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On the Pellian Like Equation $5x^2 - 7y^2 = -8$

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Abstract – *The binary quadratic equation represented by the pellian like equation $5x^2 - 7y^2 = -8$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*

Key Words: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions.

1. INTRODUCTION

The binary quadratic Diophantine equation of the form $ax^2 - by^2 = N, (a, b, N \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-14].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $5x^2 - 7y^2 = -8$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

2. Method of Analysis

The Diophantine Equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$5x^2 - 7y^2 = -8 \qquad (1)$$

Consider the linear transformations

$$x = X + 7T \quad y = X + 5T \qquad (2)$$

From (1) and (2), we have

$$X^2 = 35T^2 + 4 \qquad (3)$$

whose smallest positive integer solution is

$$X_0 = 12 \quad T_0 = 2$$

To obtain the other solutions of (3), consider the pellian equation is

$$X^2 = 35T^2 + 1 \qquad (4)$$

whose smallest positive integer solution is

$$(\tilde{X}_0, \tilde{T}_o) = (6,1)$$

The general solution of (4) is given by

$$\tilde{T}_n = \frac{1}{2\sqrt{35}} g_n \quad , \quad \tilde{X}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(6+\sqrt{35}\right)^{n+1} + \left(6-\sqrt{35}\right)^{n+1}$$
$$g_n = \left(6+\sqrt{35}\right)^{n+1} - \left(6-\sqrt{35}\right)^{n+1}$$

Applying Brahmagupta lemma between (X_0, T_0) and $(\tilde{X}_n, \tilde{T}_n)$ the other integer solutions of (3) are given by

$$\left.\begin{aligned} X_{n+1} &= 6 f_n + \sqrt{35} g_n \\ T_{n+1} &= f_n + \frac{6}{\sqrt{35}} g_n \end{aligned}\right\} \qquad (5)$$

From (2), (4) and (5) the values of x and y satisfying (1) are given by

$$x_{n+1} = 13 f_n + \frac{77}{\sqrt{35}} g_n$$

$$y_{n+1} = 11 f_n + \frac{65}{\sqrt{35}} g_n$$

The recurrence relation satisfied by the solution x and y are given by

$$x_{n+3} - 12x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 12y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x_n and y_n satisfying (1) are given in the Table :1 below

Table: 1 Numerical Examples

n	x_n	y_n
0	26	22
1	310	262
2	3694	3122
3	44018	37202
4	524522	443302
5	6250246	5282422
6	74478430	62945762
7	887490914	750066722

From the above table, we observe some interesting relations among the solutions which are presented below:

- x_n and y_n values are always even.

1. Relation among the solutions are given below:

- $x_{n+2} - 7y_{n+1} - 6x_{n+1} = 0$
- $x_{n+3} - 84y_{n+1} - 71x_{n+1} = 0$
- $y_{n+2} - 6y_{n+1} - 5x_{n+1} = 0$
- $y_{n+3} - 71y_{n+1} - 60x_{n+1} = 0$
- $x_{n+3} - 12x_{n+2} + x_{n+1} = 0$
- $x_{n+1} - 6x_{n+2} + 7y_{n+2} = 0$
- $x_{n+3} - x_{n+1} - 14y_{n+2} = 0$
- $7y_{n+3} - 71x_{n+2} + 6x_{n+1} = 0$
- $x_{n+1} - 71x_{n+3} + 84y_{n+3} = 0$
- $x_{n+3} - 6x_{n+2} - 7y_{n+2} = 0$
- $71y_{n+2} + 5x_{n+1} - 6y_{n+3} = 0$
- $7y_{n+1} + 71x_{n+2} - 6x_{n+3} = 0$
- $y_{n+1} + 5x_{n+2} - 6y_{n+2} = 0$
- $y_{n+1} + 10x_{n+2} - y_{n+3} = 0$
- $y_{n+2} + 5x_{n+3} - 6y_{n+3} = 0$
- $71y_{n+2} - 6y_{n+1} - 5x_{n+3} = 0$
- $y_{n+1} + 60x_{n+3} - 71y_{n+3} = 0$
- $y_{n+1} - 12y_{n+2} + y_{n+3} = 0$
- $7y_{n+3} - 6x_{n+3} + x_{n+2} = 0$
- $y_{n+3} - 6y_{n+2} - 5x_{n+2} = 0$

