

K-SVD: Dictionary Developing Algorithms for Sparse Representation of Signal

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Abstract - Now days trend in study of sparse representation of signals. In sparse representation having overcomplete dictionary that contain prototype signal-atoms, signals are elaborated by sparse linear combinations of these atoms. There are many applications in sparse representation which include compression, consistency in inverse problems, feature extraction, and more. We concentrated basically on study purpose of pursuit algorithms that decompose signals with respect to a given dictionary D . We are developing method the K-SVD algorithm generalizing the K-means clustering process. K-SVD is a mathematical method that develop the algorithm alternates between sparse coding data using the dictionary D and apply process for updating the dictionary atoms to get the correct data. After updated dictionary columns which is combined with an update of the sparse representations. The developed K-SVD algorithm is adaptable. It can also work with any type of pursuit method.

Key Words: Basis pursuit, dictionary, FOCUSS, K-means, K-SVD, matching pursuit.

1. INTRODUCTION

Sparse representations using learned dictionaries D are being more helpful with success in various types of data processing and machine learning applications. The accessibility of large amount of training data necessitates the development of suitable, robust and better dictionary learning algorithms. For algorithmic stability and generalization of dictionary learning algorithms we are using two cases:

1. Complete: a system $\{y_i\}$ in X is complete if every element in X can be arbitrarily well in norm by linear combinations of elements in $\{y_i\}$.
2. Overcomplete: if removal of an element from the system $\{y_i\}$ results in a complete system. The arbitrary approximation in norm can be thought as a representation somehow.

K-SVD algorithm for studying dictionaries D . We explained its development and analysis, and formalized applications to establish its usability and the advantage of trained dictionaries D . Diversities of the K-SVD algorithm for learning structural constrained dictionaries are also showcased. Out of those constraints are the non-negativity of the dictionary and shift invariance property. K-SVD deals with development of a state-of-the art image denoising algorithm. This case study is important as it nourishes the message that the general model of sparsity and redundancy, along with fitted dictionaries as also used here, it is the good practical applications in image processing.

1.1 Sparse Representation of signal

Let us consider the overcomplete dictionary matrix $D \in \mathbb{R}^{n \times K}$ that include K prototype signal atoms for columns, $\{d_j\}$ $K, j=1, \dots, K$, a signal, here $y \in \mathbb{R}^n$ can be represented as a linear combination of these atoms. That present the $y = Dx$, or $y \approx Dx$, $\|y - Dx\|_p \leq \epsilon$. The vector $x \in \mathbb{R}^k$ contains the representation coefficients of the signal y . In some method, for measurement of the deviation we are using the l^p -norms for $p = 1, 2$ and ∞ . Here we are focusing on the case of $p = 2$. If $n < K$ and D is a full-rank matrix, several alternative methods are available for the representation problem. The solution with nonzero coefficients is certainly an applicable for representation. This sparsest representation is the solution of either (P0) $\min \|x\|_0$ for finding approximating solutions have been extensively investigated and indeed, several effective decomposition algorithms are available.

2. Methodology

K-SVD Algorithm two steps, one is Sparse Coding that contain producing sparse representations matrix X , given the current dictionary D . Another one is Dictionary Update D include updating dictionary atoms, given the current sparses representations.

