Soft Pre Generalized b-Closed Sets in a Soft Topological Space

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Abstract: This paper introduces soft pre generalized b-closed sets in a soft topological space. In a soft topological space, a soft set F over X is said to be soft generalized b-closed if bcl (F) ⊆ F whenever F ⊆ F and F is a soft P-open set in X. A detail study carried out on properties of soft pre generalized b-closed sets.

Keywords: Soft Pgb- closed set, Soft Pgb- open set, Soft Pgb-closure, soft Pgb b-interior.

1. INTRODUCTION

Soft set theory was first introduced by Molodtsov [1] in 1999 as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on.

In 2010 Muhammad Shabir, Munazza Naz [2] used soft sets to define a topology namely Soft topology. In general topology the concept of generalized closed set was introduced by Levine [3]. This notation has been studied extensively in recent years by many topologists.

The investigation of generalized closed sets has led to several new and interesting concepts, e.g. new covering properties and new separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology.


2. Preliminaries

2.1. Definition [6] A soft set F over the universe X is defined by the set of ordered pairs F = { (e, F(e)) : e ∈ E, f (e) ∈ P(X)}, where E is the parameters set, A ∈ E, P(X) is the powe.r set of X, and f : A → P(X) such that f (e) = ∅ if e ∉ A. Here f (e) is called an approximate function of the soft set F. The value of f (e) may be arbitrary, some of them may be empty, some may have non-empty intersection. Note that the set of all soft set over X is denoted by S(X).

2.2. Definition [6] A soft set F over X is called a null soft set, denoted by F, if e ∈ A, F(e) = ∅.

2.3. Definition [6] A soft set F over X is called a absolute soft set, denoted by A, if e ∈ A, F(e) = X.

2.4. Definition [6] Let F ∈ S(X). A soft topology on F is denoted by τ is a collection of soft subsets of F having the following properties:

   (i) F, F ∈ τ
   (ii) (F ∈ τ : i ∈ I ⊆ N) ⊆ τ =⇒ U_{i∈I} F ∈ τ
   (iii) (F ∈ τ : i≤n∈N) ⊆ τ =⇒ ∩_{i∈N} F ∈ τ.

The pair (F, τ) is called a soft topological space.

2.5. Definition [6] Let (F, τ) be a soft topological space. Then every element of τ is called a soft open set. Clearly F and F are soft open sets. Fc is said to be soft closed if the soft set Fc is soft open in F.

2.6. Definition [5] Let (F, τ) be a soft topological space over X, a soft set F is said to be soft pre-open (soft P-open) if F ∈ cl(int(F)). F is said to be soft pre-closed (soft P-closed) if the soft set F is soft P-open in F.

2.7. Proposition [5]

   i) Every soft open set is a soft pre open set.
   ii) Every soft closed set is a soft pre closed set.

2.8. Definition [7] Let (F, τ) be a soft topological space over X, a soft set F is said to be soft b-open if F ∈ cl(int(F)) ⊆ cl(int(F)). A soft set F is said to be soft b-closed if int(cl(F)) ⊆ cl(int(F)).

2.9. Theorem[7]

In a soft topological space X and let F and F be soft sets over X. Then

   i) bcl(∅) = ∅
   ii) bint(∅) = ∅
   iii) bcl(bcl(F)) = bcl(F)
   iv) bcl(F ∪ A) = bcl(F) ∪ bcl(A)
   v) bcl(F ∩ A) = bcl(F) ∩ bcl(A)

Proof: Refer theorem 7[7]
2.10. Definition [8] Let \((F_b, \mathcal{T})\) be a soft topological space over \(X\). A soft set \(F_A\) is called a soft generalized closed (soft g-closed set) in \(U\) if \(\text{cl}(F_A) \subseteq F_0\) whenever \(F_A \subseteq F_0\) and \(F_0\) is soft open in \(X\).

2.11. Definition [5] Let \((F_b, \mathcal{T})\) be a soft topological space over \(X\). A soft set \(F_A\) is called a soft pre-generalized closed set (soft pg-closed set) in \(X\) if \(\text{pcl}(F_A) \subseteq F_0\) whenever \(F_A \subseteq F_0\) and \(F_0\) is soft pre-open in \(X\).

3. Soft Pre Generalized b-closed set

In this section we introduce soft pre generalized b-closed sets in a soft topological space and study some of their properties.

3.1. Definition Let \((F_b, \mathcal{T})\) be a soft topological space over \(X\). A soft set \(F_A\) is called a soft pre generalized b-closed set (briefly soft pgb-closed) set in \(X\) if \(\text{pcl}(F_A) \subseteq F_0\) whenever \(F_A \subseteq F_0\) and \(F_0\) is soft pre-open in \(X\).

3.2. Definition Let \((F_b, \mathcal{T})\) be a soft topological space and let \(F_b\) be soft subset of \(F_b\). Then the soft pre generalized b-closure (briefly soft pgb-closure) of \(F_A\) denoted by \(\text{pgbcl}(F_A)\) is defined as the intersection of all soft pre generalized b-closed supersets of \(F_A\).

