ANALOGUE OF ZARISKI GEOMETRY OVER COMPLEX FIELD

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Abstract – *The aim of the paper is to find an analogue of Zariski geometry which we describe as tame analytic geometry over the field of complex numbers C.*

Key Words **: Zariski geometry, analytic, locally finite, closed field**

1. INTRODUCTION

The axioms for a Zariski geometry for a set M given by Ehud Hrushovski and Boris Zilber in [3] described that a collection C of subsets of powers of M(closed sets) to behave like the algebraic sets over an algebraically closed field. In particular, the projection of a closed set in a Zariski geometry is constructible, i.e.,a Boolean combination of closed sets, as follows in algebraic geometry from the Nullstellensatz.. Another important principle is that closed sets can be decomposed into a finite number of irreducible components.[1-4].

Let us see the above concept through example, if M is a compact complex manifold and C contains all the analytic sets over M , then the requisite finiteness follows immediately from compactness, and the projection condition is given by Remmert's theorem[4].

We hope to find an analogue of Zariski geometry which we describe as tame analytic geometry over all of C. This necessarily involves relaxing the finiteness condition. In particular, the proper analytic subsets of C are locally finite (in the usual topology) and have a countable number of connected components in C.

Typical projections of countable analytic subsets of C^2, by contrast, are not Locally finite, as for example Q or Q [0,−1]. That these sets are dense in R and C respectivelyand these sets are to assign greater than zero for dimension. It shows that we cannot hope to have a Zariski-type closure for these sets which agrees with the closure operation for the usual topology.

By treating all countable sets on an equal footing, however, we are able to prove a quantifier elimination theorem analogous to that for Zariski geometries. In the consequence, we need the following definitions and theorems to prove the main result.

Definition1.1: A quasi-Zariski geometry (alternatively a QZ-structure) is a triple $\langle M, \mathcal{C}, \dim \rangle$ where:(i) M is a set;

(ii) $\mathcal{C} = \bigcup_{n\in N} \mathcal{C}_n$, where $\mathcal{C}_n$ is a sub collection of the subsets of M^n; and

(iii) $\dim : \mathcal{C} \to N \cup \{-\infty\}$

Lemma 1.1. The projection mappings $\pi : M^n \to M^{n-1}$ are continuous.

Lemma 1.2. The irreducible components of X are defined upto an enumeration uniquely.

The following proof is an analogue for Zariski geometry or an alternative proof for Zilber's proof.

Theorem 1.3.

An arbitrary intersection of closed sets is closed.

Proof. Let $\{X_\alpha : \alpha \in I\}$ be an arbitrary collection of closed set,

Let, $d = \min(\dim(X_{\alpha_1} \cap \cdots \cap X_{\alpha_k}))\ \alpha_1,\ldots,\alpha_k \in I;\ k \in N$, be the least dimension of any finite intersection among the X_α.Without loss of generality we may take $d = \dim X_0$, say, and $X_\alpha \subseteq X_0$ for all $\alpha \in I$. We shall prove the theorem by induction on d.

If $d = 0$ then X_0, and hence $\bigcap_{\alpha \in I} X_\alpha$, are all countable and therefore closed. Otherwise, we may write $X_0 = Z \cup \bigcup_{j\in\omega} W_j$ with each W_j irreducible of dimension d and $\dim Z < d$. Then by induction we know that $Z \cap T$ closed. Each W_j is irreducible, so satisfies either (i) for some $\alpha \in I$, $\dim(W_j \cap X_\alpha) < d$ or $\bigcap_{\alpha\in I} X_\alpha$; is (ii) $W_j \subseteq X_\alpha$ for all $\alpha \in I$

By induction in case (i) and obviously in case (ii), $W_j \cap \bigcap_{\alpha\in I}$

That is, X_α is closed.

Since it is countable union of closed sets, and therefore closed.

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