2. Each of the following expressions represents a nasty number:

- $3(77y_{2n+2} - 65x_{2n+2} + 4)$
- $\frac{3}{7}(77x_{2n+3} - 917x_{2n+2} + 28)$
- $\frac{1}{28}(77x_{2n+4} - 10927x_{2n+2} + 336)$
- $\frac{1}{2}(77y_{2n+3} - 775x_{2n+2} + 24)$
- $\frac{3}{71}(77y_{2n+4} - 9235x_{2n+2} + 284)$
- $\frac{1}{2}(917y_{2n+2} - 65x_{2n+3} + 24)$
- $\frac{3}{71}(10927y_{2n+2} - 65x_{2n+4} + 284)$
- $3(155y_{2n+2} - 13y_{2n+3} + 4)$
- $\frac{1}{4}(1847y_{2n+2} - 13y_{2n+4} + 48)$
- $3(131x_{2n+4} - 1561x_{2n+3} + 4)$
- $3(917y_{2n+3} - 775x_{2n+3} + 4)$
- $\frac{1}{2}(917y_{2n+4} - 9235x_{2n+3} + 24)$
- $\frac{1}{2}(10927y_{2n+3} - 775x_{2n+4} + 24)$

- $3(10927y_{2n+4}-9235x_{2n+4}+4)$
- $3(1847y_{2n+3}-155y_{2n+4}+4)$

3. Each of the following expressions represents a cubical integer:

- $\frac{1}{2}(77y_{3n+3}-65x_{3n+3}+231y_{n+1}-195x_{n+1})$
- $\frac{1}{14}\begin{pmatrix}77x_{3n+4}-917x_{3n+3}+231x_{n+2}\\-2751x_{n+1}\end{pmatrix}$
- $\frac{1}{168}\begin{pmatrix}77x_{3n+5}-10927x_{3n+3}+231x_{n+3}\\-32781x_{n+1}\end{pmatrix}$
- $\frac{1}{12}\begin{pmatrix}77y_{3n+4}-775x_{3n+3}+231y_{n+2}\\-2325x_{n+1}\end{pmatrix}$
- $\frac{1}{142}\begin{pmatrix}77y_{3n+5}-9235x_{3n+3}+231y_{n+3}\\-27705x_{n+1}\end{pmatrix}$
- $\frac{1}{12}\begin{pmatrix}917y_{3n+3}-65x_{3n+4}+2751y_{n+1}\\-195x_{n+2}\end{pmatrix}$
- $\frac{1}{142}\begin{pmatrix}10927y_{3n+3}-65x_{3n+5}+32781y_{n+1}\\-195x_{n+3}\end{pmatrix}$
- $\frac{1}{2}(155y_{3n+3}-13y_{3n+4}+465y_{n+1}-39y_{n+2})$
- $\frac{1}{24}(1847y_{3n+3}-13y_{3n+5}+5541y_{n+1}-39y_{n+3})$
- $\frac{1}{2}(131x_{3n+5}-1561x_{3n+4}+393x_{n+3}-4683x_{n+2})$
- $\frac{1}{2}\begin{pmatrix}917y_{3n+4}-775x_{3n+4}+2751y_{n+2}\\-2325x_{n+2}\end{pmatrix}$
- $\frac{1}{12}\begin{pmatrix}917y_{3n+5}-9235x_{3n+4}+2751y_{n+3}\\-27705x_{n+2}\end{pmatrix}$
- $\frac{1}{12}\begin{pmatrix}10927y_{3n+4}-775x_{3n+5}+32781y_{n+2}\\-2325x_{n+3}\end{pmatrix}$
- $\frac{1}{2}\begin{pmatrix}10927y_{3n+5}-9235x_{3n+5}+32781y_{n+3}\\-27705x_{n+3}\end{pmatrix}$
- $\frac{1}{2}\begin{pmatrix}1847y_{3n+4}-155y_{3n+5}+5541y_{n+2}\\-465y_{n+3}\end{pmatrix}$