3.3. Example Let \((F_b, \mathcal{T})\) be a soft topological space over \(X\), where \(X=\{h_1, h_2, h_3\}\), \(\mathcal{T} = \{e_1, e_2\}\), \(\mathcal{T} = \{\emptyset, \{h_1, h_2, h_3\}\}\). The set \(F_B = \{(e_1, h_1), (e_2, h_2, h_3)\}\). The soft set \(A_3 = \{(e_1, h_1, h_2, h_3)\}\) is a soft pgb-closed set.

3.4. Theorem. Every soft closed set is soft pgb-closed set.

Proof: Let \(F_A\) be any soft closed set in \(X\) such that \(F_A \subseteq F_0\) where \(F_0\) is soft P-open. Since \(\text{cl}(F_A) \subseteq F_0\) and \(\text{cl}(F_A) = F_0\) bcl(F_A) \(\subseteq F_0\) Hence \(F_A\) is soft pgb-closed set in \(X\).

The converse of the above theorem need not be true as seen from the following example.

3.5. Example

In example 3.3, consider the soft subset \(B_E = \{(e_1, h_2), (e_2, h_1, h_3)\}\). The set \(B_E\) is a soft pgb-closed but not a soft closed set.

3.6. Theorem. Every soft b-closed set is soft pre generalized b-closed set.

Proof: Let \(F_A\) be any soft b-closed set such that \(F_0\) is any soft pre-open set containing \(F_A\). Since \(F_A\) is soft b-closed, \(bcl(F_A) = F_A \subseteq F_0\) Hence \(F_A\) is soft pgb-closed set.

The converse of the above theorem need not true be as seen from the following example.

3.7. Example

In example 3.5, the soft subset \(B_E\) is soft pgb-closed but not a soft b-closed.

3.8. Theorem. Every soft generalized closed set is a soft pgb-closed set.

Proof: Let \(F_A\) be any soft g-closed set in a soft topological space \(X\) and \(F_0\) is any soft open set containing \(F_A\). Since every soft open set is a soft pre-open set, \(\text{bcl}(F_A) \subseteq F_0\) Hence \(F_A\) is soft pgb-closed set.

The converse of the above theorem need not true be as seen from the following example.

3.9. Example

In example 3.3, consider the soft subset \(C_E = \{(e_1, h_2), (e_2, h_1, h_3)\}\). The set \(C_E\) is a soft pgb-closed set but not a soft generalized closed set.

3.10. Theorem. Every soft pg-closed set is a soft pgb-closed set

Proof: Let \(F_A\) be any soft pg-closed set such that \(F_0\) is soft pg-pre open set containing \(F_A\). Hence \(F_A\) is soft pgb-closed set.

The converse of the above theorem need not true be as seen from the following example.

3.11. Example

In example 3.3, Consider the soft subset \(D_E = \{(e_1, h_2, h_3), (e_2, h_1)\}\). The set \(D_E\) is a soft pgb-closed set but not a soft pg-closed set.

3.12. Theorem. If \(F_A\) and \(F_B\) are soft pgb-closed sets in \(X\) then \(F_A \cup F_B\) is a soft pgb-closed set in \(X\).
Proof: Let $F_A$ and $F_B$ be soft pgb-closed sets in $X$ and $F_0$ be any soft pre open set containing $F_A$ and $F_B$. Therefore $bcl(F_A) \subseteq F_0$; $bcl(F_B) \subseteq F_0$. Since $F_A \subseteq F_0$, $F_B \subseteq F_0$ then $F_A \bigcup F_B \subseteq F_0$. Hence $bcl(F_A \bigcup F_B) = bcl(F_A) \cup bcl(F_B) \subseteq F_0$. Therefore $F_0 \cap bcl(F_A)$ is a soft pgb-closed set in $X$.

3.13. Theorem. A set $F_A$ is a soft pgb-closed set iff $bcl(F_A) - F_A$ contains no -empty soft pre closed set.

Proof: Assume that $F_C$ is a soft pre closed set in $X$ such that $F_C \subseteq bcl(F_A) - F_A$. Then $F_C \subseteq F_A$. Since $F_A$ is a soft pgb-closed set and $F_C$ is soft pre open then $bcl(F_A) \subseteq F_C$. ie) $F_C \subseteq \overline{bcl(F_A)}$, $F_C = \emptyset$. Hence $bcl(F_A) - F_A$ contains no soft pre closed set.

Conversely, Assume that $bcl(F_A) - F_A$ contains no non-empty soft pre closed set. Let $F_A \subseteq F_0$, $F_0$ is soft pre open. Suppose that $bcl(F_A)$ is not contained in $F_0$, $bcl(F_A) \cap F_0^c$ is a non empty soft pre closed set of $bcl(F_A) - F_A$, which is contradiction. Therefore $bcl(F_A) \subseteq F_0$. Hence $F_A$ is soft pgb-closed set.

