

W-R₀ SPACE IN MINIMAL g-CLOSED SETS OF TYPE1

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ABSTRACT - In this article we introduce and study Weakly minimal closed sets of Type1-R₀ spaces, Weakly minimal g-closed sets of Type1-R₀ spaces, Weakly minimal closed sets of Type1-i-R₀ spaces, Weakly minimal closed sets of Type1-d-R₀ spaces, Weakly minimal closed sets of Type1-b-R₀ spaces, and also discuss the inter-relationships among separation properties along with several counter examples.

Key Words: minimal open sets, minimal g-open sets, minimal open sets of Type1, minimal g-open set of Type1.

1. INTRODUCTION.

L.Nachbin[6] Topology and order, D.Van Nostrand Inc., Princeton, New Jersey studied increasing [resp. decreasing, balanced] open sets in 1965. K. Bhagya Lakshmi, J. Venkateswara Rao[13] studied W-R₀ Type spaces in topological ordered spaces in 2014. G.Venkateswarlu, V.Amarendra Babu, and M.K.R.S Veera kumar [11] introduced and studied minimal open sets of Type1 sets, minimal g-open sets of Type1 sets in 2016. G.Venkateswarlu, V.Amarendra Babu, K. Bhagya Lakshmi and V.B.V.N. Prasad [14] studied W-C₀ Spaces in 2019.

In this article we introduce new separation axioms of type Weakly- minimal closed sets Type1-R₀ spaces, Weakly minimal g-closed sets of Type1-R₀ spaces, Weakly minimal g- closed sets of Type1-i-R₀ spaces, Weakly minimal g- closed sets of Type1-d-R₀ spaces, Weakly minimal g- closed sets of Type1-b-R₀ spaces, and discuss the inter-relationships among separation properties along with several counter examples.

2. PRELIMINARIES.

DEFINITION 2.1[11]: In a topological space (X, T), an open sub set U of X is called a minimal open sets of Type1! If \exists at least one non-empty closed set F such that $F \subseteq U$ or $U = \Phi$.

DEFINITION 2.2[11]: In a topological space (X, T), an open sub set U of X is called a minimal g- open sets of Type1! If \exists at least one non-empty g- closed set F such that $F \subseteq U$ or $U = \Phi$.

3. MAIN RESULT:

Now we state and prove our first main result. Before that we first introduce the following definitions and notations.

Note:

The collections of all increasing Weakly-minimal closed sets of Type1, increasing Weakly-minimal g-closed sets Type1 is denoted by $W-i-m_i^{cl}(Z, T1)$, $W-i-m_i-g^{cl}(Z, T1)$. [resp. decreasing and balanced is denoted by $W-d-m_i^{cl}(Z, T1)$, $W-d-m_i-g^{cl}(Z, T1)$, $W-b-m_i^{cl}(Z, T1)$, $W-b-m_i-g^{cl}(Z, T1)$]. Topological ordered space is denoted by TOS, Weakly-Minimal closed sets of Type 1, Weakly-Minimal g-closed sets of Type 1, set is denoted by $W- m_i^{cl}(Z, T1)$, $W-m_i-g^{cl}(Z, T1)$ and α -closed, β -closed, Ψ -closed is denoted by $\alpha^{cl}, \beta^{cl}, \Psi^{cl}$.

WE INTRODUCE THE FOLLOWING DEFINITIONS:

DEFINITION 3.1: In a topological space (Z, T), a non empty closed subset F of Z is called a minimal closed sets of Type1 if \exists at least one non-empty open set U such that $F \subseteq U$ or $U = Z$.

DEFINITION 3.2: In a topological space (Z, T), a non empty g- closed subset F of Z is called a minimal g- closed sets of Type1 if \exists at least one non-empty g- open set U such that $F \subseteq U$ or $U = Z$.

DEFINITION 3.3: A space (Z, T) is called a minimal closed sets of Type1 R₀-space if $m_i^{cl}\{x\}$ contained in G where G is Minimal closed sets of Type1 (briefly $m_i^{cl}\{x\}$) and $x \in G \in T$

DEFINITION 3.4: A space (Z, T) is called a minimal g- closed sets of Type1 R₀-space if $m_i-g^{cl}\{x\}$ contained in G where G is Minimal g-closed sets of Type1 (briefly $m_i-g^{cl}\{x\}$) and $x \in G \in T$

DEFINITION 3.5: A space (Z, T) is called a minimal g- closed sets of Type1 α -R₀-space if for $x \in G \in m_i-g-O(Z, T)$ $\alpha m_i-g^{cl}\{x\}$ contained in G where G is Minimal g-closed sets of Type1 (briefly $m_i-g^{cl}\{x\}$) and $x \in G \in T$

DEFINITION 3.6: A space (Z, T) is called a minimal g- closed sets of Type1 Ψ -R₀-space if for $x \in G \in m_i-g-O(Z, T)$ $\Psi m_i-g^{cl}\{x\}$ contained in G where G is Minimal g-closed sets of Type1 (briefly $m_i-g^{cl}\{x\}$) and $x \in G \in T$

DEFINITION 3.7: A space (Z, T) Called Weakly minimal closed sets of Type1-- R_0 if the intersection of $\text{micl}\{x\}$ is non-empty set $\forall x \in Z$

DEFINITION 3.8: A space (Z, T) Called Weakly minimal g-closed sets of Type1-- R_0 if the intersection of $\text{mi-g}^{\text{cl}}\{x\}$ is non-empty set $\forall x \in Z$

DEFINITION 3.9: A space (Z, T) Called Weakly minimal closed sets of Type1-i- R_0 if the intersection of $\text{i-mi}^{\text{cl}}\{x\}$ is non-empty set $\forall x \in Z$

DEFINITION 3.10: A space (Z, T) Called Weakly minimal g-closed sets of Type1-i- R_0 if the intersection of $\text{i-mi-g}^{\text{cl}}\{x\}$ is non-empty set $\forall x \in Z$

DEFINITION 3.11: A space (Z, T) Called Weakly minimal closed sets of Type1-d-- R_0 if the intersection of $\text{d-mi}^{\text{cl}}\{x\}$ is non-empty set $\forall x \in Z$

DEFINITION 3.12: A space (Z, T) Called Weakly minimal g-closed sets of Type1-d-- R_0 if the intersection of $\text{d-mi-g}^{\text{cl}}\{x\}$ is non-empty set $\forall x \in Z$

DEFINITION 3.13: A space (Z, T) Called Weakly minimal closed sets of Type1-b-- R_0 if the intersection of $\text{b-mi}^{\text{cl}}\{x\}$ is non-empty set $\forall x \in Z$

DEFINITION 3.14: A space (Z, T) Called Weakly minimal g-closed sets of Type1-b- R_0 if the intersection of $\text{b-mi-g}^{\text{cl}}\{x\}$ is non-empty set $\forall x \in Z$

THEOREM 3.15: In a TOS (Z, T, \leq) , every $W\text{-mi-g}^{\text{cl}}(Z, T1) - R_0$ space is a $W\text{-}\alpha^{R_0}$ space but not converse.

Proof: Suppose (Z, T) be a $W\text{-mi-g}^{\text{cl}}(Z, T1) - R_0$ space. Then the intersection of $\text{mi-g}^{\text{cl}}\{x\}$ is empty set $\forall x \in Z$ by fact, every $\text{mi-g}^{\text{cl}}(Z, T1)$ is a $\text{mi}^{\text{cl}}(Z, T1)$ and then every $\text{mi}^{\text{cl}}(Z, T1)$ is α^{cl} set Then $\alpha^{\text{cl}}\{x\}$ contained in $\text{mi-g}^{\text{cl}}\{x\} \forall x \in Z$. That implies the intersection of $\alpha^{\text{cl}}\{x\}$ contained in $\text{mi-g}^{\text{cl}}\{x\}$. But the intersection of $\text{mi-g}^{\text{cl}}\{x\}$ empty set $\forall x \in Z$ we get the intersection of $\alpha^{\text{cl}}\{x\}$ is empty set $\forall x \in Z$. Hence (Z, T) is $W\text{-}\alpha^{R_0}$ space.

