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Numerical Analysis for Nonlinear Consolidation of Saturated Soil using Lattice Boltzmann Method

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Abstract - *In this paper, a new numerical method for one-dimensional (1D) nonlinear consolidation analysis of saturated soil is proposed on the basis of the lattice Boltzmann method. At first, the lattice Bhatnagar-Gross-Krook (LBGK) model is used for 1D nonlinear consolidation problem of saturated soil subjected to time-dependent loading under different types of boundary conditions. In addition, the multiscale Chapman-Enskog expansion is applied to recover mesoscopic lattice Boltzmann equation to macroscopic nonlinear consolidation equation. As a result of the numerical simulation for verification, the numerical results are proved to be in good agreement with the analytical solutions available in previous literature. Finally, the numerical simulation is performed to investigate the consolidation behavior of saturated soil subjected to two different types of time-dependent loading.*

Key Words: Nonlinear consolidation, Saturated soil, Lattice Boltzmann method, Time-dependent loading

1.INTRODUCTION

It is very important in predicting settlement of ground composed of soft soil to analyze one-dimensional (1D) consolidation by taking nonlinear behavior of the ground into account. Since the study on 1D nonlinear consolidation theory was started in the 1960s, many researchers have suggested different kinds of 1D nonlinear consolidation theory. Davis and Raymond [1] developed a nonlinear consolidation theroy and derived an analytical solution for a constant loading case, assuming that the decrease in permeability is proportional to the decrease in compressibility during the consolidation process and that the distribution of initial effective stress is constant with depth. Based on the relationship between the void ratio and the logarithm of effective stress and permeability (i.e. e-log σ' and e-log k_w), many scholars have solved the similar problem using finite difference method [2-4]. Gibson et al. [5, 6] proposed the general theories of 1D finite nonlinear consolidation of thin and thick homogeneous layers for a constant loading condition. Xie et al. [7] developed analytical solution for 1D consolidation of soft soil subjected to time-dependent loading on the basis of the nonlinear consolidation theory proposed by Davis and Raymond. Chen et al. [8] and Zheng et al. [9] carried out numerical analysis for 1D nonlinear consolidation of saturated soil by differential quadrature method. Cheng et al. [10] developed the finite analytic method to simulate 1D nonlinear consolidation under different time-dependent loading and initial conditions.

It is worth to note that the nonlinear consolidation of soil is governed by partial differential equation which is difficult to obtain analytical solution, except for specifec conditions, and thus numerical methods are still the most important means for analyzing the nonlinear consolidation problem. Recently, unlike conventional numerical methods based on macroscopic equation, the lattice Boltzmann method (LBM) which is based on mesoscopic equation has emerged as an alternative powerful method for solving fluid dynamics problems and achieved much success in studying nonlinear equations of complex systems [11,12]. Compared to traditional numerical methods, due to the advantages such as the simplicity of programming and the numerical efficiency, the LBM has been widely applied not only to fluid dynamics but also to many other areas, such as advection-diffusion problem [13], soil dynamics [14] and so on. More recently, Kim et al. [15] employed the LBM to analyze 1D linear consolidation of saturated clay. Previous studies show that LBM can be used in various engineering disciplines. Nevertheless, the LBM has hardly ever been used for nonlinear consolidation analysis of soil. Thus, the goal of the present study is to extend the LBM into 1D nonlinear consolidation analysis of saturated soil.

In this paper, a new numerical method for 1D nonlinear consolidation analysis of saturated soil is proposed on the basis of the lattice Boltzmann method. The lattice Bhatnagar-Gross-Krook (LBGK) model is employed for 1D nonlinear consolidation of saturated soil subjected to time-dependent loading under various boundary conditions. In order to recover mesoscopic lattice Boltzmann equation to macroscopic nonlinear consolidation equation, the multiscale Chapman-Enskog expansion is applied. As a result of the numerical simulation for verification, the numerical results are proved to be in good agreement with the analytical solutions available in previous literature. The numerical simulation is performed to investigate the consolidation behavior of saturated soil subjected to two different types of time-dependent loading.

This paper is organized as follows. In Section 2, 1D nonlinear consolidation equation of saturated soil subjected to time-dependent loading is presented and in Section 3, the lattice Boltzmann method for 1D nonlinear consolidation of saturated soil is proposed. In Section 4, the numerical results are compared with the analytical solutions available in the literature in order to verify the proposed method, and also the numerical simulation is performed to investigate the consolidation behavior of saturated soil subjected to two different time-dependent loading. Finally, Section 5 gives conclusions.

2. MATHEMATICAL MODEL

A saturated soil layer of thickness H_t ($H_t = 2H$ for double drainage condition; $H_t = H$ for single drainage condition) subjected to time-dependent loading is considered as shown in Fig. 1. Assuming the validity of the nonlinear consolidation theory proposed by Davis and Raymond [1], except for the assumption of a constant loading, the governing equation of 1D nonlinear consolidation of saturated soil subjected to time-dependent loading is as follows:

$$\frac{\partial u}{\partial t} = c_v \left[\frac{\partial^2 u}{\partial z^2} + \frac{1}{\sigma'} \left(\frac{\partial u}{\partial z} \right)^2 \right] + \frac{\mathrm{d}q}{\mathrm{d}t} \quad (1)$$

where u, q, c_v and σ' are excess pore water pressure, time-dependent loading, the coefficient of consolidatioin and the effective stress, respectively; t and z are the variables of time and space respectively.