2.1K-SVD frame

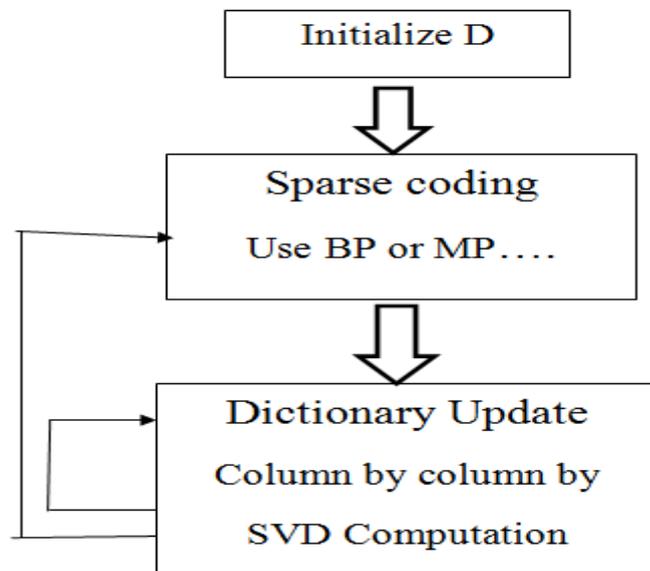


Fig: Basic Frame of K-SVD

2.2 Basic Algorithm of K-SVD

1. Input: Signal set Y , initial dictionaries D_0 , target sparsity S , number of iteration L .
2. Output: Dictionary D and sparse matrix X such that $Y=DX$.
3. Init: set $D = D_0$
4. for $n=1, \dots, L$ do
5. $\forall i: x_i = \text{Arg min} \|y_i - Dx\|$ subject to $\|x\|_0 \leq S$
6. for $j=1, \dots, K$ do
7. $d_j=0$
8. $I = \{\text{indices of the signals in } Y \text{ whose representations use } d_j\}$
9. $E = Y_i - D X_i = Y_i - \sum_{i \neq j} d_j X_{i,l}$
10. $\{d, g\} = \text{Arg min}_{d,g} \|E - dg^T\|_f^2$ subject to $\|d\|_2 = 1$
11. $d_j = d$;
12. $X_j, I = g^T$
13. end for
14. end for

Initially, we set D and to search the better coefficient matrix. According to given data searching the optimal is difficult. By using pursuit method, we will calculate coefficients, it can provide a solution data with a fixed. When that step is done, next step is showing to find for a best dictionary. This task updates only one column at a time, fixing all columns in rather than one and searching a new column and new values for that coefficients will helpful for better decreases the MSE. We change the columns

in sequence manner and permit to changing the respective coefficients. Once we modify the search column according to K-means then ready to proceed to next generalization of the K-mean

We designed dictionary for performance of K-SVD in synthetic and real image application. In recent days K-SVD algorithm have more demand, especially when the dimensions of the dictionary increase, or the number of training signals becomes large. Initially apply the K-SVD algorithm on synthetic signals method, for checking is this algorithm occupies the original dictionary D that originating from the data and to compare its results with related algorithms. Generation of the data train that describes a random matrix (referred to later as the generating dictionary) of size 20×50 was generated with uniformly distributed entries. Each column was normalized to a unit l^2 -norm. Then, 1500 data signals of dimension 20 were produced, each of them made by a linear combination of three various dictionary atoms, with uniformly distributed coefficients in random and independent locations. White Gaussian noise with varying signal-to-noise ratio (SNR) was added to the resulting data signals. Applying the K-SVD, the dictionary was initialized with data signals. The coefficients were found using OMP with a fixed number of three coefficients. The maximum number of iterations was set to 80. Comparison to other reported works we implemented the MOD algorithm and applied it on the same data, using OMP with a fixed number of three coefficients and initializing in the same way. We executed the MOD algorithm for a total number of 80 iterations. We also executed the MAP-based algorithm of Kreutz-Delgado. This algorithm was executed as is, therefore using FOCUSS as its decomposition method. Here, again, a maximum of 80 iterations were allowed.

3. CONCLUSION

According to work we proposed the problem of generating and using overcomplete dictionaries. We developed an algorithm the K-SVD for training an overcomplete dictionary which is better for group of given signals. From this we generalized K-means algorithm, implement designed for solving a same but related problem. We also proved that dictionary which found by K-SVD. On that performance we will apply for both synthetic and real image in different applications. We used all this technology for filling in missing pixels and compression and outperforms alternatives such as the non-decimated Haar and over complete or unitary DCT.

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