EXAMPLE 3.16: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}\}$.
 $\text{mi-g}^{\text{cl}}(Z, T1)$ are Φ, Z
 α^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}$
 $\text{mi-g}^{\text{cl}}\{\zeta_1\}$ is Z
 $\text{mi-g}^{\text{cl}}\{\delta_2\}$ is Z
 $\text{mi-g}^{\text{cl}}\{\Omega_3\}$ is Z
 The intersection of $\text{mi-g}^{\text{cl}}\{x\}$ is $Z \forall x \in Z$
 $\alpha^{\text{cl}}\{\zeta_1\}$ is Z
 $\alpha^{\text{cl}}\{\delta_2\}$ is $\{\delta_2\}$
 $\alpha^{\text{cl}}\{\Omega_3\}$ is $\{\Omega_3\}$
 The intersection of $\alpha^{\text{cl}}\{x\}$ is empty set not equal to $Z \forall x \in Z$

THEOREM 3.17: In a TOS (Z, T, \leq) , every $W\text{-i-mi-g}^{\text{cl}}(Z, T1) - R_0$ space is a $W\text{-}\alpha^{R_0}$ space but not converse.

Proof: Suppose (Z, T) be a $W\text{-i-mi-g}^{\text{cl}}(Z, T1) - R_0$ space. Then the intersection of $\text{i-mi-g}^{\text{cl}}\{x\}$ is empty set $\forall x \in Z$ by fact, every $\text{i-mi-g}^{\text{cl}}(Z, T1)$ is a $\text{i-mi}^{\text{cl}}(Z, T1)$ and then every $\text{i-mi}^{\text{cl}}(Z, T1)$ is α^{cl} set Then $\alpha^{\text{cl}}\{x\}$ contained in $\text{i-mi-g}^{\text{cl}}\{x\} \forall x \in Z$. That implies the intersection of $\alpha^{\text{cl}}\{x\}$ contained in $\text{i-mi-g}^{\text{cl}}\{x\}$. But the intersection of $\text{i-mi-g}^{\text{cl}}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $\alpha^{\text{cl}}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is $W\text{-}\alpha^{R_0}$ space.

EXAMPLE 3.18: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}, \{\zeta_1, \Omega_3\}\}$.
 $\leq_4 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2)\}, (\Omega_3, \zeta_1), (\Omega_3, \delta_2)\}$.
 $\text{mi-g}^{\text{cl}}(Z, T1)$ are $\Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\}$
 $\text{i-mi-g}^{\text{cl}}(Z, T1)$ are Φ, Z
 α^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}$.
 α^{cl} sets are $\Phi, Z, \{\delta_2\}$.
 $\text{i-mi-g}^{\text{cl}}\{\zeta_1\}$ is Z
 $\text{i-mi-g}^{\text{cl}}\{\delta_2\}$ is Z
 $\text{i-mi-g}^{\text{cl}}\{\Omega_3\}$ is Z
 The intersection of $\text{i-mi-g}^{\text{cl}}\{x\}$ is $Z \forall x \in Z$
 $\alpha^{\text{cl}}\{\zeta_1\}$ is Z
 $\alpha^{\text{cl}}\{\delta_2\}$ is $\{\delta_2\}$
 $\alpha^{\text{cl}}\{\Omega_3\}$ is Z
 The intersection of $\alpha^{\text{cl}}\{x\}$ is $\{\delta_2\}$ not equal to $Z \forall x \in Z$

THEOREM 3.19: In a TOS (Z, T, \leq) , every $W\text{-d-mi-g}^{\text{cl}}(Z, T1) - R_0$ space is a $W\text{-d}\alpha^{R_0}$ space but not converse.

Proof: Suppose (Z, T) be a $W\text{-d-mi-g}^{\text{cl}}(Z, T1) - R_0$ space. Then the intersection of $\text{d-mi-g}^{\text{cl}}\{x\}$ is empty set $\forall x \in Z$ by fact, every $\text{d-mi-g}^{\text{cl}}(Z, T1)$ is a $\text{d-mi}^{\text{cl}}(Z, T1)$ and then every $\text{d-mi}^{\text{cl}}(Z, T1)$ is $\text{d}\alpha^{\text{cl}}$ set Then $\text{d}\alpha^{\text{cl}}\{x\}$ contained in $\text{d-mi-g}^{\text{cl}}\{x\} \forall x \in Z$. That implies the intersection of $\text{d}\alpha^{\text{cl}}\{x\}$ contained in $\text{d-mi-g}^{\text{cl}}\{x\}$. But the intersection of $\text{d-mi-g}^{\text{cl}}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $\text{d}\alpha^{\text{cl}}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is $W\text{-d}\alpha^{R_0}$ space.

EXAMPLE 3.20: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}, \{\zeta_1, \Omega_3\}\}$.
 $\leq_2 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2)\}, (\Omega_3, \delta_2)\}$.
 $\text{mi-g}^{\text{cl}}(Z, T1)$ are Φ, Z
 $\text{d-mi-g}^{\text{cl}}(Z, T1)$ are Φ, Z
 α^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}$.
 $\text{d}\alpha^{\text{cl}}$ sets are $\Phi, Z, \{\Omega_3\}$.
 $\text{d-mi-g}^{\text{cl}}\{\zeta_1\}$ is Z
 $\text{d-mi-g}^{\text{cl}}\{\delta_2\}$ is Z
 $\text{d-mi-g}^{\text{cl}}\{\Omega_3\}$ is Z
 The intersection of $\text{d-mi-g}^{\text{cl}}\{x\}$ is $Z \forall x \in Z$
 $\text{d}\alpha^{\text{cl}}\{\zeta_1\}$ is Z
 $\text{d}\alpha^{\text{cl}}\{\delta_2\}$ is Z
 $\text{d}\alpha^{\text{cl}}\{\Omega_3\}$ is $\{\Omega_3\}$
 The intersection of $\text{d}\alpha^{\text{cl}}\{x\}$ is $\{\Omega_3\}$ not equal to $Z \forall x \in Z$

THEOREM 3.21: In a TOS (Z, T, \leq) , every W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $b\alpha^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of b - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every b - m_i - $g^{cl}(Z, T1)$ is a b - $m_i^{cl}(Z, T1)$ and then every b - $m_i^{cl}(Z, T1)$ is $b\alpha^{cl}$ set Then $b\alpha^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $b\alpha^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\}$. But the intersection of b - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $b\alpha^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $b\alpha^{R0}$ space.

EXAMPLE 3.22: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \zeta_1, \delta_2 \}, \{ \delta_2, \Omega_3 \} \}$.
 $\leq_9 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3) \}$.
 m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$
 b - m_i - $g^{cl}(Z, T1)$ are Φ, Z
 α^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \delta_2, \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$
 $b\alpha^{cl}$ sets are $\Phi, Z, \{ \zeta_1, \Omega_3 \}$.
 b - m_i - $g^{cl}\{ \zeta_1 \}$ is Z
 b - m_i - $g^{cl}\{ \delta_2 \}$ is Z
 b - m_i - $g^{cl}\{ \Omega_3 \}$ is Z
The intersection of b - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$
 $b\alpha^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1, \Omega_3 \}$
 $b\alpha^{cl}\{ \delta_2 \}$ is Z
 $b\alpha^{cl}\{ \Omega_3 \}$ is $\{ \zeta_1, \Omega_3 \}$
The intersection of $b\alpha^{cl}\{x\}$ is $\{ \zeta_1, \Omega_3 \}$ not equal to $Z \forall x \in Z$

THEOREM 3.23: In a TOS (Z, T, \leq) , every W - i - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $b\alpha^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - i - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of i - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every i - m_i - $g^{cl}(Z, T1)$ is a b - $m_i^{cl}(Z, T1)$ and then every b - $m_i^{cl}(Z, T1)$ is $b\alpha^{cl}$ set Then $b\alpha^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $b\alpha^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\}$. But the intersection of i - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $b\alpha^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $b\alpha^{R0}$ space.