According to the assumption that the coefficient of consolidation is constant while the decrease in permeability is propotional to the decrease in compressibility,

$$c_v = \frac{k_{w0}}{\gamma_w m_{v0}} = \text{const.} \quad (2)$$

where k_{w0} is the initial coefficient of permeability; γ_w is the unit weight of water; m_{v0} is the initial coeffiecient of compressibility defined as $m_{v0} = 0.434 C_c / (1+e_0)\sigma_0'$ in which C_c is the compression index, e_0 is the initial void ratio and σ_0' is the initial effective stress.

According to Terzaghi's principle of effective stress, σ' can be expressed as:

$$\sigma' = q + \sigma_0' - u \quad (3)$$

By defining a new parameter ω,

$$\omega = \ln \frac{\sigma'}{\sigma_0' + q} \quad (4)$$

Eq. (1) can be simplified to the following form:

$$\frac{\partial \omega}{\partial t} = c_v \frac{\partial^2 \omega}{\partial z^2} + R(t) \quad (5)$$

where

$$R(t) = -\frac{1}{\sigma_0' + q} \frac{\mathrm{d}q}{\mathrm{d}t} \quad (6)$$

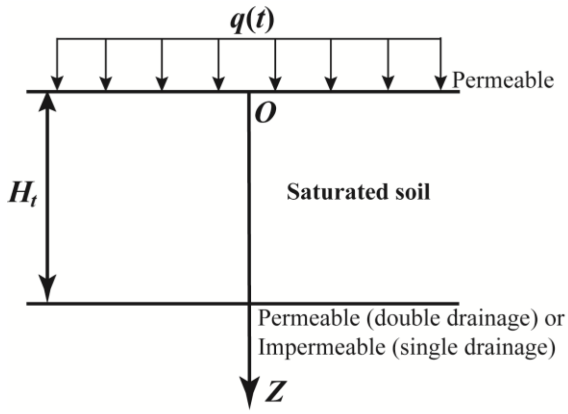


Fig. 1: A saturated soil layer subjected to time-dependent loading.

Initial and boundary conditions are considered as follows:

$$\omega|_{t=0} = 0 \quad (7)$$

$$\omega|_{z=0} = 0,\ \omega|_{z=2H} = 0 \text{ (for double drainage condition)} \quad (8)$$

$$\omega|_{z=0} = 0,\ \left.\frac{\partial \omega}{\partial z}\right|_{z=H} = 0 \text{ (for single drainage condition)} \quad (9)$$

3. LATTICE BOLTZMANN METHOD

3.1 Lattice Boltzmann Equation

In the present study, 1D consolidation analysis is carried out using lattice Bhatnagar-Gross-Krook (LBGK) D1Q3 model which is the most popular one in the 1D lattice Boltzmann models. The distribution function $f_i(z,t)$ is first defined on the basis of the general principles of lattice Boltzmann model as follows:

$$\omega = \sum_i f_i(z,t) = \sum_i f_i^{eq}(z,t),\ i = 0,\ 1,\ 2 \quad (10)$$

where f_i and f_i^{eq} are the distribution function and the equilibrium distribution function along direction i .

Based on the LBGK model, the lattice Boltzmann equation for Eq. (10) is given by He and Luo [16]:

$$f_i(z + e_i \Delta t, t + \Delta t) = f_i(z,t) - \frac{1}{\tau}\left[f_i(z,t) - f_i^{eq}(z,t) \right] + \Delta t F_i \quad (11)$$

where e_i and Δt are the discrete velocity and the discrete time step respectively; τ is the dimensionless relaxation time; F_i is the force term calculated by:

$$F_i = w_i S$$

where w_i is the weight factor with $w_0 = 2/3$ and $w_1 = w_2 = 1/6$; S is the rate of loading.

The LBKG model given by Eq. (11) is performed by two procedures, a collision and a streaming expressed as:

Collision:

$$f_i(z, t+\Delta t) - f_i(z,t) = \frac{1}{\tau}\left[f_i^{eq}(z,t) - f_i(z,t) \right] + \Delta t w_i S \quad (12)$$

Streaming:

$$f_i(z+\Delta z, t+\Delta t) = f_i(z, t+\Delta t) \qquad (13)$$

In the D1Q3 model, the discrete velocities $e_i (i = 0,1,2)$ are defined as follows:

$$e_i = \begin{cases} 0, & i=0 \\ c, & i=1 \\ -c, & i=2 \end{cases} \qquad (14)$$

$$c = \frac{\Delta z}{\Delta t}$$

where c is the lattice speed.

The equilibrium distribution function is given by:

$$f_i^{eq} = w_i \omega, \ (i = 0,1,2) \qquad (15)$$

Boundary conditions for Eqs. (8) and (9) is given by:

$$f_i(z_1,t) = 0, \ f_i(z_N,t) = 0 \quad \text{(for double drainage)} \qquad (16)$$

$$f_i(z_1,t) = 0, \ f_i(z_N,t) = f_i(z_{N-1},t) \quad \text{(for single drainage)} \qquad (17)$$

where $f_i(z_1,t)$, $f_i(z_N,t)$ and $f_i(z_{N-1},t)$ are the distribution functions for the first lattice node z_1, the last lattice node z_N and the N-1th lattice node z_{N-1}, respectively.

