Reconstruction of Sparse Signals(Speech) Using Compressive Sensing

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Abstract - *Compressive sensing is a signal processing technique for efficiently acquiring and reconstructing a signal, by finding solutions to underdetermined linear systems. Reconstruction of under determine signals possess several problems in signal processing. In this paper compressive sensing was used to reconstruct the speech signals which have sparsity in fourier domain. To optimise the signal sample reconstruction basis pursuit algorithm was used. The algorithm was implemented using MATLAB software and the simulation results in the absence of external noise shows that the basis pursuit algorithm can be used to reconstruct the speech signals which are sampled below the nyquist rate.*

Key Words Compressive sensing, sparsity, incoherent, Fourier transform, basis pursuit algorithm.

1.INTRODUCTION

The principal approach for signal reconstruction from its estimations is characterized by the Shannon – Nyquist sampling theorem expressing that the sampling rate should be atleast double the maximal signal frequency. In the discrete case, the quantity of estimations ought to be atleast equivalent to the signal length so as to be actually recovered. Anyway this methodology may require substantial requirement of power, noteworthy detecting time, heavy power consumption and vast number of sensors. Another impediment of sampling utilizing nyquist rate is that the rate at which sampling must be done, may not be viable dependably. For instance, in the event of multiband signals having wide spectral range, sampling rate proposed by nyquist basis might be orders of size higher than the specifications of best accessible analog to-digital converter (ADC). The sampling rate utilizing Nyquist theorem is chosen by the highest frequency component present in signal.

After the renowned Shannon-Nyquist sampling theorem, presentation of compressive sensing (CS) resembles a noteworthy leap forward in signal handling network. CS was presented by Donoho, Candes, Romberg, and Tao in 2004 [1]-[2]. They have built up it scientific establishment. Compressive sensing is a novel methodology that goes past the conventional methodology. It demonstrates that a sparse signal can be recovered from less number of samples.

2. MOTIVATION

Signals with N-samples can be very much depicted by utilizing just M-parameters, where M is much less than N. Without a doubt there exist a wide assortment of systems for information decrease, utilizing low dimensional models to diminish the weight of handling. compressed sensing(CS) has turned into a functioning exploration territory as of late because of its fascinating theoretical nature and its useful utility in a wide scope of uses. CS is a progressive technique for sampling that enables a signal to be obtained and precisely recovered with altogether less samples than required by Nyquist-rate sampling.

3. RESTRICTIONS

3.1 Sparsity: compressive sensing strategy is connected for signals which pursues a few limitations. The confinements are sparsity and incoherence. Numerous natural signals are compressible by changing them to some domain—for example Sounds are minimally represented in the frequency domain and pictures in the wavelet domain.

Advances in compressive sensing recommend that if the signal is meager or compressible, the sampling Sprocedure would itself be able to be structured to gain just fundamental data. Changing a signal to another domain may enable us to represent to a sample all the more compactly. Sparse signal models enable us to accomplish high rates of compression and utilize the information to recuperate the original signal from few number of estimations.

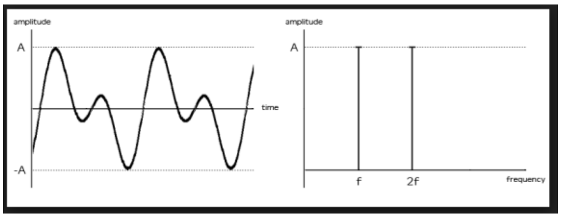

Fig1: Sparse representation of signal

3.2. Incoherence: The concept of coherence was introduced in a slightly less general framework and has since been used extensively in the field of sparse representations of signals. In particular, it is used as a measure of the ability of suboptimal algorithms such as matching pursuit and basis pursuit to correctly identify the true representation of a sparse signal. To be formal, one defines the coherence or the mutual coherence of a matrix *A* is defined as the maximum absolute value of the cross-correlations between the columns of *A*. Formally, let $a_1, a_2, \ldots, a_N$ be the columns of the matrix *A*, which are assumed to be normalized such that $aa^T=1$. The mutual coherence of A is then defined as μ(A).

$$\mu(A) = \max_{1\le i\ne j\le m} |a_i{}^T a_j| \qquad (1)$$

We say that a dictionary is incoherent if μ(*A*) is small. Standard results then require that the measurement matrix satisfy a strict incoherence property. Coherence is in some sense a natural property in the compressed sensing framework, for if two columns are closely correlated, it will be impossible in general to distinguish whether the energy in the signal comes from one or the other.