EXAMPLE 3.24: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \} \}$.
 $\leq_9 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3) \}$.
 m_i - $g^{cl}(Z, T1)$ are Φ, Z
 i - m_i - $g^{cl}(Z, T1)$ are Φ, Z
 α^{cl} sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$.
 $b\alpha^{cl}$ sets are $\Phi, Z, \{ \delta_2 \}$.
 i - m_i - $g^{cl}\{ \zeta_1 \}$ is Z
 i - m_i - $g^{cl}\{ \delta_2 \}$ is Z
 i - m_i - $g^{cl}\{ \Omega_3 \}$ is Z
The intersection of i - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$.
 $b\alpha^{cl}\{ \zeta_1 \}$ is Z
 $b\alpha^{cl}\{ \delta_2 \}$ is $\{ \delta_2 \}$
 $b\alpha^{cl}\{ \Omega_3 \}$ is Z
The intersection of $b\alpha^{cl}\{x\}$ is $\{ \delta_2 \}$ not equal to $Z \forall x \in Z$.

THEOREM 3.25: In a TOS (Z, T, \leq) , every W - i - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $d\alpha^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - i - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of i - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every i - m_i - $g^{cl}(Z, T1)$ is a d - $m_i^{cl}(Z, T1)$ and then every d - $m_i^{cl}(Z, T1)$ is $d\alpha^{cl}$ set Then $d\alpha^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $d\alpha^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\}$. But the intersection of i - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $d\alpha^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $d\alpha^{R0}$ space.

EXAMPLE 3.26: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \zeta_1, \delta_2 \}, \{ \zeta_1, \Omega_3 \} \}$.
 $\leq_1 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3), (\delta_2, \Omega_3) \}$
 m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$
 i - m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \Omega_3 \}$
 α^{cl} sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$
 $d\alpha^{cl}$ sets are Φ, Z
 i - m_i - $g^{cl}\{ \zeta_1 \}$ is Z
 i - m_i - $g^{cl}\{ \delta_2 \}$ is Z
 i - m_i - $g^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$
The intersection of i - m_i - $g^{cl}\{x\}$ is $\{ \Omega_3 \} \forall x \in Z$.
 $d\alpha^{cl}\{ \zeta_1 \}$ is Z
 $d\alpha^{cl}\{ \delta_2 \}$ is Z
 $d\alpha^{cl}\{ \Omega_3 \}$ is Z
The intersection of $d\alpha^{cl}\{x\}$ is Z not equal to $\{ \Omega_3 \} \forall x \in Z$.

THEOREM 3.27: In a TOS (Z, T, \leq) , every W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $i\alpha^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of b - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every b - m_i - $g^{cl}(Z, T1)$ is a i - $m_i^{cl}(Z, T1)$ and then every i - $m_i^{cl}(Z, T1)$ is $i\alpha^{cl}$ set Then $i\alpha^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $i\alpha^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\}$. But the intersection of b - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $i\alpha^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $i\alpha^{R0}$ space.

EXAMPLE 3.28: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \zeta_1, \delta_2 \}, \{ \delta_2, \Omega_3 \} \}$.
 $\leq_1 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3), (\delta_2, \Omega_3) \}$.
 m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$
 b - m_i - $g^{cl}(Z, T1)$ are Φ, Z
 α^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$
 $i\alpha^{cl}$ sets are $\Phi, Z, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$
 b - m_i - $g^{cl}\{ \zeta_1 \}$ is Z
 b - m_i - $g^{cl}\{ \delta_2 \}$ is Z
 b - m_i - $g^{cl}\{ \Omega_3 \}$ is Z
The intersection of b - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$.
 $i\alpha^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1 \}$
 $i\alpha^{cl}\{ \delta_2 \}$ is $\{ \delta_2, \Omega_3 \}$
 $i\alpha^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$

The intersection of $i\alpha^{cl}\{x\}$ is empty set not equal to $Z \forall x \in Z$.

THEOREM 3.29: In a TOS (Z, T, \leq) , every W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $d\alpha^{R0}$ space but not converse.

Proof. Suppose (Z, T) be a W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of b - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every b - m_i - $g^{cl}(Z, T1)$ is a d - $m_i^{cl}(Z, T1)$ and then every d - $m_i^{cl}(Z, T1)$ is $d\alpha^{cl}$ set Then $d\alpha^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $d\alpha^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\}$. But the intersection of b - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$.

we get the intersection of $d\alpha^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $d\alpha^{R0}$ space.

EXAMPLE 3.30: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \zeta_1, \delta_2 \}, \{ \delta_2, \Omega_3 \} \}$.

$\leq_2 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2) \}, (\Omega_3, \delta_2) \}$

m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$

b - m_i - $g^{cl}(Z, T1)$ are Φ, Z

α^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$

$d\alpha^{cl}$ sets are $\Phi, Z, \{ \zeta_1, \Omega_3 \}$

b - m_i - $g^{cl}\{ \zeta_1 \}$ is Z

b - m_i - $g^{cl}\{ \delta_2 \}$ is Z

b - m_i - $g^{cl}\{ \Omega_3 \}$ is Z

The intersection of b - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$.

$d\alpha^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1 \}$

$d\alpha^{cl}\{ \delta_2 \}$ is Z

$d\alpha^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$

The intersection of $d\alpha^{cl}\{x\}$ is empty set not equal to $Z \forall x \in Z$.

THEOREM 3.31: In a TOS (Z, T, \leq) , every W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $i\alpha^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of d - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every d - m_i - $g^{cl}(Z, T1)$ is a i - $m_i^{cl}(Z, T1)$ and then every i - $m_i^{cl}(Z, T1)$ is

$i\alpha^{cl}$ set Then $i\alpha^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $i\alpha^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\}$. But the intersection of d - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$.

we get the intersection of $i\alpha^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $i\alpha^{R0}$ space.

EXAMPLE 3.32: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \zeta_1, \Omega_3 \} \}$.

$\leq_3 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3) \}$.

m_i - $g^{cl}(Z, T1)$ are Φ, Z

d - m_i - $g^{cl}(Z, T1)$ are Φ, Z

α^{cl} sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$

$i\alpha^{cl}$ sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$.

d - m_i - $g^{cl}\{ \zeta_1 \}$ is Z

d - m_i - $g^{cl}\{ \delta_2 \}$ is Z

d - m_i - $g^{cl}\{ \Omega_3 \}$ is Z

The intersection of d - m_i - $g^{cl}\{Z\}$ is $Z \forall x \in Z$

$i\alpha^{cl}\{ \zeta_1 \}$ is Z

$i\alpha^{cl}\{ \delta_2 \}$ is $\{ \delta_2 \}$

$i\alpha^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$

The intersection of $i\alpha^{cl}\{x\}$ is empty set not equal to $Z \forall x \in Z$

THEOREM 3.33: In a TOS (Z, T, \leq) , every W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $b\alpha^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of d - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every d - m_i - $g^{cl}(Z, T1)$ is a b - $m_i^{cl}(Z, T1)$ and then every b - $m_i^{cl}(Z, T1)$ is

$b\alpha^{cl}$ set Then $b\alpha^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $b\alpha^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\}$. But the intersection of d - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$.

we get the intersection of $b\alpha^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $b\alpha^{R0}$ space.