4. Each of the following expressions represents a biquadratic integer:

- $\frac{1}{2}\begin{pmatrix}77y_{4n+4}-65x_{4n+4}+308y_{2n+2}\\-260x_{2n+2}+12\end{pmatrix}$
- $\frac{1}{14}\begin{pmatrix}77y_{4n+5}-917x_{4n+4}+308x_{2n+3}\\-3668x_{2n+2}+84\end{pmatrix}$
- $\frac{1}{168}\begin{pmatrix}77y_{4n+6}-10927x_{4n+4}+308x_{2n+4}\\-43708x_{2n+2}+1008\end{pmatrix}$
- $\frac{1}{12}\begin{pmatrix}77y_{4n+5}-775x_{4n+4}+308y_{2n+3}\\-3100x_{2n+2}+72\end{pmatrix}$
- $\frac{1}{142}\begin{pmatrix}77y_{4n+6}-9235x_{4n+4}+308y_{2n+4}\\-36940x_{2n+2}+852\end{pmatrix}$
- $\frac{1}{12}\begin{pmatrix}917y_{4n+4}-65x_{4n+5}+3668y_{2n+2}\\-260x_{2n+3}+72\end{pmatrix}$
- $\frac{1}{142}\begin{pmatrix}10927y_{4n+4}+43708y_{2n+2}\\-65x_{4n+6}-260x_{2n+4}+852\end{pmatrix}$
- $\frac{1}{2}\begin{pmatrix}155y_{4n+4}-13x_{4n+5}+620y_{2n+2}\\-52y_{2n+3}+12\end{pmatrix}$
- $\frac{1}{24}\begin{pmatrix}1847y_{4n+4}-13y_{4n+6}+7388y_{2n+2}\\-52y_{2n+4}+144\end{pmatrix}$
- $\frac{1}{14}\begin{pmatrix}917x_{4n+6}-10927x_{4n+5}+3668x_{2n+4}\\-43708x_{2n+3}+84\end{pmatrix}$

- $\frac{1}{2}\begin{pmatrix}917y_{4n+5}-775x_{4n+5}+3668y_{2n+3}\\-3100x_{2n+3}+12\end{pmatrix}$

- $\frac{1}{12}\begin{pmatrix}917y_{4n+6}-9235x_{4n+5}+3668y_{2n+4}\\-36940x_{2n+3}+72\end{pmatrix}$

- $\frac{1}{12}\begin{pmatrix}10927y_{4n+5}-775x_{4n+6}+43708y_{2n+3}\\-3100x_{2n+4}+72\end{pmatrix}$

- $\frac{1}{2}\begin{pmatrix}10927y_{4n+6}-9235x_{4n+6}+43708y_{2n+4}\\-36940x_{2n+4}+12\end{pmatrix}$

- $\frac{1}{2}\begin{pmatrix}1847y_{4n+5}-155y_{4n+6}+7388y_{2n+3}\\-620y_{2n+4}+12\end{pmatrix}$

5. Each of the following expressions represents a quintic integer:

- $\frac{1}{2}\begin{pmatrix}77y_{5n+5}-65x_{5n+5}+385y_{3n+3}-325x_{3n+3}\\+770y_{n+1}-650x_{n+1}\end{pmatrix}$

- $\frac{1}{14}\begin{pmatrix}77x_{5n+6}-917x_{5n+5}+385x_{3n+4}\\-4585x_{3n+3}+770x_{n+2}-9170x_{n+1}\end{pmatrix}$

- $\frac{1}{168}\begin{pmatrix}77x_{5n+7}-10927x_{5n+5}+385y_{3n+5}\\-54635x_{3n+3}+770x_{n+3}-109270x_{n+1}\end{pmatrix}$

- $\frac{1}{12}\begin{pmatrix}77y_{5n+6}-775x_{5n+5}+385y_{3n+4}\\-3875x_{3n+3}+770y_{n+2}-7750x_{n+1}\end{pmatrix}$

- $\frac{1}{142}\begin{pmatrix}77y_{5n+7}-9235x_{5n+5}+385y_{3n+5}\\-46175x_{3n+3}+770y_{n+3}-92350x_{n+1}\end{pmatrix}$

- $\frac{1}{12}\begin{pmatrix}917y_{5n+5}-65x_{5n+6}+4585y_{3n+3}\\-325x_{3n+4}+9170y_{n+1}-650x_{n+2}\end{pmatrix}$