3.2 Recovery of 1D nonlinear consolidation equation

The multiscale Chapman-Enskog expansion is used to recover the macroscopic 1D nonlinear consolidation equation of saturated soil. The 1D nonlinear consolidation equation can be scaled spatially as, z is set to z_1/ε, t is set to t_1/ε^2 where ε is a small parameter.

For D1Q3 model, ω can be expressed by Eqs. (10) and (15) as follows:

$$\omega = \sum_i f_i(z,t) = f_0(z,t) + f_1(z,t) + f_2(z,t) \qquad (18)$$

$$\omega = \sum_i f_i^{eq}(z,t) = w_0\omega(z,t) + w_1\omega(z,t) + w_2\omega(z,t) \qquad (19)$$

The distribution function and the source term can be expressed in terms of a small parameter ε as:

$$f_i = f_i^{eq} + \varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)} \qquad (20)$$

$$F_i = \varepsilon^2 F_i^{(2)} \qquad (21)$$

where $f_i^{(k)}$ and $F_i^{(2)}$ are the non-equilibrium distribution functions and non-equilibrium force term defined by:

$$\sum_i F_i^{(2)} = F^{(2)} \qquad (22)$$

Since

$$\omega = \sum_i f_i = \sum_i \left[f_i^{eq} + \varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)} \right] \qquad (23)$$

and $\omega = \sum_i f_i^{eq}$, hence other expanded term in the above equation should be zero, i.e.,

$$\sum_i f_i^{(k)} = 0, \ (k \geq 1) \qquad (24)$$

Applying the Taylor series, the updated distribution function is expanded:

$$f_i(z+e_i\Delta t, t+\Delta t) = f_i(z,t) + \frac{\partial f_i}{\partial z} e_i \Delta t + \frac{\partial f_i}{\partial t}\Delta t + \frac{\Delta t^2}{2}\left(\frac{\partial^2 f_i}{\partial z^2} e_i e_i + 2\frac{\partial^2 f_i}{\partial z \partial t} e_i + \frac{\partial^2 f_i}{\partial t^2} \right) + O\left(\Delta t^3\right) \qquad (25)$$

Introducing scaling for the above equation, i.e., $1/\partial z$ is replaced by $\varepsilon/\partial z_1$, $1/\partial t$ is replaced by $\varepsilon^2/\partial t_1$:

$$f_i(z+e_i\Delta t, t+\Delta t) = f_i(z,t) + \varepsilon\frac{\partial f_i}{\partial z_1} e_i \Delta t + \varepsilon^2\frac{\partial f_i}{\partial t_1}\Delta t + \frac{\Delta t^2}{2}\left(\frac{\varepsilon^2\partial^2 f_i}{\partial z_1^2} e_i e_i + 2\frac{\varepsilon^3\partial^2 f_i}{\partial z_1 \partial t_1} e_i + \frac{\varepsilon^4\partial^2 f_i}{\partial t_1^2} \right) + O\left(\Delta t^3\right) \qquad (26)$$

Substituting Eqs. (20) and (26) into Eq. (11) and retaining terms up to order of ε^2, yields:

$$-\frac{1}{\tau\Delta t}\left[\varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)} \right] + \varepsilon^2 F_i^{(2)} = \varepsilon\frac{\partial f_i^{eq}}{\partial z_1} e_i + \varepsilon^2\frac{\partial f_i^{(1)}}{\partial z_1} e_i + \varepsilon^2\frac{\partial f_i^{eq}}{\partial t_1} + \frac{\Delta t}{2}\frac{\varepsilon^2\partial^2 f_i^{eq}}{\partial z_1^2} e_i e_i + O\left(\varepsilon^3\right) + O\left(\Delta t^2\right) \qquad (27)$$

Comparing the two sides of Eq. (27) and treating terms in order of ε and ε^2, yields:

Terms order of ε:

$$-\frac{1}{\tau\Delta t} f_i^{(1)} = \frac{\partial f_i^{eq}}{\partial z_1} e_i \qquad (28)$$

Terms order of ε^2:

$$-\frac{1}{\tau\Delta t} f_i^{(2)} + F_i^{(2)} = \frac{\partial f_i^{(1)}}{\partial z_1} e_i + \frac{\partial f_i^{eq}}{\partial t_1} + \frac{\Delta t}{2}\frac{\partial^2 f_i^{eq}}{\partial z_1^2} e_i e_i \qquad (29)$$

Applying Eq. (28) to the right side of Eq. (29), yields:

$$-\frac{1}{\tau\Delta t} f_i^{(2)} + F_i^{(2)} = \frac{\partial f_i^{eq}}{\partial t_1} - (\tau\Delta t - \frac{\Delta t}{2})\frac{\partial^2 f_i^{eq}}{\partial z_1^2} e_i e_i \qquad (30)$$

For recovering the 1D nonlinear consolidation equation Eq. (5), Eq. (30) is summed over all states, i.e.,

$$\sum_i\left(-\frac{1}{\tau\Delta t} f_i^{(2)} \right) + \sum_i F_i^{(2)} = \sum_i \frac{\partial f_i^{eq}}{\partial t_1} - (\tau\Delta t - \frac{\Delta t}{2})\sum_i\left(\frac{\partial^2 f_i^{eq}}{\partial z_1^2} e_i e_i \right) \qquad (31)$$