EXAMPLE 3.34: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \zeta_1, \delta_2 \}, \{ \zeta_1, \Omega_3 \} \}$.

$\leq_4 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\Omega_3, \zeta_1), (\Omega_3, \delta_2) \}$.

m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$

d - m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \zeta_1, \Omega_3 \}$

α^{cl} sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$

$b\alpha^{cl}$ sets are Φ, Z

d - m_i - $g^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1, \Omega_3 \}$

d - m_i - $g^{cl}\{ \delta_2 \}$ is Z

d - m_i - $g^{cl}\{ \Omega_3 \}$ is $\{ \zeta_1, \Omega_3 \}$

The intersection of d - m_i - $g^{cl}\{x\}$ is $\{ \zeta_1, \Omega_3 \} \forall x \in Z$.

$b\alpha^{cl}\{ \zeta_1 \}$ is Z

$b\alpha^{cl}\{ \delta_2 \}$ is Z

$b\alpha^{cl}\{ \Omega_3 \}$ is Z

The intersection of $b\alpha^{cl}\{x\}$ is Z not equal to $\zeta_1, \Omega_3 \forall x \in Z$.

THEOREM 3.35: In a TOS (Z, T, \leq) , every W - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - β^{R0} space but not converse.

Proof: Suppose (Z, T) be a W - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of m_i - $g^{cl}\{x\}$ is empty set $\forall x \in Z$ by fact, every m_i - $g^{cl}(Z, T1)$ is a $m_i^{cl}(Z, T1)$ and then every $m_i^{cl}(Z, T1)$ is

β^{cl} set Then $\beta^{cl}\{x\}$ contained in m_i - $g^{cl}\{x\} \forall x \in Z$.

That implies the intersection of $\beta^{cl}\{x\}$ contained in m_i - $g^{cl}\{x\}$. But the intersection of m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$

we get the intersection of $\beta^{cl}\{x\}$ is empty set $\forall x \in Z$. Hence (Z, T) is W - β^{R0} space.

EXAMPLE 3.36: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \zeta_1, \delta_2 \} \}$.

m_i - $g^{cl}(Z, T1)$ are Φ, Z

β^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$

m_i - $g^{cl}\{ \zeta_1 \}$ is Z

m_i - $g^{cl}\{ \delta_2 \}$ is Z

m_i - $g^{cl}\{ \Omega_3 \}$ is Z

The intersection of m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$.

$\beta^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1 \}$

$\beta^{cl}\{ \delta_2 \}$ is $\{ \delta_2 \}$

$\beta^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$

The intersection of $\beta^{cl}\{x\}$ is empty set not equal to $Z \forall x \in Z$.

THEOREM 3.37: In a TOS (Z, T, \leq) , every W-i- $m_i\text{-}g^{cl}(Z, T1)$ – R_0 space is a W- $i\beta^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W-i- $m_i\text{-}g^{cl}(Z, T1)$ – R_0 space. Then the intersection of i- $m_i\text{-}g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every i- $m_i\text{-}g^{cl}(Z, T1)$ is a $i\text{-}m_i^{cl}(Z, T1)$ and then every $i\text{-}m_i^{cl}(Z, T1)$ is $i\beta^{cl}$ set Then $i\beta^{cl}\{x\}$ contained in $i\text{-}m_i\text{-}g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $i\beta^{cl}\{x\}$ contained in $i\text{-}m_i\text{-}g^{cl}\{x\}$. But the intersection of $i\text{-}m_i\text{-}g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $i\beta^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W- $i\beta^{R0}$ space.

EXAMPLE 3.38: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}, \{\zeta_1, \Omega_3\}\}$.

$\leq_4 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2)\}, (\Omega_3, \zeta_1), (\Omega_3, \delta_2)\}$.

$m_i\text{-}g^{cl}(Z, T1)$ are Φ, Z

$i\text{-}m_i\text{-}g^{cl}(Z, T1)$ are Φ, Z

β^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}$.

$i\beta^{cl}$ sets are $\Phi, Z, \{\delta_2\}$.

$i\text{-}m_i\text{-}g^{cl}\{\zeta_1\}$ is Z

$i\text{-}m_i\text{-}g^{cl}\{\delta_2\}$ is Z

$i\text{-}m_i\text{-}g^{cl}\{\Omega_3\}$ is Z

The intersection of $i\text{-}m_i\text{-}g^{cl}\{x\}$ is $Z \forall x \in Z$.

$i\beta^{cl}\{\zeta_1\}$ is Z

$i\beta^{cl}\{\delta_2\}$ is $\{\delta_2\}$

$i\beta^{cl}\{\Omega_3\}$ is Z

The intersection of $i\beta^{cl}\{x\}$ is $\{\delta_2\}$ not equal to $Z \forall x \in Z$.

THEOREM 3.39: In a TOS (Z, T, \leq) , every W-d- $m_i\text{-}g^{cl}(Z, T1)$ – R_0 space is a W- $d\beta^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W-d- $m_i\text{-}g^{cl}(Z, T1)$ – R_0 space. Then the intersection of d- $m_i\text{-}g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every d- $m_i\text{-}g^{cl}(Z, T1)$ is a $d\text{-}m_i^{cl}(Z, T1)$ and then every $d\text{-}m_i^{cl}(Z, T1)$ is $d\beta^{cl}$ set Then $d\beta^{cl}\{x\}$ contained in $d\text{-}m_i\text{-}g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $d\beta^{cl}\{x\}$ contained in $d\text{-}m_i\text{-}g^{cl}\{x\}$. But the intersection of $d\text{-}m_i\text{-}g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $d\beta^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W- $d\beta^{R0}$ space.

EXAMPLE 3.40: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}\}$,

$\leq_4 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2)\}, (\Omega_3, \zeta_1), (\Omega_3, \delta_2)\}$.

$m_i\text{-}g^{cl}(Z, T1)$ are Φ, Z

$d\text{-}m_i\text{-}g^{cl}(Z, T1)$ are Φ, Z

β^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}$.

$d\beta^{cl}$ sets are $\Phi, Z, \{\Omega_3\}$.

$d\text{-}m_i\text{-}g^{cl}\{\zeta_1\}$ is Z

$d\text{-}m_i\text{-}g^{cl}\{\delta_2\}$ is Z

$d\text{-}m_i\text{-}g^{cl}\{\Omega_3\}$ is Z

The intersection of $d\text{-}m_i\text{-}g^{cl}\{x\}$ is $Z \forall x \in Z$.

$d\beta^{cl}\{\zeta_1\}$ is Z

$d\beta^{cl}\{\delta_2\}$ is $\{\delta_2\}$

$d\beta^{cl}\{\Omega_3\}$ is Z

The intersection of $d\beta^{cl}\{x\}$ is $\{\delta_2\}$ not equal to $Z \forall x \in Z$.

THEOREM 3.41: In a TOS (Z, T, \leq) , every W-b- $m_i\text{-}g^{cl}(Z, T1)$ – R_0 space is a W- $b\beta^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W-b- $m_i\text{-}g^{cl}(Z, T1)$ – R_0 space. Then the intersection of b- $m_i\text{-}g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every b- $m_i\text{-}g^{cl}(Z, T1)$ is a $b\text{-}m_i^{cl}(Z, T1)$ and then every $b\text{-}m_i^{cl}(Z, T1)$ is $b\beta^{cl}$ set Then $b\beta^{cl}\{x\}$ contained in $b\text{-}m_i\text{-}g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $b\beta^{cl}\{x\}$ contained in $b\text{-}m_i\text{-}g^{cl}\{x\}$. But the intersection of $b\text{-}m_i\text{-}g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $b\beta^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W- $b\beta^{R0}$ space.

