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Domination and Total Domination in an Undirected Graph $G_{m,n}$

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Abstract *– In this paper we discuss about the undirected graph* $G_{m,n}$ *whose set of vertices is given as* $V = I_n = \{1,2,3, \ldots\}$ *where u, v* $\in V$ *are adjacent if and only if* $u \neq v$ *and u+ v is not divisible by m where m belongs to Natural numbers greater than 1.We determine minimal dominating sets, domination number, the total domination for the graph* $G_{m,n}$.

Key Words: Minimal dominating set, domination number, the total domination

1. INTRODUCTION

Graph theory is considered as one of the most prospering branch both in modern mathematics and computer applications. In the recent times Theory of domination became an area which attracted many researchers due to its wide scope. Historically the domination type problem originated from chess. The historical roots of domination in graph theory dates back to 1862, when the n × n chess board. The concept of domination number was given by Berge in 1958.Ore gave the name for the same concept as "dominating set" in 1962. The book [1] lists over 1200 papers on domination in graphs. More than 75 variations on dominations were cited chess master C.F. de Jaenisch wrote a treatise [2], in which he suggested the number of queens required to attack every square on a in [1].

In this paper we consider an undirected graph $G_{m,n}$ as defined in[3] whose set of vertices is given as $V = I_n = \{1,2,3, \ldots\}$ where u, v $\in V$ are adjacent if and only if $u \neq v$ and u+ v is not divisible by m where m belongs to Natural numbers greater than 1.We determine the domination number and minimal dominating set for $G_{m,n}$ 'Then with respect to order, minimum and maximum degree of $G_{m,n}$ we determine the bounds on domination number $\gamma(G_{m,n})$ .We also determine the total domination number for $G_{m,n}$.

2. UNDIRECTED GRAPH $G_{m,n}$

An Undirected graph $G_{m,n}$ was first introduced and studied by Dr.Ivy Chakrabarty[3], some of the properties of $G_{m,n}$ are :

2.1.The graph $G_{m,n}$ **is** connected where m,n $\in N$ $and\ m,n > 1$.

2.2. $G_{m,n} \cong k_3$ if and only if n=3 and $m \geq 6$.

2.3.If *n=m-1 where m is odd and k=n/2 then* $G_{m,n}$ is a complete k-partite graph.

2.4.The graph $G_{m,n}$ is Eulerian where m is odd and n=m-1.

2.5.The graph $G_{m,n}$ is complete when $m \geq 2n$.

2.6.The graph $G_{m,n}$ is Hamiltonian when $m \geq 2n\ with\ n \geq 3$.

3. DOMINATION IN $G_{m,n}$:

***Definition*3.1:** A set $S \subset V$, where V is the set of vertices of $G_{m,n}$, is said to be a dominating set of $G_{m,n}$ if for every vertex $u \in V$ is an element of S or adjacent to an element of S.

Definition*3.2:** A dominating set is said to be ***Minimal dominating set [MDS] if no vertex is removed without destroying its dominance property.

The figure 1 illustrates a graph $G_{3,5}$ with its minimal dominating sets of cardinality two (the set$\{v_3, v_4\}$) ,three(the set $\{v_2, v_3, v_5\}$)

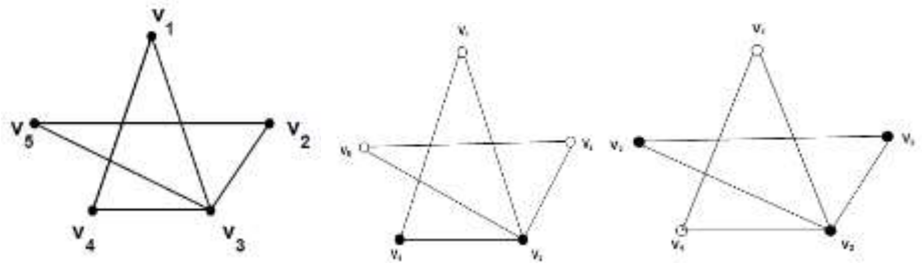

Figure 1

The following theorem is obvious from [4] hence we omit its proof.

Theorem 3.3: *The necessary and sufficient condition for a dominating set D to be minimal is:*
If for each vertex p in D there exists a vertex q in V-D such that$N(q) \cap D = \{p\}$.

Theorem 3.4: *If* $G_{m,n}$ *is a graph with no isolated vertices, then the complement V-S of every minimal dominating set S is a dominating set.*

Proof: Let us consider S as a minimal dominating set of $G_{m,n}$.

Assume that V-S is not a dominating set.

$\exists\ v_i$ *such that* v_i is not dominated by any vertex in V-S.

Since $G_{m,n}$ has no isolated vertices $\Rightarrow v_i$ should be dominated by at least one vertex in S-$\{v_i\}$

$\Rightarrow$ S-$\{v_i\}$ is a dominating set which contradicts the minimalism of S.

Hence every vertex in S is dominated by at least one vertex in V-S.
Therefore V-S is a dominating set.

Definition 3.5: The domination number $\gamma(G_{m,n})$ **of the graph** $G_{m,n}$ is the minimum cardinality of a dominating set in $G_{m,n}$.

Definition 3.6: *If* $|D| = \gamma(G_{m,n})$ *then the dominating set D is called a Minimum dominating set of* $G_{m,n}$.