3.14. Theorem. The intersection of any two soft pgb-closed sets in $X$ is also a soft pgb-closed set in $X$.

Proof: Let $F_A$ and $F_B$ be any two soft pgb-closed sets. $F_A \subseteq F_0$, $F_0$ is any soft pre open and $F_B \subseteq F_0$, $F_0$ is soft pre open. Then $bcl(F_A) \subseteq F_0$, $bcl(F_B) \subseteq F_0$, therefore $bcl(F_A) \cap bcl(F_B) \subseteq bcl(F_A \cap F_B) \subseteq F_0$. $F_0$ is soft pre open in $X$. Hence $F_A \cap F_B$ is a soft pgb-closed set.

3.15. Theorem. If $F_A$ is a soft pgb-closed set in $X$ and $F_A \subseteq F_0 \subseteq bcl(F_A)$. Then $F_B$ is a soft pgb-closed set in $X$.

Proof: Since $F_0 \subseteq bcl(F_A)$, we have $bcl(F_B) \subseteq bcl(F_A)$. Clearly $bcl(F_B) - F_B \subseteq bcl(F_A) - F_A$. By theorem 3.13, $bcl(F_A) - F_A$ contains no non-empty soft pre closed set. Hence $bcl(F_B) - F_B$ contains no non-empty soft pre closed set. Therefore $F_B$ is soft pgb-closed set in $X$.

4. Soft Pre Generalized $b$-open set

In this section we introduce soft pre generalized $b$-open sets in a soft topological space and study some of their properties.

4.1. Definition. Let $(F_b, \tau)$ be a soft topological space over $X$. A soft set $F_A$ is called a soft generalization of a $b$-open set (pgb-open set) if the complement $F_A^c$ is a soft pgb-closed set in $X$. Equivalently, $F_A$ is called a soft pgb-open set if $F \subset \overline{bint(F_A)}$ whenever $F \subset F_A$ and $F_C$ is soft pre closed set in $X$.

4.2. Definition. Let $(F_b, \tau)$ be a soft topological space and let $F_b$ be a soft subset of $F_0$. Then the soft pre generalized $b$-interior (soft pgb-interior) of $F_b$, denoted by $pgbint(F_A)$ is defined as the soft union of all soft pre generalized open subsets of $F_b$.

4.3. Example

In example 3.3, consider the soft subset $F_{E_2} = \{(e_1, \{h_3\}), (e_2, \emptyset)\}$. The soft set $F_{E_2}$ is a soft pgb-open set.

4.4. Theorem. Every soft b-open set is a soft pre generalized b-open set.

Proof: Let $F_A$ be any soft b-open set such that $F_C$ is any soft pre closed set contained in $F_A$. Since $F_A$ is soft b-open, $bint(F_A) = \overline{F_A} \cap F_C$. Therefore $bint(F_A) \subseteq F_C$. Hence $F_A$ is a soft pgb-open set.

The converse of the above theorem need not true be as seen from the following example.

4.5. Example. In example 3.3, consider the soft subset $H_E = \{(e_1, \overline{X}), (e_2, \{h_1\})\}$. The soft set $H_E$ is a soft pgb-open set but not a soft b-open set.

4.6. Theorem. The intersection of two soft pgb-open sets in $X$ is also a soft pgb-open set in $X$.

Proof: Let $F_A$ and $F_B$ be soft pgb-open sets in $X$. Then $F_A^c$ and $F_B^c$ are soft pgb-closed sets. By theorem 3.12, $(F_A^c \cap F_B^c)$ is also a soft pgb-closed set in $X$. (ie) $(F_A^c \cap F_B^c) = (F_A \cup F_B)^c$ is a soft pgb-closed set in $X$. Therefore $F_A \cap F_B$ is a soft pgb-open set in $X$.

4.7. Theorem. If $bint(F_A) \subseteq F_b \subseteq F_A$ and $F_A$ is soft pgb-open in $X$, then $F_B$ is soft pgb-open in $X$.

Proof: Suppose that $bint(F_A) \subseteq F_b \subseteq F_A$ and $F_A$ is soft pgb-open in $X$. $F_A^c \subseteq F_B^c \subseteq bcl(F_A^c)$. Since $F_A^c$ is soft pgb-closed in $X$ by theorem 3.15, $F_B^c$ is soft pgb-closed in $X$. Hence $F_B$ is soft pgb-open in $X$.

Conclusion

Soft set theory was introduced by D.Molodtsov[1] in 1999.[2] Muhammad shahir and Munazza Naz introduced the concept of soft topological space. In 2014,J.Subhashini and C.Sekar[5] introduced the concept of soft pre generalized closed sets. In this paper, we introduce soft pre generalized b- closed sets based on the soft pre open sets. Also we introduce soft pre generalized b- open sets and study some theorems.

References