EXAMPLE 3.42: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\zeta_1, \delta_2\}, \{\delta_2, \Omega_3\}\}$.

$\leq_9 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3)\}$.

$m_i\text{-}g^{cl}(Z, T1)$ are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\zeta_1, \Omega_3\}$

$b\text{-}m_i\text{-}g^{cl}(Z, T1)$ are $\Phi, Z, \{\delta_2\}$

β^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}, \{\zeta_1, \Omega_3\}$

$b\beta^{cl}$ sets are $\Phi, Z, \{\delta_2\}, \{\zeta_1, \Omega_3\}$.

$b\text{-}m_i\text{-}g^{cl}\{\zeta_1\}$ is Z

$b\text{-}m_i\text{-}g^{cl}\{\delta_2\}$ is $\{\delta_2\}$

$b\text{-}m_i\text{-}g^{cl}\{\Omega_3\}$ is Z

The intersection of $b\text{-}m_i\text{-}g^{cl}\{x\}$ is $\{\delta_2\} \forall x \in Z$.

$b\beta^{cl}\{\zeta_1\}$ is $\{\zeta_1, \Omega_3\}$

$b\beta^{cl}\{\delta_2\}$ is $\{\delta_2\}$

$b\alpha^{cl}\{\Omega_3\}$ is $\{\zeta_1, \Omega_3\}$

The intersection of $b\text{-}\beta^{cl}\{x\}$ is Φ not equal to $\{\delta_2\} \forall x \in Z$.

THEOREM 3.43: In a TOS (Z, T, \leq) , every W-i- $m_i\text{-}g^{cl}(Z, T1)$ – R_0 space is a W- $b\beta^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W-i- $m_i\text{-}g^{cl}(Z, T1)$ – R_0 space. Then the intersection of i- $m_i\text{-}g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every i- $m_i\text{-}g^{cl}(Z, T1)$ is a $b\text{-}m_i^{cl}(Z, T1)$ and then every $b\text{-}m_i^{cl}(Z, T1)$ is $b\beta^{cl}$ set Then $b\beta^{cl}\{x\}$ contained in $i\text{-}m_i\text{-}g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $b\beta^{cl}\{x\}$ contained in $i\text{-}m_i\text{-}g^{cl}\{x\}$. But the intersection of $i\text{-}m_i\text{-}g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $b\beta^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W- $b\beta^{R0}$ space.

EXAMPLE 3.44: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\}\}$.

$\leq_5 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3), (\delta_2, \Omega_3)\}$

$m_i\text{-}g^{cl}(Z, T1)$ are $\Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\}$

$i\text{-}m_i\text{-}g^{cl}(Z, T1)$ are $\Phi, Z, \{\delta_2, \Omega_3\}$

β^{cl} sets are $\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\Omega_3\}, \{\zeta_1, \delta_2\}, \{\delta_2, \Omega_3\}, \{\zeta_1, \Omega_3\}$

$b\beta^{cl}$ sets are Φ, Z

$i\text{-}m_i\text{-}g^{cl}\{\zeta_1\}$ is Z

$i\text{-}m_i\text{-}g^{cl}\{\delta_2\}$ is $\{\delta_2, \Omega_3\}$

$i\text{-}m_i\text{-}g^{cl}\{\Omega_3\}$ is $\{\delta_2, \Omega_3\}$

The intersection of $i\text{-}m_i\text{-}g^{cl}\{x\}$ is $\{\delta_2, \Omega_3\} \forall x \in Z$.

$b\beta^{cl}\{\zeta_1\}$ is Z

$b\beta^{cl}\{\delta_2\}$ is Z

$b\beta^{cl}\{\Omega_3\}$ is Z

The intersection of $b\beta^{cl}\{x\}$ is Z not equal to $\{\delta_2, \Omega_3\} \forall x \in Z$.

THEOREM 3.45: In a TOS (Z, T, \leq) , every W - i - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $d\beta^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - i - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of i - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every i - m_i - $g^{cl}(Z, T1)$ is a d - $m_i^{cl}(Z, T1)$ and then every d - $m_i^{cl}(Z, T1)$ is $d\beta^{cl}$ set Then $d\beta^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $d\beta^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\}$. But the intersection of i - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $d\beta^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $d\beta^{R0}$ space.

EXAMPLE 3.46: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \zeta_1, \delta_2 \}, \{ \zeta_1, \Omega_3 \} \}$.
 $\leq_1 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3), (\delta_2, \Omega_3) \}$
 m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}$
 i - m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \Omega_3 \}$
 β^{cl} sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$
 $d\beta^{cl}$ sets are Φ, Z
 i - m_i - $g^{cl}\{ \zeta_1 \}$ is Z
 i - m_i - $g^{cl}\{ \delta_2 \}$ is Z
 i - m_i - $g^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$
The intersection of i - m_i - $g^{cl}\{x\}$ is $\{ \Omega_3 \} \forall x \in Z$.
 $d\beta^{cl}\{ \zeta_1 \}$ is Z
 $d\beta^{cl}\{ \delta_2 \}$ is Z
 $d\beta^{cl}\{ \Omega_3 \}$ is Z
The intersection of $d\beta^{cl}\{x\}$ is Z not equal to $\{ \Omega_3 \} \forall x \in Z$.

THEOREM 3.47: In a TOS (Z, T, \leq) , every W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $i\beta^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of b - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every b - m_i - $g^{cl}(Z, T1)$ is a i - $m_i^{cl}(Z, T1)$ and then every i - $m_i^{cl}(Z, T1)$ is $i\beta^{cl}$ set Then $i\beta^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $i\beta^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\}$. But the intersection of b - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$.

EXAMPLE 3.48: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \} \}$.
 $\leq_1 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\delta_2, \zeta_1), (\zeta_1, \Omega_3), (\delta_2, \Omega_3) \}$
 m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$
 b - m_i - $g^{cl}(Z, T1)$ are Φ, Z
 β^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \zeta_1, \delta_2 \}, \{ \zeta_1, \Omega_3 \}$
 $i\beta^{cl}$ sets are $\Phi, Z, \{ \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$
 b - m_i - $g^{cl}\{ \zeta_1 \}$ is Z
 b - m_i - $g^{cl}\{ \delta_2 \}$ is Z
 b - m_i - $g^{cl}\{ \Omega_3 \}$ is Z
The intersection of b - m_i - $g^{cl}\{Z\}$ is $Z \forall x \in Z$.
 $i\beta^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1, \Omega_3 \}$
 $i\beta^{cl}\{ \delta_2 \}$ is Z
 $i\beta^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$
The intersection of i - $\beta^{cl}\{Z\}$ is $\{ \Omega_3 \}$ not equal to $Z \forall x \in Z$.

THEOREM 3.49: In a TOS (Z, T, \leq) , every W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $d\beta^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - b - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of b - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every b - m_i - $g^{cl}(Z, T1)$ is a d - $m_i^{cl}(Z, T1)$ and then every d - $m_i^{cl}(Z, T1)$ is $d\beta^{cl}$ set Then $d\beta^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $d\beta^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\}$. But the intersection of b - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $d\beta^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $d\beta^{R0}$ space.

