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Analyzing vehicle handling using BICYCLE MODEL in MATLAB/OCTAVE

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Abstract – *Vehicle handling characteristics defines how a vehicle reacts to the driver's response during cornering. Bicycle model is the simplest model considering only two degrees of freedom neglecting the effect of longitudinal direction as it doesn't have much impact and makes the analysis easier. This paper mainly focuses on mathematical modelling of the governing equations and simulation in MATLAB/OCTAVE using an approach of state space representation and discussing the advantages of this method to other virtual analysis methods.*

Key Words: Vehicle handling characteristics, Cornering, degrees of freedom, Longitudinal direction, Mathematical modelling and State space representation.

1. INTRODUCTION

Handling is one of the most important modules of vehicle dynamics. A good handling characteristic of a vehicle improves car safety and also ergonomics. Many approaches are followed for analyzing vehicle behavior in cornering but most of them follow Modelling the vehicle and then simulating, which consumes a lot of time. Hence, the introduction of mathematical modelling is done and for this a simpler bicycle model is considered which can further be extended if needed. The equations which are obtained are converted in a state space form for easy MATLAB coding and also helps us to iterate countless times improving the accuracy. This simulation builds a strong foundation in the initial stage of designing.

2. VELOCITY ANALYSIS IN BCC

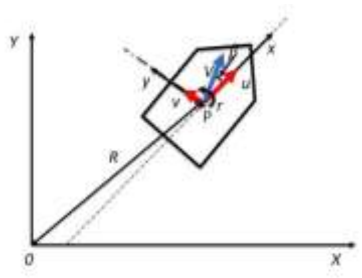


Figure1

P: center gravity of the vehicle.

β: side slip angles of the vehicle.

r: yaw rate of the vehicle

u the longitudinal velocity of point P at the x axis.

v: the lateral velocity of point P at the y-axis.

R: position vector from the fixed coordinate on the ground to the fixed coordinate at the vehicle.

The position vector of the vehicle from the reference on the ground (coordinate system $X-Y$), is defined as R.

Then, the velocity vector $\dot{R}$ can be written as:

$$\dot{R}=ui+vj \text{ (Eq.1)}$$

Where i and j are the unit vectors in x and y directions respectively. Where i and j are the unit vectors in x and y directions respectively. On differentiating the velocity $\dot{R}$ with respect to time gives acceleration of the vehicle $\ddot{R}$

$$\ddot{R}=\dot{u}i+ui+\dot{v}j+v\dot{j} \text{ (Eq.2)}$$

Now, considered that the unit vectors at the fixed coordinate on the ground as iF and jF.

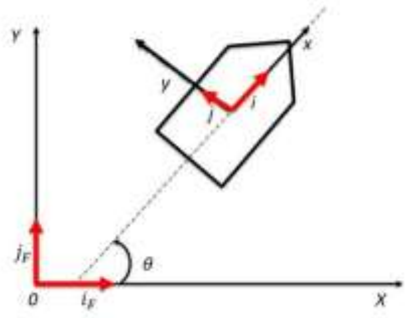


Figure2

The relation of the unit vector at the x and y axis (i and j), to the reference X and Y axis on the ground (iF and jF) can be expressed as:

$$i = cos\theta\ if + sin\theta\ jf \text{ (Eq.3)}$$

$$j = -sin\theta\ if + cos\theta\ jf \text{ (Eq.4)}$$

Then, differentiate i and j with the time we get:

$\ddot{j}=-r$i　　$\ddot{i}=rj$

Substitute them in Eq 2.

$$\ddot{R}= (\dot{u}\text{-vr})\ i+ (\dot{v}\text{+ur})\ j$$

Therefore, the longitudinal and lateral acceleration of the vehicle at point P can be expressed as:

ax= $(u^{\cdot}-vr)$

$ay=(v^{\cdot}+ur)$

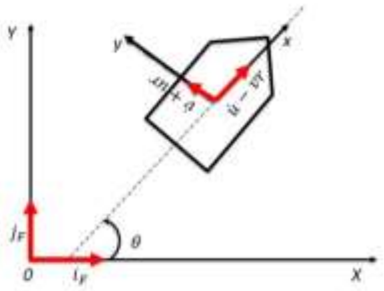


Figure3

3. BICYCLE MODEL

Bicycle model is the utmost basic vehicle dynamics model with two degree of freedom, representing the lateral and yaw motions. The cause for considering only two degree of freedom is that it is not compulsory to include the longitudinal direction, because it does not affect the lateral or yaw stability of the vehicle.