According to Eqs. (22) and (24), $\sum_i F_i^{(2)} = F^{(2)}$ and $\sum_i\left(-\frac{1}{\tau\Delta t} f_i^{(2)} \right) = 0$. Then, both terms of the right side of the above equation are written as, respectively:

$$\sum_i \frac{\partial f_i^{eq}}{\partial t_1} = \frac{\partial \sum_i f_i^{eq}}{\partial t_1} = \frac{\partial \omega}{\partial t_1} = \frac{1}{\varepsilon^2}\frac{\partial \omega}{\partial t} \qquad (32)$$

$$\sum_i \left(\frac{\partial^2 f_i^{eq}}{\partial z_1^{\ 2}} e_i e_i \right) = \frac{\partial^2}{\partial z_1^{\ 2}} \sum_i \left(f_i^{eq} e_i e_i \right) = \frac{c^2}{\varepsilon^2} \frac{\partial^2 \omega}{\partial z^2} \quad (33)$$

Meanwhile, the source term F is taken as:

$$F = \sum_i F_i = \sum_i w_i S = R(t) \quad (34)$$

Hence, Eq. (31) can be simplified and the 1D nonlinear consolidation equation can be recovered as:

$$\frac{\partial \omega}{\partial t} = c^2 \Delta t (\tau - \frac{1}{2}) \frac{\partial^2 \omega}{\partial z^2} + R(t) \quad (35)$$

By comparing Eq. (5) with Eq. (35), the relationship between the coefficient of consolidation c_v and the dimensionless relaxation time τ can be obtained.

$$c_v = c^2 \Delta t (\tau - \frac{1}{2}) \quad (36)$$

4. RESULTS AND DISCUSSION

4.1 Verification

In order to verify the proposed LBGK model for 1D nonlinear consolidation analysis, the numerical simulation is performed for two cases in terms of instantaneous and ramp loading. And the numerical results are compared with the analytical solutions available in previous literature. For the numerical simulation, the domain is discretized into 100 nodes; the dimensionless relaxation time, the discrete lattice spacing and the time step are set as 1.

4.1.1 Verification under instantaneous loading

In order to verify the proposed LBGK model for 1D nonlinear consolidation analysis under instantaneous loading, analytical solutions derived by Davis and Raymond [1] is used for reference. Figs. 2 and 3 show the excess pore water pressure isochrones and the average degree of consolidation with time factor defined by $T_v = c_v t / H^2$ for double drainage condition. It can be seen that the numerical results are in good agreement with analytical solution.

4.1.2 Verification under time-dependent loading

For time-dependent loading, analytical solutions for ramp loading given by Xie et al. [7] is selected for reference. The ramp loading as shown in Fig. 4 can be expressed as follows:

$$q(t) = \begin{cases} \frac{q_u}{t_c} \cdot t, & t < t_c \\ q_u, & t \geq t_c \end{cases} \quad (37)$$

where q_u is the ultimate loading; t_c is the time of application of any load (i.e. construction time).

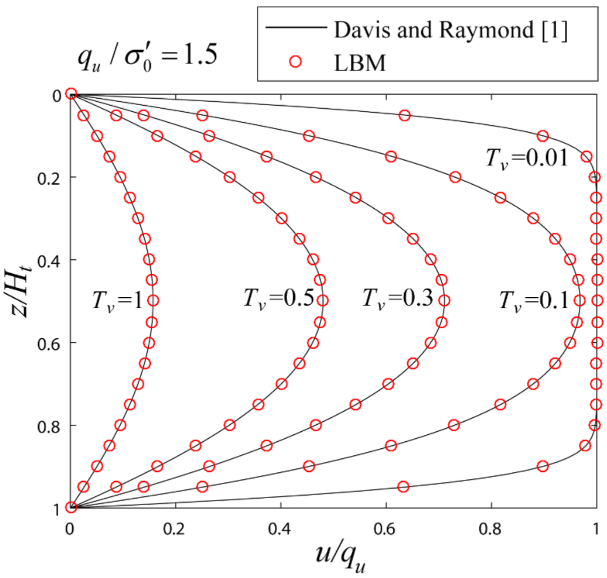


Fig. 2: The excess pore water pressure isochrones.

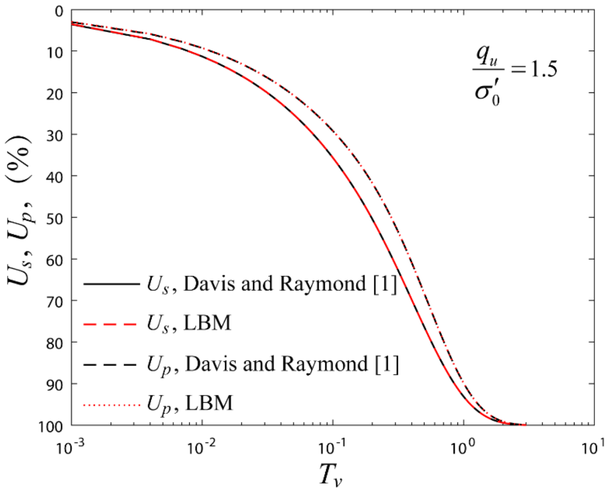


Fig. 3: The average degree of consolidation with time factor.