EXAMPLE 3.50: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \zeta_1, \delta_2 \} \}$.
 $\leq_{10} = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\Omega_3, \zeta_1), (\delta_2, \Omega_3), (\delta_2, \zeta_1) \}$.
 m_i - $g^{cl}(Z, T1)$ are Φ, Z
 b - m_i - $g^{cl}(Z, T1)$ are Φ, Z
 β^{cl} sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$
 $d\beta^{cl}$ sets are $\Phi, Z, \{ \zeta_1, \Omega_3 \}$
 b - m_i - $g^{cl}\{ \zeta_1 \}$ is Z
 b - m_i - $g^{cl}\{ \delta_2 \}$ is Z
 b - m_i - $g^{cl}\{ \Omega_3 \}$ is Z
The intersection of b - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$.
 $d\beta^{cl}\{ \zeta_1 \}$ is Z
 $d\beta^{cl}\{ \delta_2 \}$ is $\{ \delta_2 \}$
 $d\beta^{cl}\{ \Omega_3 \}$ are $\{ \delta_2, \Omega_3 \}$
The intersection of d - $\beta^{cl}\{x\}$ is $\{ \delta_2 \}$ not equal to $Z \forall x \in Z$.

THEOREM 3.51: In a TOS (Z, T, \leq) , every W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $i\beta^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of d - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every d - m_i - $g^{cl}(Z, T1)$ is a i - $m_i^{cl}(Z, T1)$ and then every i - $m_i^{cl}(Z, T1)$ is $i\beta^{cl}$ set Then $i\beta^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $i\beta^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\}$. But the intersection of d - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $i\beta^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $i\beta^{R0}$ space.

EXAMPLE 3.52: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \} \}$, $\leq_7 = \{ (\zeta_1, \zeta_1), (\delta_2, \Omega_3), (\Omega_3, \Omega_3), (\delta_2, \zeta_1) \}$.
 m_i - $g^{cl}(Z, T1)$ are Φ, Z
 d - m_i - $g^{cl}(Z, T1)$ are Φ, Z
 β^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \zeta_1, \delta_2 \}$
 $i\beta^{cl}$ sets are $\Phi, Z, \{ \zeta_1 \}, \{ \zeta_1, \delta_2 \}$
 d - m_i - $g^{cl}\{ \zeta_1 \}$ is Z
 d - m_i - $g^{cl}\{ \delta_2 \}$ is Z
 d - m_i - $g^{cl}\{ \Omega_3 \}$ is Z
The intersection of d - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$.
 $i\beta^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1 \}$
 $i\beta^{cl}\{ \delta_2 \}$ is $\{ \zeta_1, \delta_2 \}$
 $i\beta^{cl}\{ \Omega_3 \}$ is Z
The intersection of i - $\beta^{cl}\{x\}$ is $\{ \zeta_1 \}$ not equal to $Z \forall x \in Z$.

THEOREM 3.53: In a TOS (Z, T, \leq) , every W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $b\beta^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of d - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every d - m_i - $g^{cl}(Z, T1)$ is a b - $m_i^{cl}(Z, T1)$ and then every b - $m_i^{cl}(Z, T1)$ is $b\beta^{cl}$ set Then $b\beta^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $b\beta^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\}$. But the intersection of d - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $b\beta^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $b\beta^{R0}$ space.

EXAMPLE 3.54: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \zeta_1, \delta_2 \}, \{ \delta_2, \Omega_3 \} \}$, $\leq_3 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3) \}$. m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$ d - m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \zeta_1 \}$ β^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$ $b\beta^{cl}$ sets are Φ, Z d - m_i - $g^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1 \}$ d - m_i - $g^{cl}\{ \delta_2 \}$ is Z d - m_i - $g^{cl}\{ \Omega_3 \}$ is Z The intersection of d - m_i - $g^{cl}\{x\}$ is $\{ \zeta_1 \} \forall x \in Z$. $b\beta^{cl}\{ \zeta_1 \}$ is Z $b\beta^{cl}\{ \delta_2 \}$ is Z $b\beta^{cl}\{ \Omega_3 \}$ is Z The intersection of $b\beta^{cl}\{x\}$ is Z not equal to $\{ \zeta_1 \} \forall x \in Z$.

THEOREM 3.55: In a TOS (Z, T, \leq) , every W - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - Ψ^{R0} space but not converse.

Proof: Suppose (Z, T) be a W - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of m_i - $g^{cl}\{x\}$ is empty set $\forall x \in Z$ by fact, every m_i - $g^{cl}(Z, T1)$ is a $m_i^{cl}(Z, T1)$ and then every $m_i^{cl}(Z, T1)$ is Ψ^{cl} set Then $\beta^{cl}\{x\}$ contained in m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $\Psi^{cl}\{x\}$ contained in m_i - $g^{cl}\{x\}$. But the intersection of m_i - $g^{cl}\{x\}$ is empty set $\forall x \in Z$. we get the intersection of $\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$. Hence (Z, T) is W - Ψ^{R0} space.

EXAMPLE 3.56: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \} \}$ m_i - $g^{cl}(Z, T1)$ are Φ, Z Ψ^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \zeta_1, \delta_2 \}, \{ \zeta_1, \Omega_3 \}$ m_i - $g^{cl}\{ \zeta_1 \}$ is Z m_i - $g^{cl}\{ \delta_2 \}$ is Z m_i - $g^{cl}\{ \Omega_3 \}$ is Z The intersection of m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$. $\Psi^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1 \}$ $\Psi^{cl}\{ \delta_2 \}$ is $\{ \delta_2 \}$ $\Psi^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$ The intersection of $\Psi^{cl}\{x\}$ is empty set but not equal to $Z \forall x \in Z$.

THEOREM 3.57: In a TOS (Z, T, \leq) , every W - i - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $i\Psi^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - i - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of i - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every i - m_i - $g^{cl}(Z, T1)$ is a i - $m_i^{cl}(Z, T1)$ and then every i - $m_i^{cl}(Z, T1)$ is $i\Psi^{cl}$ set Then $i\Psi^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $i\Psi^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\}$. But the intersection of i - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $i\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $i\Psi^{R0}$ space.

EXAMPLE 3.58: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \} \}$. $\leq_7 = \{ (\zeta_1, \zeta_1), (\delta_2, \Omega_3), (\Omega_3, \Omega_3), (\delta_2, \zeta_1) \}$. m_i - $g^{cl}(Z, T1)$ are Φ, Z i - m_i - $g^{cl}(Z, T1)$ are Φ, Z Ψ^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \zeta_1, \delta_2 \}, \{ \zeta_1, \Omega_3 \}$. $i\Psi^{cl}$ sets are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \zeta_1, \delta_2 \}, \{ \zeta_1, \Omega_3 \}$ i - m_i - $g^{cl}\{ \zeta_1 \}$ is Z i - m_i - $g^{cl}\{ \delta_2 \}$ is Z i - m_i - $g^{cl}\{ \Omega_3 \}$ is Z The intersection of i - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$. $i\Psi^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1 \}$ $i\Psi^{cl}\{ \delta_2 \}$ is $\{ \zeta_1, \delta_2 \}$ $i\Psi^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$ The intersection of i - $\Psi^{cl}\{x\}$ is empty but not equal to $Z \forall x \in Z$.

THEOREM 3.59: In a TOS (Z, T, \leq) , every W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space is a W - $d\Psi^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - d - m_i - $g^{cl}(Z, T1)$ – R_0 space. Then the intersection of d - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every d - m_i - $g^{cl}(Z, T1)$ is a d - $m_i^{cl}(Z, T1)$ and then every d - $m_i^{cl}(Z, T1)$ is $d\Psi^{cl}$ set Then $d\Psi^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $d\Psi^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\}$. But the intersection of d - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $d\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $d\Psi^{R0}$ space.