3.1 Assumptions

The height of vehicle CG is at ground level. No longitudinal load transfers. No lateral load transfer (since no track width). Linear range tires. Constant forward velocity. No chassis or suspension compliance effect

3.2 Derivation of Equation of motions

The equations of motion for the two degree-of-freedom vehicles are derived using basic principles of Newtonian mechanics for rigid body motion relative to translating and rotating coordinate systems. The basic equations relating the forces and moments acting on a rigid body to the acceleration of the body are

F=ma T=Iα

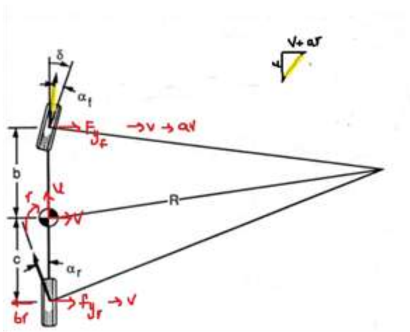


Figure4

Force balance equation.

$$F_{y_f}+F_{y_r}=m(\dot{v}+ur)$$

Moment balance equation

$$af_{y_f}-bf_{y_r}=I_z r$$

4. STATE SPACE REPRESENTATION (SSR)

It is the linear representation of the dynamic system in either continuous or discrete form or It is a representation of mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations.

x˙=Ax+Bu y˙=Cx+Du

A- System matrix B- Input matrix
C- Output matrix D-Feed forward matrix

It can be expressed in the form of differential equations

$$\frac{dx_1}{dt}=a_{11}x_1+a_{12}x_2+b_1v(t)$$

$$\frac{dx_2}{dt}=a_{21}x_1+a_{22}x_2+b_2v(t)$$

$$y=C_1x_1+C_2x,+d_1v(t)$$

$$\begin{bmatrix}\dot{x}_1\\x_2\end{bmatrix}=\begin{bmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}b_1\\b_2\end{bmatrix}v(t)$$

$$y=[C_1C_2]\begin{bmatrix}x_1\\x_2\end{bmatrix}+d_1v(t)$$

5. CONVERTING INTO SSR

$$\frac{d}{dt}\begin{bmatrix}V\\r\end{bmatrix}=\begin{bmatrix}\frac{-(C_{\propto f}+C_{\propto r})}{mu}&\frac{bC_{\propto r}-aC_{\propto f}}{mu}-u\\\frac{bC_{\propto r}-aC_{\propto f}}{I_{zu}}&\frac{-(a^2C_{\propto f}+b^2C_{\propto r})}{I_{zu}}\end{bmatrix}\begin{bmatrix}V\\r\end{bmatrix}+\begin{bmatrix}\frac{C_{\propto f}}{m}\\\frac{aC_{\propto f}}{I_z}\end{bmatrix}\delta f$$

There are three outputs for this model: **yaw rate (r), lateral speed(v) and lateral acceleration(**dv/dt**)**

$$[Yyawrate]=[0\ 1]\begin{bmatrix}V\\r\end{bmatrix}+[0]\delta$$

$$[Y\_(s)]=[1\ 0]\begin{bmatrix}V\\r\end{bmatrix}+[0]\delta$$

$$[Y\_(a)]=\begin{bmatrix}\frac{-(C_{\propto f}+C_{\propto r})}{mu}&\frac{bC_{\propto r}-aC_{\propto f}}{mu}-u\end{bmatrix}+u\,[0\ 1]$$