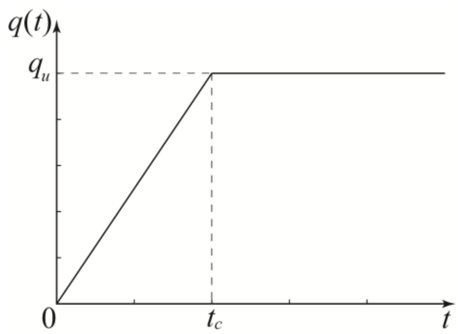


Fig. 4: Ramp loading

Fig. 5 presents the normalized excess pore water pressure with time factor T_v at half the depth for double drainage condition and different construction time factors. The construction time factor is defined by $T_{vc} = c_v t_c / H^2$.

It is found that two solutions are in excellent agreement. Fig. 6 shows the average degree of consolidation under the different construction time factors. It is also shown that the LBM results are agree well with the analytical solutions at all construction time factors.

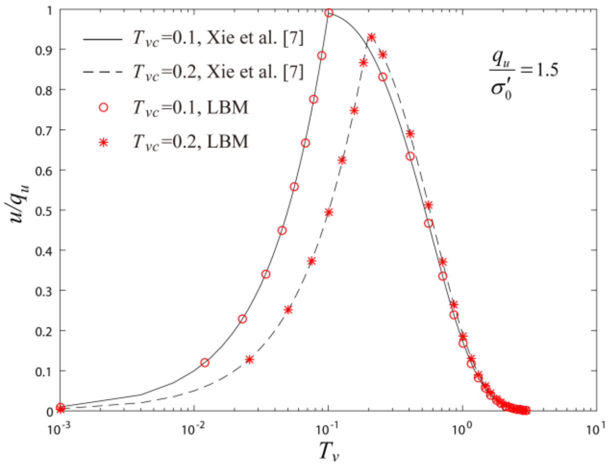


Fig. 5: Variation of the excess pore water pressure under different construction time factors.

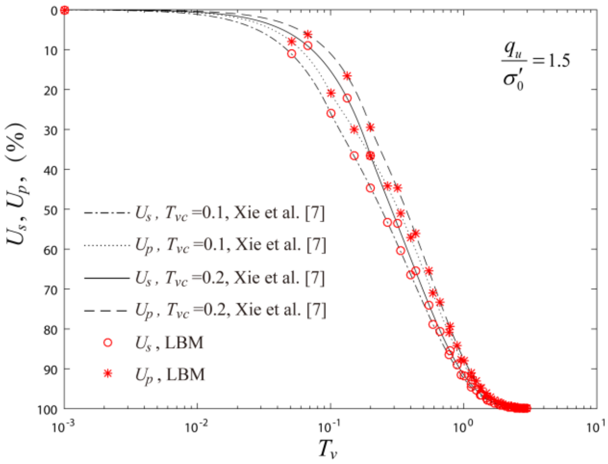


Fig. 6: The average degree of consolidation under different construction time factors

4.2 Example and discussion

Two different types of time-dependent loadings shown in Fig. 7, i.e., exponential and haversine cyclic loadings are considered and the loading functions are expressed as follows.

The exponential loading is expressed as

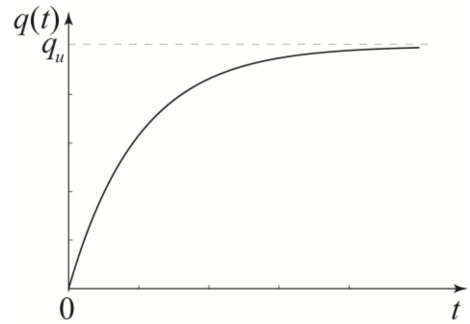
$$q(t) = q_u(1-e^{-bt}) \qquad (38)$$

where b is the loading parameter controlling the rate of exponential loading.

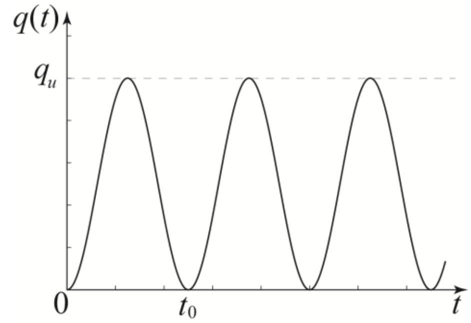
The haversine cyclic loading is expressed as

$$q(t) = q_u \sin^2 \frac{\pi t}{t_0} \qquad (39)$$

where t_0 is the loading parameter which is period of haversine cyclic loading.



(a): Exponential loading



(b): Haversine cyclic loading

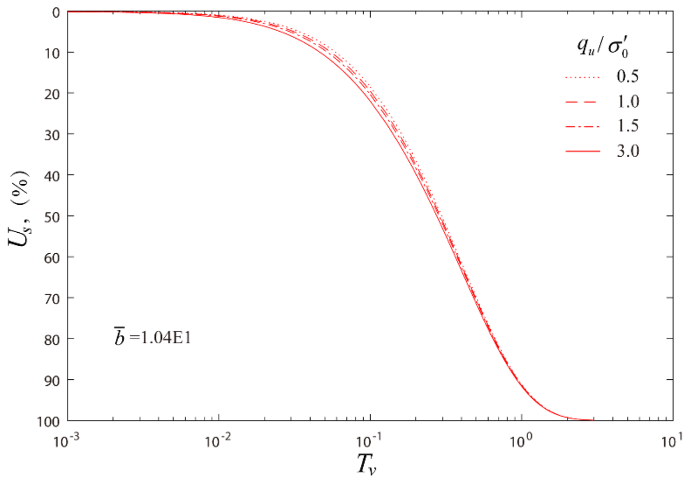
Fig. 7: Example loading type.