4. FOURIER TRANSFORM

Discrete Fourier Transform (DFT) is a fundamental transform in digital signal processing with applications in frequency analysis, signal processing etc. DFT is the transformation of the discrete signal taking in time domain into its discrete frequency domain representation. The periodicity and symmetry properties of DFT are useful for compression. The *n*th DFT coefficient of length *N* sequence *x*(*n*) is defined as follows:

$$X(k)= \sum_{n=0}^{N-1} x(n)\, e^{-2\pi jnk/N} \;;k=0,1..,N-1 \qquad (2)$$

And its inverse transform(IDFT) as:

$$x(n)=\frac{1}{N}\sum_{k=0}^{N-1} X(n)\, e^{2\pi jnk/N}, n=0,1..,N-1. \qquad (3)$$

The number of complex multiplications and additions to compute DFT is N^2. Moreover fast algorithms exist that makes it possible to compute DFT efficiently. This algorithm is popularly known as Fast Fourier Transform (FFT) which reduces the computational burden to $N\log_2 N$. FFT is computational efficient algorithms to compute the DFT and its inverse.

5. GENERATION OF COMPRESSED SIGNAL

Consider a real-valued, finite length, one-dimensional, discrete-time signal *x*, which can be viewed as an *N* ×1 column vector in *RN* with elements *x*[*n*], *n* = 1, 2,….., *N* . The image or higher-dimensional data can be treated by vectorizing it into a long one dimensional vector. For simplicity, assume that the basis is orthonormal. we can express any signal *x* as:

$$x=\sum_{i=1}^{N} s_i \Psi_i \;\text{ or }\; x=\Psi s \qquad (4)$$

where *s* is the *N* ×1 column vector, *x* and *s* are equivalent representations of the same signal, with *x* in the time domain and *s* in the Ψ domain. We will focus on signals that have a sparse representation which is fourier transform domain here.

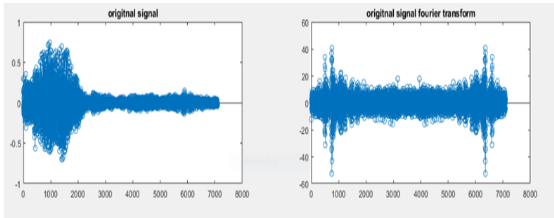

Fig 2: Signal in different domains

The CS acquisition model can be described mathematically by

$$y=\Phi x \qquad (5)$$

where y is the compressed signal of length m, Φ is Gaussian random matrix and x is the input signal represented in transformed domain shown in eq (1). The compressed signal is recovered back by using only m samples.

6. RECONSTRUCTION OF COMPRESSED SIGNAL

The inputs to the reconstruction algorithm are the measurement vector y and reconstruction matrix Φ and

$$y= \Phi\Psi s = \Theta s \qquad (6)$$

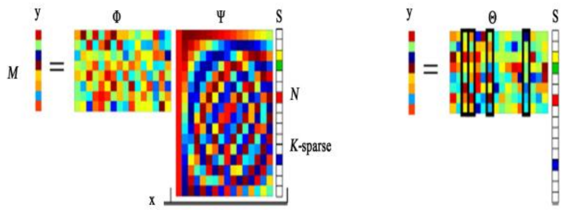

Fig 3: Representation of equation (6)

Where Θ = ΦΨ is an *M* × *N* matrix. The measurement process is non-adaptive; that is, Φ does not depend in any way on the signal *x*.

The original signal can be recovered back from compressive measurements which is an underdetermined system of linear equations and have infinite number of possible solutions. In such cases, the unique solution can be obtained by posing the reconstruction problem as an l_0-optimization problem .The l_0-optimization problem searches for a solution having minimum l_0-norm subject to the given constraints. This is equivalent to trying all the possibilities to and the desired solution.

$$\hat{S}=\arg\min_s ||s||_1 \qquad (7)$$

where $||s||_1$ denotes the l_1-norm of s, which represents the absolute sum of elements of a vector. The generalized expression of a norm is given by (8), from which definition of l_1 and other relevant norms can be obtained wherever required.

$$l_p: ||x||_p = \sqrt[p]{\sum_i |x|^p} \qquad (8)$$

Inorder to reconstruct the signal an algorithm should be required. In this context basis pursuit algorithm is used.

7. BASIS PURSUIT ALGORITHM

Basis Pursuit (BP) is a convex optimization problem, which searches for a solution having minimum l_1-norm, subject to the equality constraint given in (4). BP is used in CS to find the sparse approximation $\hat{S}$ of input signal x, in dictionary or reconstruction matrix Θ, from compressive measurements y. BP can recover faithfully only if, the measurements are noise-free.