A dominating set of minimum cardinality is called a $\gamma - set\ of\ G_{m,n}$.

From Figure 1, $\gamma(G_{m,n}) = 2$. Also the set D= $\{v_3, v_4\}$ is a dominating set such that $|D| = \gamma(G_{m,n})$. Hence it is a Minimum dominating set of $G_{m,n}$ and also a $\gamma - set\ of\ G_{m,n}$.

Therefore the following observation is obvious:

Theorem 3.7: *Every Minimum dominating set of* $G_{m,n}$ *is a minimal dominating set.*

Theorem 3.8: *For any graph $G_{m,n}$ if m is even and n is odd then the domination number* $\gamma(G_{m,n}) = 1$

Proof: For any graph $G_{m,n}$ if m is even and n is odd

⇒ the maximum degree for any vertex is n-1.
Hence the vertex with maximum degree is adjacent to the other vertices in the graph.
Let S be a set of all such vertices.

⇒ at least one element say, u $\in S$ dominates other vertices of $G_{m,n}$.

Therefore , the number $\gamma(G_{m,n}) = 1$.

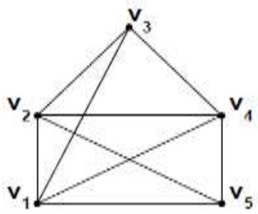

Figure 2: G8;5

The minimum cardinality of a set in MDS($G_{m,n}$) is called as ***domination number*** $\gamma(G_{m,n})$.

Lemma 3.9: *In a graph $G_{m,n}$, the domination number $\gamma(G_{m,n})$, is bounded above with $\gamma(G_{m,n}) = 1$.*

Proof: The proof follows from Theorem 3.8.

Lemma 3.10: *In a graph $G_{m,n}$, the lower bound does not exists for domination number $\gamma(G_{m,n})$.*

Proof: For any m ,n the graph $G_{m,n}$ cannot be edgeless .Thus the lower bound does not exists for domination number $\gamma(G_{m,n})$
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Theorem 3.11: *For any undirected graph $G_{m,n}$, $\gamma(G_{m,n}) \leq n - \Delta(G_{m,n})$.*

Proof: Let the vertex set of $(G_{m,n})$ is V=$\{v_1, v_2, v_3, v_4, \ldots. v_n\}$ .Let v_i be a vertex of maximum degree $\Delta(G_{m,n})$.
Obviously v_i is adjacent to $N(v_i)$ vertices such that $\Delta(G_{m,n})= N(v_i)$.
Therefore V- $N(v_i)$ is a dominating set.
Hence $\gamma(G_{m,n}) \leq |V - N(v_i)|$.
Thus $\gamma(G_{m,n}) \leq n - \Delta(G_{m,n})$.

Theorem 3.12: *If m=2n where n is odd then the graph $G_{m,n}$ is a complete graph with $\gamma(G_{m,n}) = 1$.*

Proof: From above property 2.5, $G_{m,n}$ is a complete graph with m=2n and n odd.
We know that in a complete graph each vertex has degree n-1.
Hence from Theorem 3.4 the graph $G_{m,n}$ is a complete graph with $\gamma(G_{m,n}) = 1$

4. TOTAL DOMINATION IN $G_{m,n}$

Definition 4.1: A dominating set D is a *Total dominating set [TDS]* if the induced sub graph $\langle D \rangle$ has no isolated vertices.

The minimum cardinality of TDS is denoted as $\gamma_t(G_{m,n})$.Clearly a TDS exist for any graph $G_{m,n}$ without isolated vertices. A $\gamma_t - set$ is a minimum total dominating set.

Theorem 4.1: *For a graph $G_{m,n}$ where m is even and n is odd, the total domination number $\gamma_t(G_{m,n}) = 2$.*

Proof: Clearly from Theorem 3.12 $G_{m,n}$ is a complete graph with vertex set V= I_n ={1,2...n}.
Let T= $\{v_i\} \cup S$ where S is a set of vertices v_j such that $v_i \neq v_j$.

Also the number of elements in S is n-1.
Clearly v_i dominates every vertex in S.

Therefore T becomes a dominating set of $G_{m,n}$.

Now for any S, v_i and a vertex in S are adjacent in $G_{m,n}$.

Hence T forms a TDS of $G_{m,n}$.

We know by definition of TDS that the minimum cardinality of any TDS is at least 2.
It follows T is a Minimum total dominating set of $G_{m,n}$.

Hence $\gamma_t(G_{m,n}) = 2$.

Corollary 4.2: *If $G_{m,n}$ is an undirected connected graph with no isolated vertices then* $\gamma_t(G_{m,n}) \leq n - \Delta(G_{m,n}) + 1$.

Proof: Since $G_{m,n}$ is a connected graph implies that there exists at least one vertex with maximum degree, $\Delta(G_{m,n})$**= n-1.**

Also the minimum cardinality of any TDS is at least 2.
i.e, $\gamma_t(G_{m,n}) = 2$

Now let us consider $n - \Delta(G_{m,n}) + 1 = n - (n-1) + 1 = 2$.

Hence $\gamma_t(G_{m,n}) \leq n - \Delta(G_{m,n}) + 1$.

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