Furthermore, based on the proposed method, the numerical simulation is carried out to investigate the influence of the ratio of ultimate loading intensity to initial effective stress and the loading parameters on 1D nonlinear consolidation of saturated soil under double drainage condition.

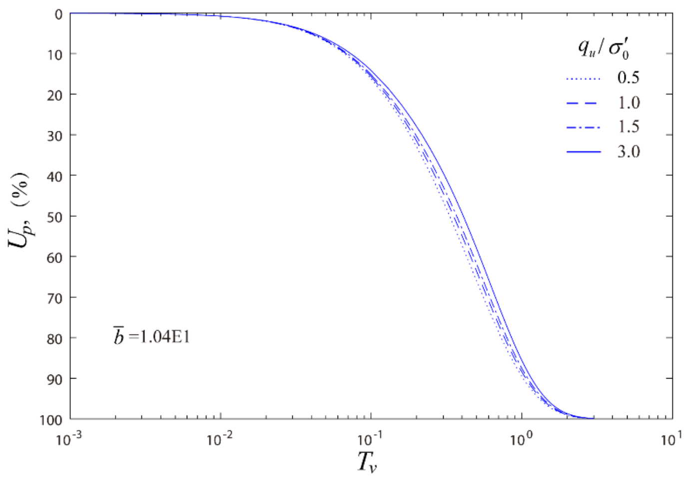
4.2.1 Consolidation under different values of the ratio of ultimate loading intensity to initial effective stress

Fig. 8 shows the variation of average degree of consolidation with time factor under different ratio of ultimate loading intensity to initial effective stress for the exponential loading with the dimensionless loading parameter $\bar{b} = 1.04E1$, which is defined by $\bar{b} = H^2 b / c_v$. Figs. 8(a) and 8(b) present the changes of average degree of consolidation defined by settlement and effective stress with time factor, respectively. It can be seen that the average degree of consolidation U_s defined by settlement increases with the increase of the ratio q_u / σ'_0, but the average degree of consolidation U_p defined by effective stress decreases with the increase of the ratio q_u / σ'_0.

Fig. 9 shows the results at the different ratio q_u / σ'_0 under the haversine cyclic loading with the dimensionless loading parameter $T_0 = 0.05$, which is defined by $T_0 = c_v t_0 / H^2$. Similar to the results from the exponential loading, it can be found that the average degree of consolidation U_s increases with the increase of the ratio q_u/σ_0', but the average degree of consolidation U_p decreases.



(a): U_s-T_v curves



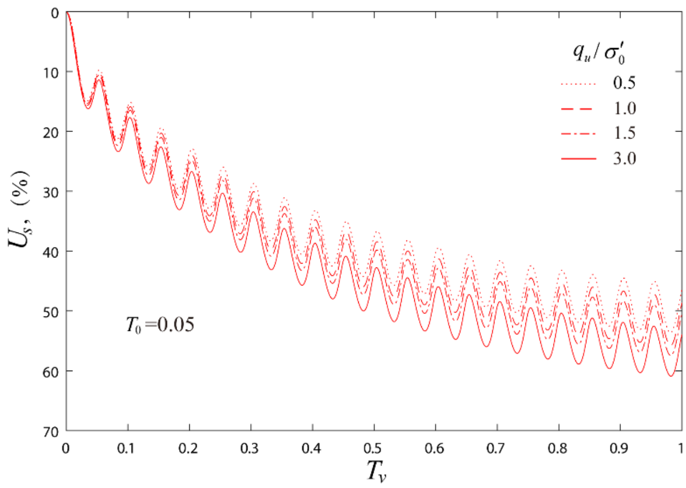
(b): U_p-T_v curves

Fig. 8: Variation of average degree of consolidation with time factor under different ratio q_u/σ_0' for exponential loading.

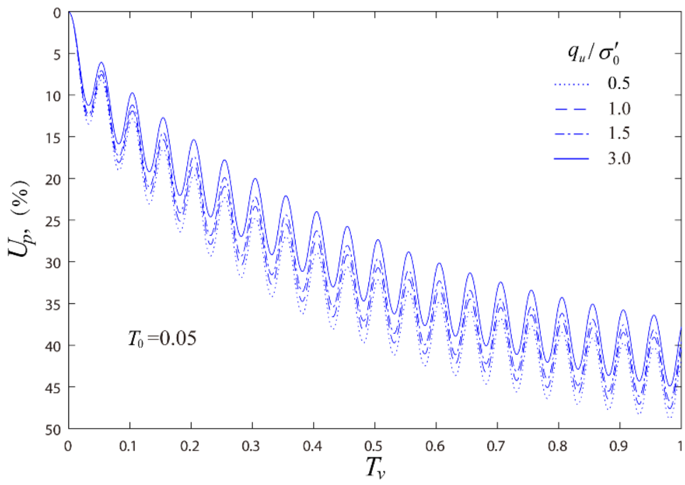
4.2.2. Consolidation under different loading parameters

Fig. 10 shows the variation of average degree of consolidation with time factor at different dimensionless loading parameter $\bar{b}$ under exponential loading with the ratio q_u/σ_0'=1.5. It can be found that the dimensionless loading parameter $\bar{b}$ has significant effects on the rate of settlement and the rate of dissipation of excess pore water pressure, and a bigger dimensionless loading parameter $\bar{b}$ leads to a faster rate of settlement and dissipation of excess pore water pressure. Moreover, both the settlement and the dissipation of excess pore water pressure tend to proceed more quickly at the early stage of consolidation as the demensionless loading parameter $\bar{b}$ increases.