- $\frac{1}{142}\begin{pmatrix}10927y_{5n+5}-65x_{5n+7}+54635y_{3n+3}\\-325x_{3n+5}+109270y_{n+1}-650x_{n+3}\end{pmatrix}$

- $\frac{1}{2}\begin{pmatrix}155y_{5n+5}-13y_{5n+6}+775y_{3n+3}\\-65y_{3n+4}+1550y_{n+1}-130y_{n+2}\end{pmatrix}$

- $\frac{1}{24}\begin{pmatrix}1847y_{5n+5}-13y_{5n+7}+9235y_{3n+3}\\-65y_{3n+5}+18470y_{n+1}-130y_{n+3}\end{pmatrix}$

- $\frac{1}{14}\begin{pmatrix}917x_{5n+7}-10927x_{5n+6}+4585x_{3n+5}\\-54635x_{3n+4}+9170x_{n+3}-109270x_{n+2}\end{pmatrix}$

- $\frac{1}{2}\begin{pmatrix}917y_{5n+6}-775x_{5n+6}+4585y_{3n+4}\\-3875x_{3n+4}+9170y_{n+2}-7750x_{n+2}\end{pmatrix}$

- $\frac{1}{12}\begin{pmatrix}917y_{5n+7}-9235x_{5n+6}+4585y_{3n+5}\\-46175x_{3n+4}+9170y_{n+3}-923500x_{n+2}\end{pmatrix}$

- $\frac{1}{12}\begin{pmatrix}10927y_{5n+6}-775x_{5n+7}+54635y_{3n+4}\\-3875x_{3n+5}+109270y_{n+2}-7750x_{n+3}\end{pmatrix}$

- $\frac{1}{2}\begin{pmatrix}10927y_{5n+7}-9235x_{5n+7}+54635y_{3n+5}\\-46175x_{3n+5}+109270y_{n+3}-92350x_{n+3}\end{pmatrix}$

- $\frac{1}{2}\begin{pmatrix}1847y_{5n+6}-155y_{5n+7}+9235y_{3n+4}\\-775y_{3n+5}+18470y_{n+2}-1550y_{n+3}\end{pmatrix}$

REMARKABLE OBSERVATIONS

I. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbolas which are presented in Table: 2 below:

Table: 2 Hyperbolas

S.NO	Hyperbola	(X,Y)
1	$Y^2-35X^2=16$	$\begin{pmatrix}11x_{n+1}-13y_{n+1}\ ,\\77y_{n+1}-65x_{n+1}\end{pmatrix}$
2	$Y^2-35X^2=784$	$\begin{pmatrix}155x_{n+1}-13x_{n+2}\ ,\\77x_{n+2}-917x_{n+1}\end{pmatrix}$
3	$Y^2-35X^2=112896$	$\begin{pmatrix}1847x_{n+1}-13x_{n+3}\ ,\\77x_{n+3}-10927x_{n+1}\end{pmatrix}$
4	$Y^2-35X^2=576$	$\begin{pmatrix}131x_{n+1}-13y_{n+1}\ ,\\77y_{n+2}-775x_{n+1}\end{pmatrix}$
5	$Y^2-35X^2=80656$	$\begin{pmatrix}1561x_{n+1}-13y_{n+3}\ ,\\77y_{n+3}-9235x_{n+1}\end{pmatrix}$

6	$Y^2-35X^2=576$	$\left(11x_{n+2}-155y_{n+1},\ 917y_{n+1}-65x_{n+2}\right)$
7	$Y^2-35X^2=80656$	$\left(11x_{n+3}-1847y_{n+1},\ 10927y_{n+1}-65x_{n+3}\right)$
8	$Y^2-35X^2=400$	$\left(11y_{n+2}-131y_{n+1},\ 775y_{n+1}-65y_{n+2}\right)$
9	$Y^2-35X^2=57600$	$\left(11y_{n+3}-1561y_{n+1},\ 9235y_{n+1}-65y_{n+3}\right)$
10	$Y^2-35X^2=784$	$\left(1847x_{n+2}-155x_{n+3},\ 917x_{n+3}-10927x_{n+2}\right)$
11	$Y^2-35X^2=16$	$\left(131x_{n+2}-155y_{n+2},\ 917y_{n+2}-775x_{n+2}\right)$
12	$Y^2-35X^2=576$	$\left(1561x_{n+2}-155y_{n+3},\ 917y_{n+3}-9235x_{n+2}\right)$
13	$Y^2-35X^2=576$	$\left(131x_{n+3}-1847y_{n+2},\ 10927y_{n+2}-775x_{n+3}\right)$
14	$Y^2-35X^2=16$	$\left(1561x_{n+3}-1847y_{n+3},\ 10927y_{n+3}-9235x_{n+3}\right)$
15	$Y^2-35X^2=400$	$\left(131y_{n+3}-1561y_{n+2},\ 9235y_{n+2}-775y_{n+3}\right)$

II. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in Table: 3 below:

Table: 3 Parabolas

S.NO	Parabola	(X,Y)
1	$2Y-35X^2=16$	$\left(11x_{n+1}-13y_{n+1},\ 77y_{2n+2}-65x_{2n+2}+4\right)$
2	$2Y-5X^2=112$	$\left(155x_{n+1}-13x_{n+2},\ 77x_{2n+3}-917x_{2n+2}+28\right)$
3	$24Y-5X^2=16128$	$\left(1847x_{n+1}-13x_{n+3},\ 77x_{2n+4}-10927x_{2n+2}+336\right)$
4	$12Y-35X^2=576$	$\left(131x_{n+1}-13y_{n+1},\ 77y_{2n+3}-775x_{2n+2}+24\right)$
5	$142Y-35X^2=80656$	$\left(1561x_{n+1}-13y_{n+3},\ 77y_{2n+4}-9235x_{2n+2}+284\right)$
6	$12Y-35X^2=576$	$\left(11x_{n+2}-155y_{n+1},\ 917y_{2n+2}-65x_{2n+3}+24\right)$
7	$142Y-35X^2=80656$	$\left(11x_{n+3}-1847y_{n+1},\ 10927y_{2n+2}-65x_{2n+4}+284\right)$
8	$2Y-7X^2=80$	$\left(11y_{n+2}-131y_{n+1},\ 775y_{2n+2}-65y_{2n+3}+20\right)$
9	$120Y-7X^2=11520$	$\left(11y_{n+3}-1561y_{n+1},\ 1847y_{2n+2}-13y_{2n+4}+48\right)$
10	$2Y-5X^2=112$	$\left(1847x_{n+2}-155x_{n+3},\ 917x_{2n+4}-10927x_{2n+3}+28\right)$
11	$2Y-35X^2=16$	$\left(131x_{n+2}-155y_{n+2},\ 917y_{2n+3}-775x_{2n+3}+4\right)$
12	$12Y-35X^2=576$	$\left(1561x_{n+2}-155y_{n+3},\ 917y_{2n+4}-9235x_{2n+3}+24\right)$
13	$12Y-35X^2=576$	$\left(131x_{n+3}-1847y_{n+2},\ 10927y_{2n+3}-775x_{2n+4}+24\right)$
14	$2Y-35X^2=16$	$\left(1561x_{n+3}-1847y_{n+3},\ 10927y_{2n+4}-9235x_{2n+4}+4\right)$
15	$2Y-7X^2=80$	$\left(131y_{n+3}-1561y_{n+2},\ 9235y_{2n+3}-775y_{2n+4}+20\right)$

III. Let p,q be two non-zero distinct integers such that $p>q>0$. Treat p,q as the generators of the Pythagorean triangle $T(\alpha,\beta,\gamma)$

where $\alpha = 2pq$, $\beta = p^2 - q^2$, $\gamma = p^2 + q^2$, $p > q > 0$

Taking $p = x_{n+1} + y_{n+1}, q = y_{n+1}$, it is observed that $T(\alpha, \beta, \gamma)$ is satisfied by the following relations:

- $3\gamma + 7\beta - 10\alpha = -16$
- $4\frac{A}{p} = \alpha + \beta - \gamma$
- $2\frac{A}{p} = x_{n+1} y_{n+1}$

Where A, P represent the area and perimeter of $T(\alpha, \beta, \gamma)$.

3. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the Pellian like equation $5x^2 - 7y^2 = -8$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of Pell equations and determine their integer solutions along with suitable properties.

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