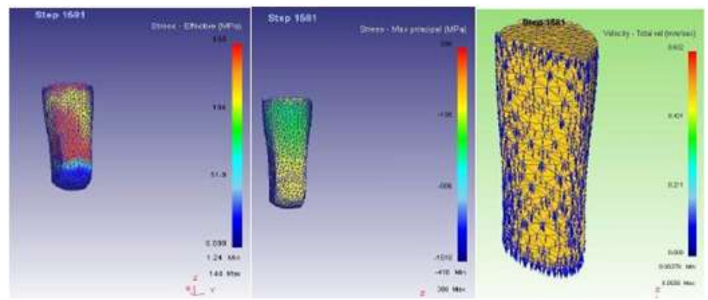
Comparison of extrusion load with punch travel in lubricated condition for solid and split die with 90% reduction

FEM simulation of Alluminium-1100:

FEM Simulation using aluminum-1100 as work material was done with non-linear converging die (cosine die) and linear converging die. It is found that the extrusion load is less in case of cosine die as shown in fig.32. Some properties like effective stress, velocity and principal stress at last step are shown in fig



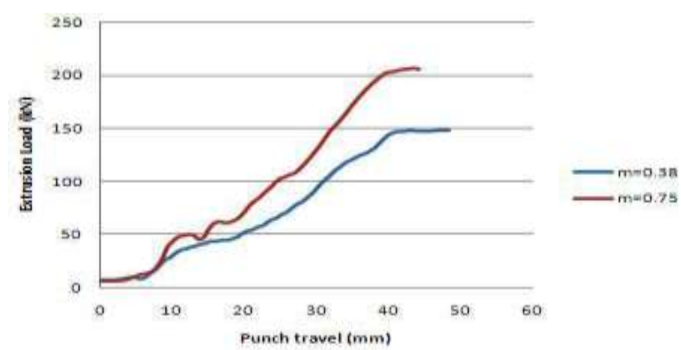
Comparison of extrusion load with punch travel in lubricated condition with cosine and taper die



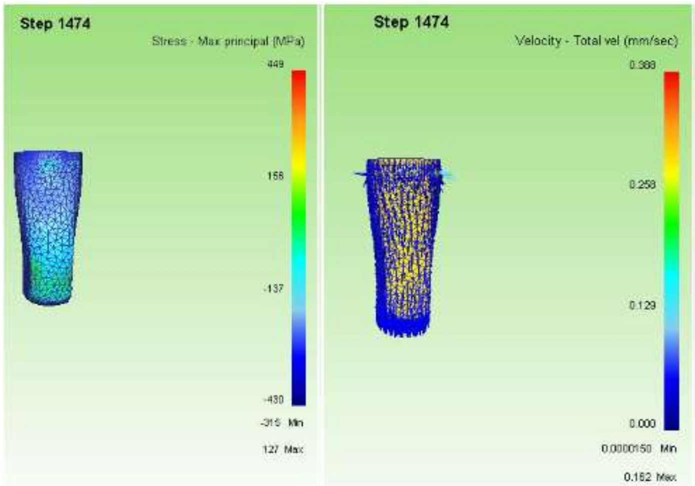
Effective stress, principal stress and velocity field at last step of extrusion

FEM simulation of pure lead:

FEM simulation using lead was done for two frictional conditions 0.38 and 0.75 as derived from experiment using ring compression test. Extrusion load with punch travel for friction factor m=0.38 and m= 0.75 are shown in fig



Extrusion load with punch travel for dry and lubricated condition using cosine die



Principal stress and velocity field at last step of extrusion

CONCLUSIONS:

A non-linear converging die profile has been designed for extrusion of square section from round billet using cosine function.

- The extrusion load increases with increase in reduction and friction factor.
- Load in case of split test is less as compared to solid work material under same experimental condition.
- The extrusion load for non-linear converging die is less as compared to linear converging die under same simulation condition.
- The flow of material in non-linear converging die appears to be gradual specially in higher reduction.

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