Fig. 11 shows the results at different dimensionless loading parameter T_0 under the harversine cyclic loading with the ratio q_u/σ_0'=1.5. It can be seen that a smaller value of the parameter T_0 induces more cycles of oscillation, while a bigger value of the parameter T_0 results in a larger oscillation in the settlement and the dissipation of excess pore water pressure.



(a): U_s-T_v curves



(b): U_p-T_v curves

Fig. 9. Variation of average degree of consolidation with time factor under different ratio q_u/σ_0' for haversine cyclic loading.

5. CONCLUSION

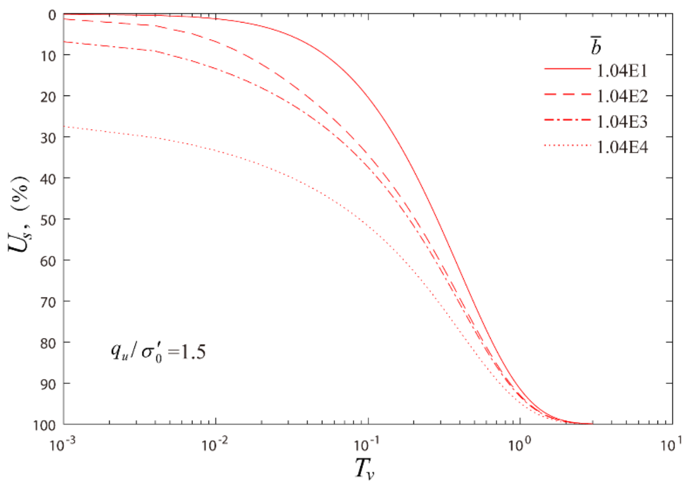
In this paper, a new numerical method for 1D nonlinear consolidation analysis of saturated soil is proposed on the basis of the lattice Boltzmann method. The lattice Bhatnagar-Gross-Krook (LBGK) model is employed for 1D nonlinear consolidation analysis of saturated soil subjected to time-dependent loading under different types of boundary conditions. In addition, the multiscale Chapman-Enskog expansion is applied to recover mesoscopic lattice Boltzmann equation to macroscopic consolidation equation. As a result of the numerical simulation for verification, the numerical results are proved to be in good agreement with the analytical solutions available in previous literature.

The consolidation behavior of saturated soil subjected to exponential loading and haversine cyclic loading is investigated through the numerical simulation. The following conclusions can be drawn:

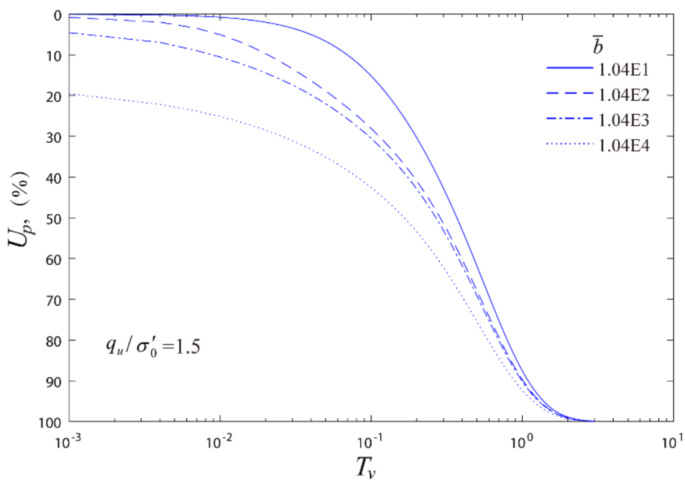
(1) As the ratio q_u/σ_0' increases, the rate of settlement increases but the rate of dissipation of excess pore water pressure decreases.

(2) A bigger dimensionless loading parameter $\bar{b}$ induces a faster rate of the settlement and the dissipation of excess pore water pressure. Moreover, both the settlement and the dissipation of excess pore water pressure tend to proceed more quickly at the early stage of consolidation as the demensionless loading parameter $\bar{b}$ increases.

(3) A smaller value of the dimensionless loading parameter T_0 induces more cycles of oscillation, while a bigger value of the parameter T_0 results in a larger oscillation in the settlement and the dissipation of excess pore water pressure.