6. MATLAB CODE FOR FRONT WHEEL DRIVE PASSENGER CAR AND F1 RACE CAR

```
%Parameters for Passenger Car
a= 2.92; %Distance between CG and front axle (ft)
L= 8.33; %Wheel Base(ft)
m= 2000*0.65; %mass (lb.)
Iz= 8158.84; %yaw moment of inertia (lb.-ft/s^2)
Caf=13752; %cornering stiffness-front axle (N/rad)
Car=11460; %cornering stiffness- rear axle (N/rad)
```

```
%Parameters for F-1 Car
 %a= 5.71; %Distance between CG and front axle (ft)
%L= 9.52; %Wheel Base(ft)
%m= 1719*0.4; %mass (lb.)
%Iz= 6256.56; %yaw moment of inertia (lb.-ft/s^2)
%Caf= 62228; %cornering stiffness-front axle (lb./rad)
%Car= 96608; %cornering stiffness- rear axle (lb./rad)
u= 65.62; %ft/sec
b=L-a; %distance between CG and rear axle

A=[-(Caf+Car)/(m*u), ((b*Car-a*Caf)/(m*u))-u;
 (b*Car-a*Caf)/(Iz*u), -(a^2*Caf+b^2*Car)/(Iz*u)];
%System Matrix
B= [Caf/m; a*Caf/Iz]; %Input Matrix
C_lat= [1 0]; D_lat=0; %Output Matrix Lateral Speed
C_yaw= [0 1]; D_yaw=0; %Output Matrix Yaw Rate
C_acc= [-(Caf+Car)/(m*u), ((b*Car-a*Caf)/(m*u))-u] +
u*[0,1]; %Output Matrix Lateral Acceleration
D_acc=B (1);
C= [C_lat; C_yaw; C_acc]; %Combining the output matrix
into one matrix
D= [D_lat; D_yaw; D_acc]; %Combining the feed-forward
matrix
t= [0:0.01:6];
U=(5*pi/180) *sin(7*2*pi*t); %5 degree,7Hz sine steering
input
%U=pi/180*sin (0.5*2*pi*t); %1 degree,0.5Hz sine
steering input
sys=ss (A,B,C,D); %state space representation
Y=lsim(sys,U,t); %Linear simulation which generates
sinusoidal input
sys.inputname={'steering'};
sys.outputname= {'lateral speed';'yaw';'lateral acc'};
[mag,w]=bode(sys('yaw','steering'));
loglog(abs(w(1,:))/6.28,abs(mag(1,:)),'linewidth',3);
grid on
set (gca,'fontsize',16,'fontweight','bold')
title('Yaw Rate/Steer Angle Frequency Response-
Passenger Car');
xlabel('Frequency (Hz)');
ylabel('Yaw Rate (deg/sec)');
hold on
figure(2) %Step Response
step(sys);
hold on
%Sine Response for lateral speed input
figure(3)
subplot (3,1,1)
grid on
plot(t,Y(:,1),'r','linewidth',2);
set(gca,'fontweight','bold')
title('Lateral Speed Sine Response- Passenger
Car','fontsize',15)
xlabel('Time (sec)');
ylabel('Lateral speed (ft/sec)')
hold on
%Sine Response for yaw rate
subplot(2,1,2)
grid on
plot(t,Y(:,2)*180/pi,'r','linewidth',2);
set(gca,'fontweight','bold')
title('Yaw Rate Sine Response- Passenger
Car','fontsize',15)
xlabel('Time (sec)');
ylabel('Yaw Rate (deg/sec)')
hold on
%Sine Response for lateral acceleration
Figure (4)
Subplot (2,1,1)
grid on
plot(t,Y(:,3),'r','linewidth',2);
set(gca,'fontweight','bold')
hold on
%Sine response for steering
Subplot (2,1,2)
grid on
plot(t,U*180/pi,'r','linewidth',2);
set(gca,'fontweight','bold')
title ('Steering Angle- Passenger Car','fontsize',15)
xlabel ('Time (sec)');
ylabel ('Steering Angle(deg)')
```

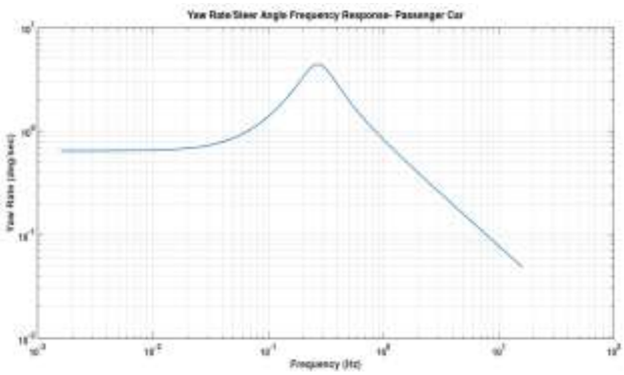
Note: - The only change in the code for Rear wheel and All wheel steer vehicles is the B matrix.