EXAMPLE 3.60: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \zeta_1, \Omega_3 \} \}$. $\leq_6 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\delta_2, \zeta_1) \}, (\zeta_1, \Omega_3), \{ \delta_2, \Omega_3 \} \}$. m_i - $g^{cl}(Z, T1)$ are Φ, Z d - m_i - $g^{cl}(Z, T1)$ are Φ, Z Ψ^{cl} sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$. $d\Psi^{cl}$ sets are $\Phi, Z, \{ \delta_2 \}$. d - m_i - $g^{cl}\{ \zeta_1 \}$ is Z d - m_i - $g^{cl}\{ \delta_2 \}$ is Z d - m_i - $g^{cl}\{ \Omega_3 \}$ is Z The intersection of d - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$. $d\Psi^{cl}\{ \zeta_1 \}$ is Z $d\Psi^{cl}\{ \delta_2 \}$ is $\{ \delta_2 \}$ $d\Psi^{cl}\{ \Omega_3 \}$ is Z The intersection of $d\alpha^{cl}\{x\}$ is $\{ \delta_2 \}$ not equal to $Z \forall x \in Z$.

THEOREM 3.61: In a TOS (Z, T, \leq) , every W - b - m_i - $g^{cl}(Z, T1) - R_0$ space is a W - $b\Psi^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - b - m_i - $g^{cl}(Z, T1) - R_0$ space. Then the intersection of b - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every b - m_i - $g^{cl}(Z, T1)$ is a b - $m_i^{cl}(Z, T1)$ and then every b - $m_i^{cl}(Z, T1)$ is $b\Psi^{cl}$ set Then $b\Psi^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $b\Psi^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\}$. But the intersection of b - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $b\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $b\Psi^{R0}$ space.

EXAMPLE 3.62: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}, \{\zeta_1, \Omega_3\}\}$. $\leq_9 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3)\}$. m_i - $g^{cl}(Z, T1)$ are Φ, Z
 b - m_i - $g^{cl}(Z, T1)$ are Φ, Z
 Ψ^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}$
 $b\Psi^{cl}$ sets are $\Phi, Z, \{\delta_2\}$.
 b - m_i - $g^{cl}\{\zeta_1\}$ is Z
 b - m_i - $g^{cl}\{\delta_2\}$ is Z
 b - m_i - $g^{cl}\{\Omega_3\}$ is Z
 The intersection of b - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$.
 $b\Psi^{cl}\{\zeta_1\}$ is Z
 $b\Psi^{cl}\{\delta_2\}$ is $\{\delta_2\}$
 $b\Psi^{cl}\{\Omega_3\}$ is Z
 The intersection of b - $\Psi^{cl}\{x\}$ is $\{\delta_2\}$ is not equal to $Z \forall x \in Z$.

THEOREM 3.63: In a TOS (Z, T, \leq) , every W - i - m_i - $g^{cl}(Z, T1) - R_0$ space is a W - $b\Psi^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - i - m_i - $g^{cl}(Z, T1) - R_0$ space. Then the intersection of i - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every i - m_i - $g^{cl}(Z, T1)$ is a b - $m_i^{cl}(Z, T1)$ and then every b - $m_i^{cl}(Z, T1)$ is $b\Psi^{cl}$ set Then $b\Psi^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $b\Psi^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\}$. But the intersection of i - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $b\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $b\Psi^{R0}$ space.

EXAMPLE 3.64: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\zeta_1, \Omega_3\}\}$. $\leq_1 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3), (\delta_2, \Omega_3)\}$. m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\zeta_1, \Omega_3\}$
 i - m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{\Omega_3\}$
 Ψ^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}, \{\zeta_1, \Omega_3\}$
 $b\Psi^{cl}$ sets are Φ, Z
 i - m_i - $g^{cl}\{\zeta_1\}$ is Z
 i - m_i - $g^{cl}\{\delta_2\}$ is Z
 i - m_i - $g^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$
 The intersection of i - m_i - $g^{cl}\{x\}$ is $\{\Omega_3\} \forall x \in Z$.
 $b\Psi^{cl}\{\zeta_1\}$ is Z
 $\Psi^{cl}\{\delta_2\}$ is Z
 $b\Psi^{cl}\{\Omega_3\}$ is Z
 The intersection of $b\Psi^{cl}\{x\}$ is Z but not equal to $\{\Omega_3\} \forall x \in Z$.

THEOREM 3.65: In a TOS (Z, T, \leq) , every W - i - m_i - $g^{cl}(Z, T1) - R_0$ space is a W - $d\Psi^{R0}$ space but not converse.

Proof. Suppose (Z, T) be a W - i - m_i - $g^{cl}(Z, T1) - R_0$ space. Then the intersection of i - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every i - m_i - $g^{cl}(Z, T1)$ is a d - $m_i^{cl}(Z, T1)$ and then every d - $m_i^{cl}(Z, T1)$ is $d\Psi^{cl}$ set Then $d\Psi^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $d\Psi^{cl}\{x\}$ contained in i - m_i - $g^{cl}\{x\}$. But the intersection of i - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $d\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $d\Psi^{R0}$ space.

EXAMPLE 3.66: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\zeta_1, \delta_2\}\}$. $\leq_4 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\Omega_3, \zeta_1), (\Omega_3, \delta_2)\}$. m_i - $g^{cl}(Z, T1)$ are Φ, Z
 i - m_i - $g^{cl}(Z, T1)$ are Φ, Z
 Ψ^{cl} sets are $\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}, \{\zeta_1, \Omega_3\}$
 $d\Psi^{cl}$ sets are $\Phi, Z, \{\Omega_3\}, \{\zeta_1, \Omega_3\}$
 i - m_i - $g^{cl}\{\zeta_1\}$ is Z
 i - m_i - $g^{cl}\{\delta_2\}$ is Z
 i - m_i - $g^{cl}\{\Omega_3\}$ is Z
 The intersection of i - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$.
 $d\Psi^{cl}\{\zeta_1\}$ is $\{\zeta_1, \Omega_3\}$
 $d\Psi^{cl}\{\delta_2\}$ is Z
 $d\Psi^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$
 The intersection of $d\Psi^{cl}\{x\}$ is $\{\Omega_3\}$ not equal to $Z \forall x \in Z$.

THEOREM 3.67: In a TOS (Z, T, \leq) , every W - b - m_i - $g^{cl}(Z, T1) - R_0$ space is a W - $i\Psi^{R0}$ space but not converse.

Proof: Suppose (Z, T) be a W - b - m_i - $g^{cl}(Z, T1) - R_0$ space. Then the intersection of b - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every b - m_i - $g^{cl}(Z, T1)$ is a i - $m_i^{cl}(Z, T1)$ and then every i - $m_i^{cl}(Z, T1)$ is $i\Psi^{cl}$ set Then $i\Psi^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $i\Psi^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\}$. But the intersection of b - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $i\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $i\Psi^{R0}$ space.

EXAMPLE 3.68: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\}\}$. $\leq_2 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2)\}$. m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\}$
 b - m_i - $g^{cl}(Z, T1)$ are Φ, Z
 Ψ^{cl} sets are $\Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\}$
 $i\Psi^{cl}$ sets are $\Phi, Z, \{\delta_2, \Omega_3\}$
 b - m_i - $g^{cl}\{\zeta_1\}$ is Z
 b - m_i - $g^{cl}\{\delta_2\}$ is Z
 b - m_i - $g^{cl}\{\Omega_3\}$ is Z
 The intersection of b - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$.
 $i\Psi^{cl}\{\zeta_1\}$ is Z
 $i\Psi^{cl}\{\delta_2\}$ is $\{\delta_2, \Omega_3\}$
 $i\Psi^{cl}\{\Omega_3\}$ is $\{\delta_2, \Omega_3\}$
 The intersection of $i\Psi^{cl}\{x\}$ is $\{\delta_2, \Omega_3\}$ not equal to $Z \forall x \in Z$.

THEOREM 3.69: In a TOS (Z, T, \leq) , every W - b - m_i - $g^{cl}(Z, T1) - R_0$ space is a W - $d\Psi^{R_0}$ space but not converse.