Basis pursuit is the equality-constrained l_1 minimization problem minimize $||x||_1$ subject to $Ax = b$, with variable $x \in \mathbf{R}^n$, data $A \in \mathbf{R}^{m\times n}$, $b \in \mathbf{R}^m$, with $m<n$. Basis pursuit is often used as a heuristic for finding a sparse solution to an underdetermined system of linear equations. It plays a central role in modern statistical signal processing, particularly the theory of compressed sensing.

In ADMM form, basis pursuit can be written as

$$\text{Minimize } f(x)+||z||_1 \qquad (9)$$
$$\text{subject to } x-z=0 \qquad (10)$$

where f is the indicator function of $\{x\in R^n \,|Ax=b\}$. The ADMM algorithm is then

$$x^{k+1} : ¶(z^k - u^k) \qquad (11)$$

$$Z^{k+1} : s(x^{k+1}+u^k) \qquad (12)$$

$$u^{k+1} : u^k + x^{k+1} - z^{k+1} \qquad (13)$$

where ¶ is the projection on the $\{x\in R^n \,|Ax=b\}$. The x-update, when involves solving a linearly constrained minimum Euclidean norm problem can be written explicitly as

$$x^{k+1}: (I-A^T(AA^T)^{-1}A)(z^k-u^k)+A^T(AA^T)^{-1}*b. \quad (14)$$

Again the comments on efficient computation by caching a factorization of AA^T, subsequent projections are much cheaper than the first one. We can interpret ADMM for basis pursuit as reducing the solution of a least l_1 norm problem to solving a sequence of minimum Euclidean norm problems.

8. RESULTS AND DISCUSSION

The speech signal is taken as input. The representation of speech signal in MATLAB is shown in fig. 4

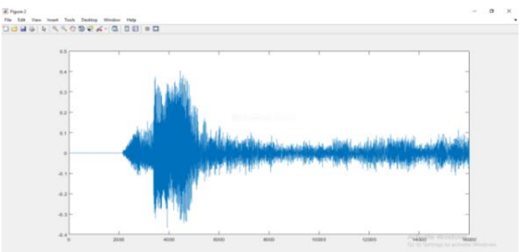

Fig 4: Input speech signal

In the above signal, significant information is mainly concentrated in frequency ranges between 3500 Hz to10600 Hz. Hence considering the signal within the range the above signal will shown in Fig 5.

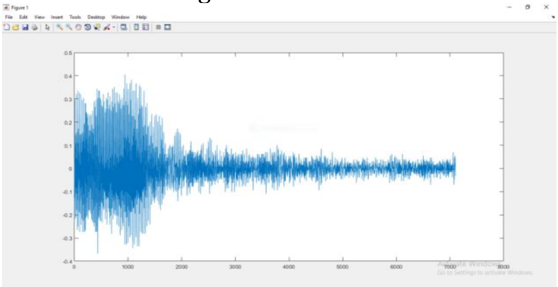

Fig 5: Required range of fig 4

Fig 5 is given as input, which contains 7100 samples. We were reduced them to 3000 samples and basis pursuit algorithm is implemented and signal is recovered in MATLAB (in the absence of external noise).

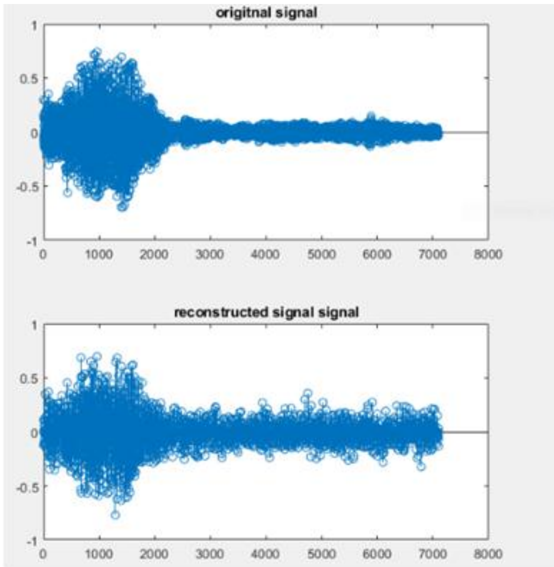

Fig 6: Results obtained after implementation of algorithm

9. CONCLUSIONS

The undersampled speech signals were successfully reconstructed using compressive sensing. The algorithm used for optimising the signal was basis pursuit. The algorithm uses l_1-norm minimisation of overall energy of the samples. The simulation results shows that more than 90% of the samples were recovered.

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