$$B=\begin{bmatrix}\frac{C_{\propto r}}{m}\\ \frac{aC_{\propto r}}{I_z}\end{bmatrix}\delta f \text{ – Rear wheel steering}$$

$$B=\begin{bmatrix}\frac{C_{\propto f}}{m} & \frac{C_{\propto r}}{m}\\ \frac{aC_{\propto f}}{I_z} & \frac{aC_{\propto r}}{I_z}\end{bmatrix}\begin{bmatrix}\delta f\\ \delta r\end{bmatrix} \text{ – All wheel steering}$$

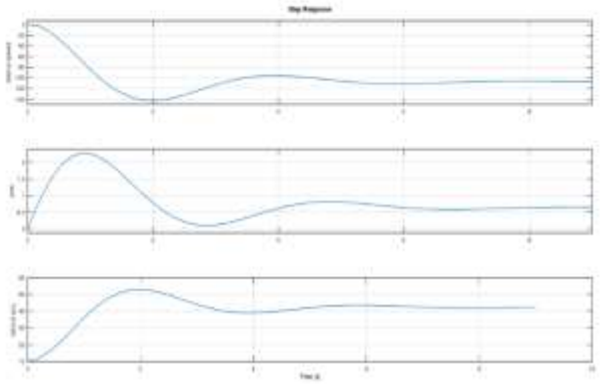
7. RESULTS FOR PASSENGER CAR

7.1 Frequency response

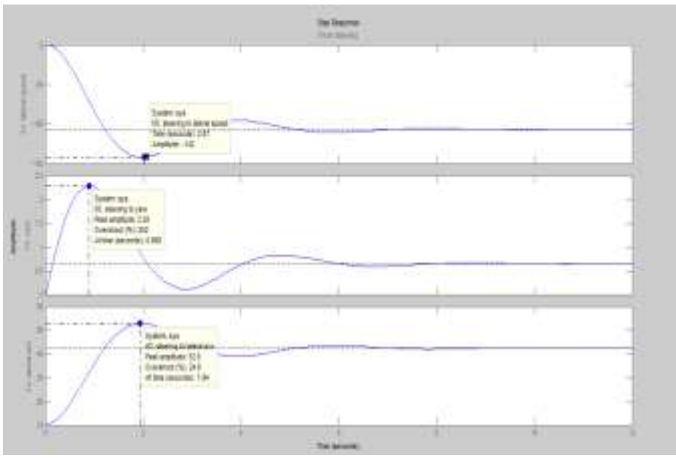


Graph1 – Yaw rate vs Frequency

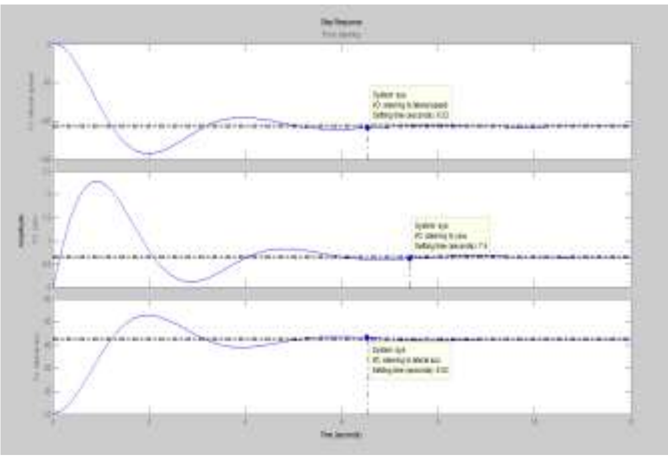
7.2 Step response



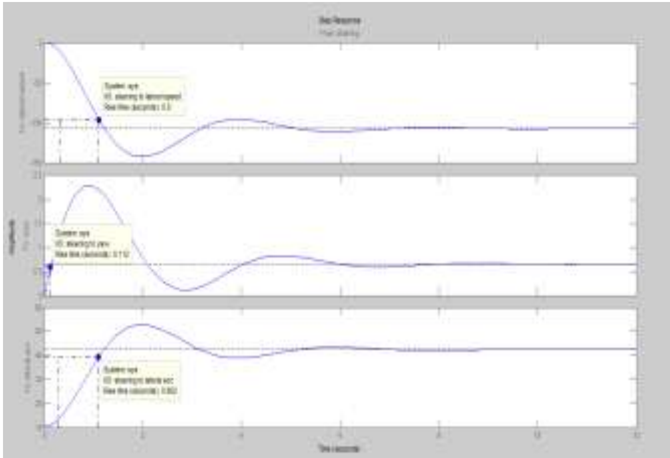
Graph2 – Step response



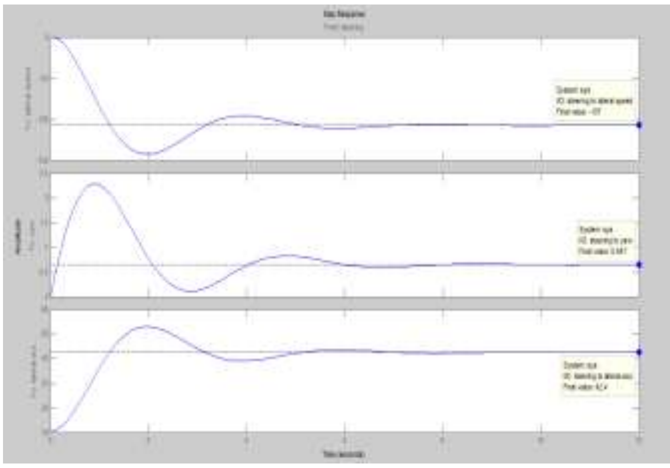
