DEVELOPED METHOD FOR OPTIMAL SOLUTION OF TRANSPORTATION PROBLEM

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Abstract – Transportation model is a special case of linear programming problem in which the main objective is to transport homogeneous commodity from various origins to different destinations at a total minimum cost. In this paper, a new developed method is proposed to find an optimal solution, the algorithm for proposed method gives solution in less iteration compare to Vogel's approximation method.

Key Words: Transportation Problem, Basic Feasible Solution, Optimal Solution, Linear Programming

1. INTRODUCTION

Transportation is a logistical problem for organizations especially for manufacturing and transport companies. This method is a useful tool in decision-making process which deals with the distribution of goods from several points, such as factories often known as sources, to a number of points of demand, such as warehouses, often known as destinations. Transportation cost has significant impact on the cost and the pricing of raw materials and goods, supplier try to control the cost of transportation. The way how the desirable minimum transportation cost can be obtained is the subject matter of transportation problems in linear programming. Some conventional methods to find the minimum transportation cost are North West Corner Method (NWCM), Least Cost Method (LCM), Vogel's Approximation Method (VAM) and to find the optimality we are using the MODI method. In this paper, we are introducing a new proposed method for finding an Initial Basic Feasible Solution. Two numerical examples are provided to prove the claim with stepwise procedure of this new method.

1.1 MATHEMATICAL REPRESENTATION

In a transportation model which involves ‘m’ sources, each having availability of Ai (i = 1,2,3,...,m) units of product, ‘n’ destinations, each having demand of Bj (j=1,2,3,...,n) units of that product. The cost Cij is the transportation cost. It is associated with the transportation of on unit of product from i-th source to the j-th destination for each i and j. The objective is to determine the number of units to be transformed from source i to destination j, so that the total transportation cost is minimum.

The equivalent Linear Programming can be written as follows:

Minimize \[ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} \], Objective Function

Subject to, \[ \sum_{j=1}^{n} X_{ij} \leq A_i, \quad i = 1,2,3,...,m \], Constraints

\[ \sum_{i=1}^{m} X_{ij} \leq B_j, \quad j = 1,2,3,...,n. \]

Where, \( X_{ij} \geq 0 \)  Non Negativity

Here, \( X_{ij} \) is the number of units shipped from source i to destination j.

2. ALGORITHM FOR PROPOSED METHOD

STEP1: Examine whether the transportation problem is balanced or not. If it is balanced, then go to next step.

STEP2: Find ‘Second minimum value’ from each row as well as column.

STEP3: Select the row or column from the matrix which has maximum value of ‘Second Minimum Value’, select the cell which has minimum cost coefficient. Allocate smallest value of demand or supply in the cell.

STEP4: Repeat step 2 and step 3 until 2*2 matrix. Apply VAM’s Penalty approach in 2*2 matrix.

STEP5: Now total minimum cost is calculated as sum of the product of cost and corresponding allocate value of supply or demand.

IMPORTANT POINTS OF ALGORITHM:

1) Here, ‘Second lowest value’ from row or column means if 2,3,4,5 and 2,2,3,5 given then 3 and 2 is Second lowest value respectively.

2) In case of tie of maximum value of second minimum between any two or more rows and columns then find average of that rows and columns and allocate the cell according to smallest value of supply or demand having maximum value of average of row or column.

3) Here for 2*2 matrix apply VAM’s penalty approach means, take row difference and column difference called penalty value of each row and column, allocate cell in row or column having maximum value of penalty with minimum cost according to smallest value of supply or demand.
2.1 NUMERICAL EXAMPLES

Example 1: Illustrate

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>150</td>
</tr>
<tr>
<td>S2</td>
<td>7(25)</td>
<td>11</td>
<td>11</td>
<td>175</td>
</tr>
<tr>
<td>S3</td>
<td>4</td>
<td>5</td>
<td>12</td>
<td>275</td>
</tr>
</tbody>
</table>

Demand 200 100 300

Solution: Since $\sum A_i = \sum B_i = 600$, the given transportation problem is balanced; therefore, exist a basic feasible solution to Proposed Method problem.