Proof: Suppose (Z, T) be a W - b - m_i - $g^{cl}(Z, T1) - R_0$ space. Then the intersection of b - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every b - m_i - $g^{cl}(Z, T1)$ is a d - $m_i^{cl}(Z, T1)$ and then every d - $m_i^{cl}(Z, T1)$ is $d\Psi^{cl}$ set Then $d\Psi^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $d\Psi^{cl}\{x\}$ contained in b - m_i - $g^{cl}\{x\}$. But the intersection of b - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $d\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $d\Psi^{R_0}$ space.

EXAMPLE 3.70: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \zeta_1, \delta_2 \}, \{ \delta_2, \Omega_3 \} \}$. $\leq_7 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3), (\delta_2, \Omega_3) \}$ m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$ b - m_i - $g^{cl}(Z, T1)$ are Φ, Z Ψ^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$ $d\Psi^{cl}$ sets are $\Phi, Z, \{ \zeta_1 \}$ b - m_i - $g^{cl}\{ \zeta_1 \}$ is Z b - m_i - $g^{cl}\{ \delta_2 \}$ is Z b - m_i - $g^{cl}\{ \Omega_3 \}$ is Z The intersection of b - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$. $d\Psi^{cl}\{ \zeta_1 \}$ is $\{ \zeta_1 \}$ $d\Psi^{cl}\{ \delta_2 \}$ is Z $d\Psi^{cl}\{ \Omega_3 \}$ is Z The intersection of $\Psi^{cl}\{x\}$ is $\{ \zeta_1 \}$ not equal to $Z \forall x \in Z$.

THEOREM 3.71: In a TOS (Z, T, \leq) , every W - d - m_i - $g^{cl}(Z, T1) - R_0$ space is a W - $i\Psi^{R_0}$ space but not converse.

Proof: Suppose (Z, T) be a W - d - m_i - $g^{cl}(Z, T1) - R_0$ space. Then the intersection of d - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every d - m_i - $g^{cl}(Z, T1)$ is a i - $m_i^{cl}(Z, T1)$ and then every i - $m_i^{cl}(Z, T1)$ is $i\Psi^{cl}$ set Then $i\Psi^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $i\Psi^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\}$. But the intersection of d - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $i\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $i\Psi^{R_0}$ space.

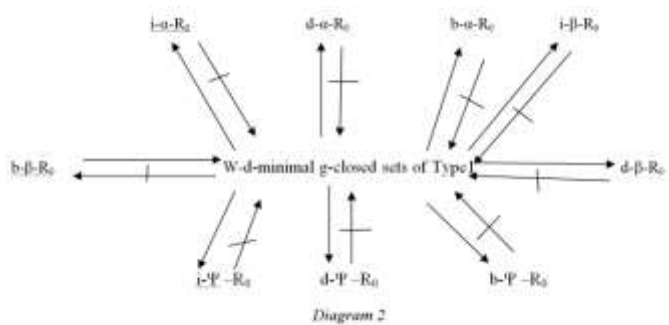
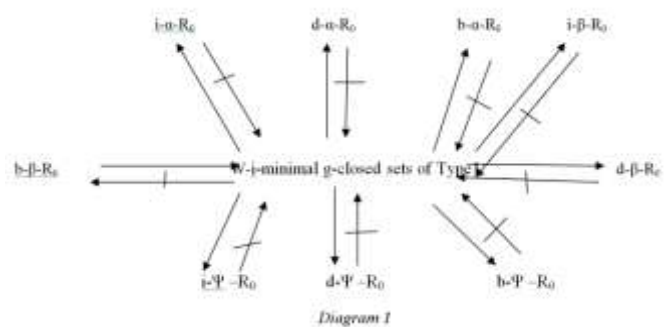
EXAMPLE 3.72: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \} \}$. $\leq_3 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3) \}$ m_i - $g^{cl}(Z, T1)$ are Φ, Z d - m_i - $g^{cl}(Z, T1)$ are Φ, Z Ψ^{cl} are $\Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \zeta_1, \delta_2 \}, \{ \zeta_1, \Omega_3 \}$ $i\Psi^{cl}$ sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}$ d - m_i - $g^{cl}\{ \zeta_1 \}$ is Z d - m_i - $g^{cl}\{ \delta_2 \}$ is Z d - m_i - $g^{cl}\{ \Omega_3 \}$ is Z The intersections of d - m_i - $g^{cl}\{x\}$ is $Z \forall x \in Z$. $i\Psi^{cl}\{ \zeta_1 \}$ is Z $i\Psi^{cl}\{ \delta_2 \}$ is $\{ \delta_2 \}$ $i\Psi^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$ The intersection of $i\Psi^{cl}\{x\}$ is empty not equal to $Z \forall x \in Z$.

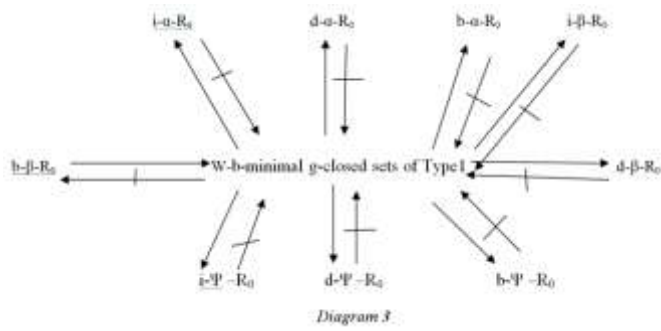
THEOREM 3.73: In a TOS (Z, T, \leq) , every W - d - m_i - $g^{cl}(Z, T1) - R_0$ space is a W - $b\Psi^{R_0}$ space but not converse.

Proof: Suppose (Z, T) be a W - d - m_i - $g^{cl}(Z, T1) - R_0$ space. Then the intersection of d - m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every d - m_i - $g^{cl}(Z, T1)$ is a b - $m_i^{cl}(Z, T1)$ and then every b - $m_i^{cl}(Z, T1)$ is $b\Psi^{cl}$ set Then $b\Psi^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $b\Psi^{cl}\{x\}$ contained in d - m_i - $g^{cl}\{x\}$. But the intersection of d - m_i - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $b\Psi^{cl}\{x\}$ is empty set $\forall x \in Z$ Hence (Z, T) is W - $b\Psi^{R_0}$ space.

EXAMPLE 3.74: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \zeta_1 \}, \{ \zeta_1, \Omega_3 \} \}$. $\leq_7 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\delta_2, \zeta_1) \}$ m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \zeta_1 \}, \{ \delta_2, \Omega_3 \}$ d - m_i - $g^{cl}(Z, T1)$ are $\Phi, Z, \{ \delta_2, \Omega_3 \}$ Ψ^{cl} sets are $\Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$ $b\Psi^{cl}$ sets are $\Phi, Z, \{ \Omega_3 \}$ d - m_i - $g^{cl}\{ \zeta_1 \}$ is Z d - m_i - $g^{cl}\{ \delta_2 \}$ is $\{ \delta_2, \Omega_3 \}$ d - m_i - $g^{cl}\{ \Omega_3 \}$ is $\{ \delta_2, \Omega_3 \}$ $b\Psi^{cl}\{ \zeta_1 \}$ is Z $b\Psi^{cl}\{ \delta_2 \}$ is Z $b\Psi^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$ The intersection of d - m_i - $g^{cl}\{x\}$ is $\{ \delta_2, \Omega_3 \}$ The intersection of $b\Psi^{cl}\{x\}$ is $\{ \Omega_3 \}$ not equal to $\{ \delta_2, \Omega_3 \}$ $b\Psi^{cl}\{ \zeta_1 \}$ is Z $b\Psi^{cl}\{ \delta_2 \}$ is Z $b\Psi^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$ The intersection of $b\Psi^{cl}\{x\}$ is $\{ \Omega_3 \}$ not equal to $\{ \delta_2, \Omega_3 \}$

The following diagrams shows the above results.





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