Graph 3- Peak value



Graph 4- Settling time

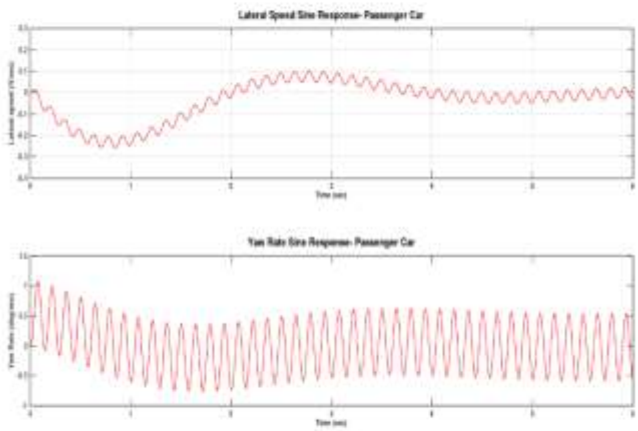


Graph5- Rise time

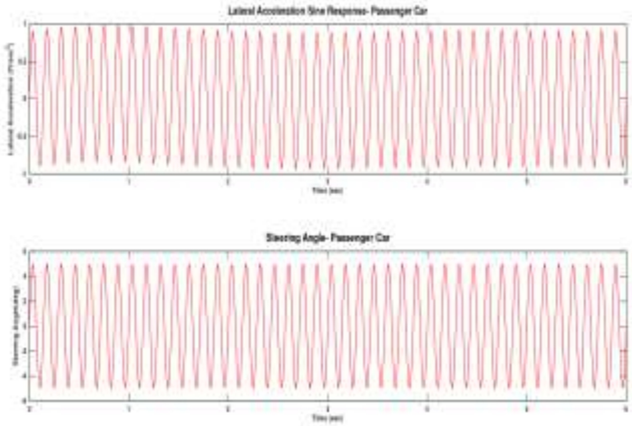


Graph6- Steady state

7.3 Sine response



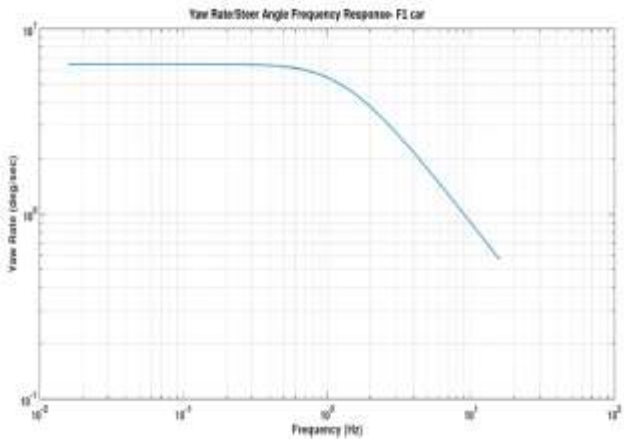
Graph7- Lateral speed, Yaw rate vs Time



Graph8- Lateral acceleration, Steering angle vs time

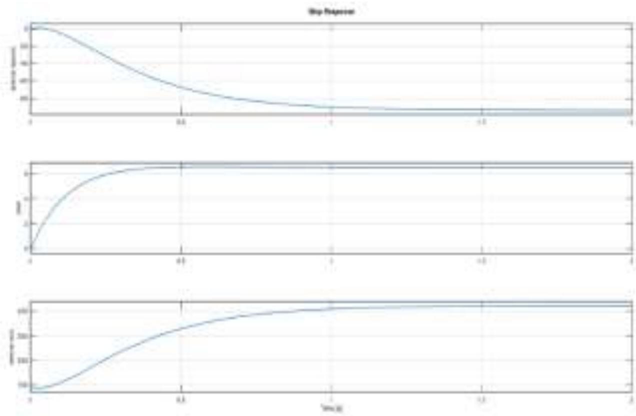
8. RESULTS FOR F1 RACE CAR

8.1 Frequency response

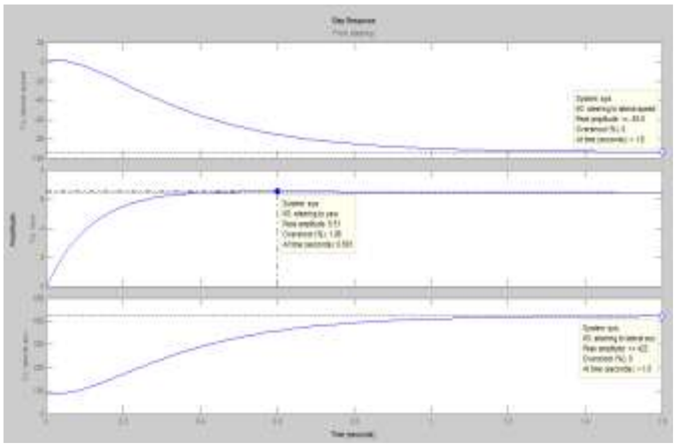


Graph9- Yaw rate vs Frequency

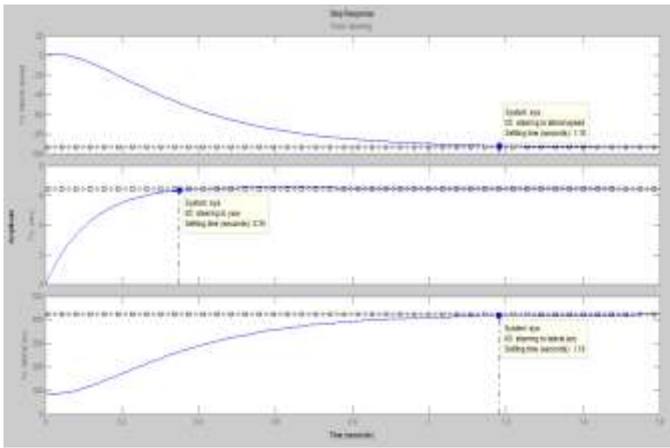
8.2 Step response



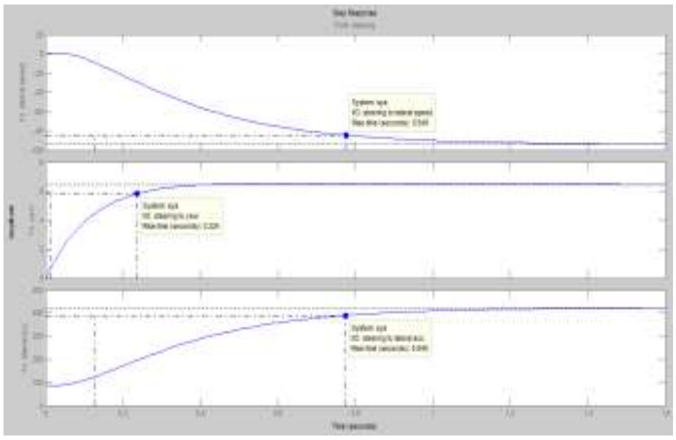
Graph10- Step response



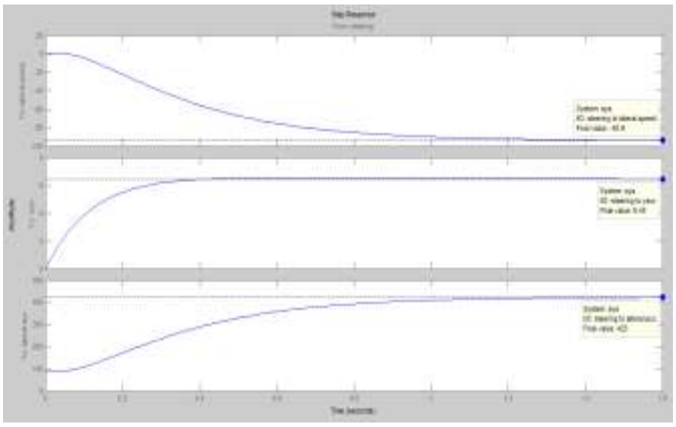
Graph11- Peak value