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>S</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
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</thead>
<tbody>
<tr>
<td>S1</td>
<td>9</td>
<td>5(20)</td>
<td>8</td>
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<td>5</td>
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<td>8</td>
</tr>
<tr>
<td>S4</td>
<td>13</td>
<td>6(5)</td>
<td>9</td>
<td>2</td>
<td>35</td>
<td>40</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>

D 0 0(15) 0(5) 0 20

D 30 0 45 20 35

The transportation cost is:

$$Z = 7*25 + 4*175 + 5*100 + 10*150 + 11*150 = 4525$$

Explanation:

1) Here, in first step we have maximum value of ‘second minimum value’ is 11, which appears in second row and third column. So, we will take average of second row and third column and we got maximum value of average in third column, so we will give allocation to cell C13 of 150 units and eliminate row1.

2) In second step there is maximum value of ‘second minimum’s’ is 12 in column 3, so we will give allocation to cell C23 of 150 units and eliminate column3.

3) In final step for 2*2 matrix, we will take difference of row and column, we get maximum value in column2, so we will give allocation to cell C32 of 100 units and by fulfilling supply and demand constraints we will allocate cell C21 of 25 units and cell C31 of 175 units.

Example 2: Illustrate

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<tr>
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<th>D4</th>
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<tr>
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<td>6</td>
<td>9</td>
<td>2</td>
<td>40</td>
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</tbody>
</table>

Demand 30 45 20 35

Solution: Since $\sum A_i \neq \sum B_i$, hence adding dummy supply of 20 units so that $\sum A_i = \sum B_i = 130$ and the given transportation problem is balanced; therefore, exist a basic feasible solution to Proposed Method problem.

<table>
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<tr>
<th></th>
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<tr>
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<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>

D 0 0(15) 0(5) 0 20

D 30 0 45 20 35

The transportation cost is:

$$Z = 3*30 + 5*20 + 8*5 + 6*5 + 0*15 + 7*15 + 0*5 + 2*35 = 435$$

Explanation:

1) Here, in first step we get maximum value of ‘second minimum value’ of 11, so we will give allocation to cell C23 of 15 units and eliminate row2.

2) In second step, we have maximum value of ‘second minimum value’ is 8, which appears in first row and third column. So, we will take average of first row and third column and we got maximum value of average in third column, so we will give allocation to cell C53 of 5 units and eliminate column3.

3) In third step, we get maximum value of ‘second minimum value’ of 9, so we will give allocation to cell C12 of 20 units and eliminate row1.

4) In fourth step, we have maximum value of ‘second minimum value’ is 6, which appears in fourth row and second column. So, we will take average of fourth row and second column and we got maximum value of average in fourth row, so we will give allocation to cell C44 of 35 units and eliminate column4.

5) In fifth step, we get maximum value of ‘second minimum value’ of 13, so we will give allocation to cell C42 of 5 units and eliminate row4.

6) In final step, for 2*2 matrix, we will take difference of row and column, we get maximum value in column2, so we will give allocation to cell C52 of 15 units and by fulfilling supply and demand constraints we will allocate cell C32 of 5 units and cell C31 of 175 units.
2.2 COMPARISON OF THE NUMERICAL RESULTS

After proving the results by this proposed method, the obtained result is compared with solution of other existing methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Example 1</th>
<th>Example 2</th>
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<tbody>
<tr>
<td>Proposed Method</td>
<td>4525</td>
<td>435</td>
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<tr>
<td>North-west corner Method</td>
<td>5925</td>
<td>895</td>
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<tr>
<td>Least cost Method</td>
<td>4550</td>
<td>585</td>
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<td>VAM</td>
<td>5125</td>
<td>455</td>
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<tr>
<td>MODI-Method</td>
<td>4525</td>
<td>435</td>
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</table>

3. CONCLUSION

The proposed method considers ‘second minimum value’ of each row and each column which is totally new concept. The solution obtained by the proposed method is optimal with less iteration compared to other methods. The developed method is effective for both the large and small size transportation problem.

REFERENCES


BIOGRAPHIES

Poojan Davda student at L.D. college of Engineering, Ahmedabad in B.E final year.