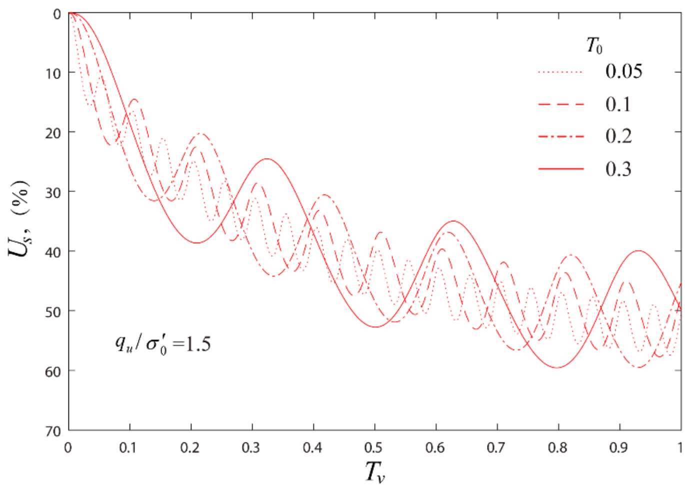


(a): U_s-T_v curves

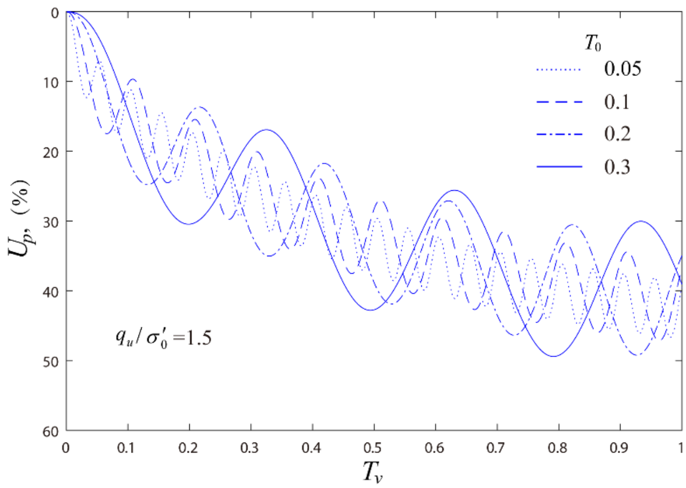


(b): U_p-T_v curves

Fig. 10. Variation of average degree of consolidation with time factor under different loading parameter for exponential loading.



(a): U_s-T_v curves



(b): U_p-T_v curves

Fig. 11. Variation of average degree of consolidation with time factor under different loading parameter for haversine cyclic loading.

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REFERENCES

[1] E.H. Davis and G.P. Raymond, A non-linear theory of consolidation, Geotechnique 15(2), 1965, pp. 161-173.

[2] L. Barden and P.L. Berry, Consolidation of normally consolidated clay, Journal of the Soil Mechanics and Foundation Division, ASCE 91 (SM5), 1965, pp. 5–35.

[3] T.I. Poskitt, The numerical solution of non-linear consolidation problems, International Journal for Numerical Methods in Engineering, Vol. 3, 1971, pp. 5-11.

[4] G. Mesri and A. Rokhsar, Theory of consolidation for clays. ASCE 100(GT8), 1974, pp. 889–903.

[5] R.E. Gibson, G.L. England and M.J.L. Hussey, The theory of one-dimensional consolidation of saturated clays. I. Finite non-linear consolidation of thin homogeneous layers [J]. Geotechnique 17(2), 1967, pp. 261-273.

[6] R.E. Gibson, R.L. Schiffman and K.W. Cargill, The theory of onedimensional soil consolidation of saturated clays: II. Finite nonlinear consolidation of thick homogeneous layers, Can. Geotech J. 18, 1981, pp. 280–293.

[7] K.H. Xie, B.H. Li and Q.L. Li, A nonlinear theory of consolidation under time-dependent loading, In: Proceedings of the Second International Conference on Soft Soil Engineering, Nanjing, China, 27–30 May 1996, pp. 193–196.

[8] R.P. Chen et al., One-dimensional nonlinear consolidation of multi-layered soil by differential quadrature method, Computers and Geotechnics 32 (2005), pp. 358–369.

[9] G.Y. Zheng et al., Analysis of non-linear consolidation of soft clay by differential quadrature method, Applied Clay Science 79(2013), pp. 2-7.

[10] D. Cheng et al., Finite Analytic Method for One-Dimensional Nonlinear Consolidation under Time-Dependent Loading. Shock and Vibration, Vol. 2017, 2017, pp. 12, Article ID 4071268.

[11] R. Benzi, S. Succi, M. Vergassola, The Lattice Boltzmann Equation: Theory and Application, Physics Reports, Vol. 222, No. 3, 1992, pp. 145-197.

[12] B.C. Shi, Z.L. Guo, Lattice Boltzmann Model for Nonlinear Convection-Diffusion Equations. Physics Review E, Vol. 79, 2009, 016701, 1-13, DOI:https://doi.org/10.1103/PhysRevE.79.016701.

[13] Z.L. Guo, B.C. Shi, N.C. Wang, Fully Lagrangian and Lattice Boltzmann Method for the Advection-Diffusion Equation. Journal of Scientific Computing Vol. 14, No. 3, 1999, pp. 291-300.

[14] U.E. Shamy, Y. Abdelhamid, Modeling granular soils liquefaction using coupled lattice Boltzmann method and discrete element method. Soil Dynamics and Earthquake Engineering 67 (2014), 2014, pp. 119–132.

[15] P. Kim et al., Lattice Boltzmann method for consolidation analysis of saturated clay, Journal of Ocean Engineering and Science, 2019, DOI: 10.1016/j.joes.2019.04.004.

[16] X. He and L.S. Luo, Lattice Boltzmann model for the incompressible Navier– Stokes equation, J. Stat. Phys. 88, 1997, pp. 927–944.