Graph12- Settling time

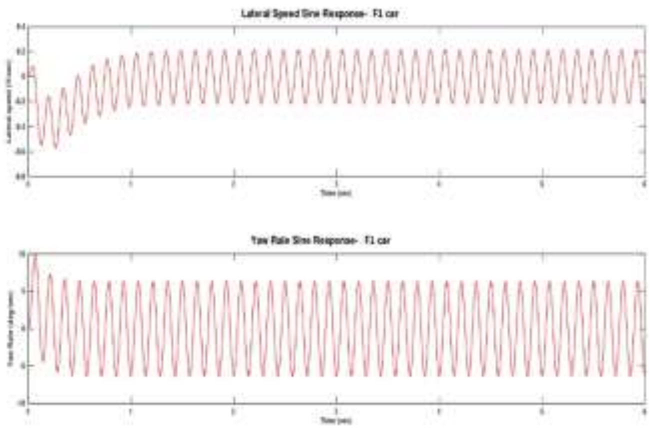


Graph13- Rise time

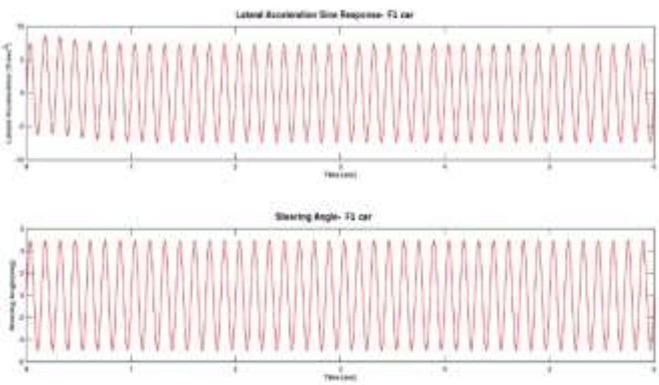


Graph 14- Steady state

8.3 Sine response



Graph15- Lateral speed, Yaw rate vs Time



Graph16- Lateral acceleration, Steering angle vs Time

9. COMPARISON OF RESULTS FOR PASSENGER CAR AND RACE CAR

The step response for passenger car is unstable and has a greater overshoot and steady state value than that for the F1 Car. The responses take larger time to stabilize to a steady-state value. The frequency response plot for passenger car show steep increase in the yaw rate while F1 car plot has a smooth increase in yaw. Basically, sine response is used for comparing the results with step plot and frequency response plot.

10. CONCLUSION

In this paper we have analyzed the handling characteristics of a front wheel drive passenger and a race car using the application of mathematical modelling in MATLAB. For that we have done force balancing and moment balancing to derive the governing equations, these equations are future simplified to state space representation for easy coding. A function named “SS” in MATLAB is used for this purpose. The difference in the results of various applications are also shown which concludes the factors that effects to improve handling characteristics.

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