A STUDY ON n-POWER CLASS(Q) OPERATOR

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Abstract - *In this paper we introduce the new class n-power class(Q) operators acting on a Hilbert space H. An operator $T \in L(H)$ is n-power class(Q) if $(T^{*2}T^{2n} = (T^*T^n)^2)$. We investigate some basic properties of such operator. In general a n-power class(Q) operator need not be a normal operator.*

Key Words: Normal, n-normal, n-power quasinormal, Hilbert space, class(Q).

1. INTRODUCTION

Throughout this chapter H is a Hilbert space and L(H) is the algebra of all bounded linear operators acting on H. S.Panayappan [2] defined a new class n- power class(Q) operator acting on a Hilbert space H. In this chapter, some basic properties of such operator and a n-power class(Q) operator need not be a normal operator are investigated.

2. DEFINITION

2.1 n -POWER CLASS(Q):

An operator $T \in B(H)$ is said to be n-power class(Q) if

$$(T^{*2}T^{2n} = (T^*T^n)^2)$$

3. RELATED THEOREMS AND EXAMPLES TO *n*-POWER CLASS(Q) OPERATOR

Theorem 3.1

If $T \in n\ power\ class(Q)$ then so are

(i) kT for any real number k.
(ii) Any $S \in L(H)$ that is unitary equivalent to T.
(iii) The restriction $T/_M$ of T to any closed subspace $M\ of\ H$ that reduces T.

Proof

(i) let T is *(n,m)-power class(Q)* then

$$T^{*2}T^{2n} = (T^*T^n)^2$$

$$(k\,T)^{*2}(kT)^{2n} = \bar{k}^2T^{*2}k^{2n}T^{2n}$$

$$= \bar{k}^2k^{2n}T^{*2}T^{2n}$$

$$= (\bar{k}\quad k^n)^2(T^*T^n)^2$$

$$= (\bar{k}\quad k^nT^*T^n)^2$$

$$= (\bar{k}\quad T^*k^nT^n)^2$$

$$= ((kT)^*(kT)^n)^2$$

(ii) Let $S \in L(H)$ be unitarily equivalent to T then there is a unitary operator $U \in L(H)$ such that $S^{2n} = U^*T^{2n}U$ which implies that $S^* = U^*T^*U$.

Thus $$S^{*2}S^{2n} = U^*T^*UU^*T^*US^{2n}$$

$$= U^*T^*UU^*T^*UU^*T^{2n}U$$

$$= U^*(T^*)^2T^{2n}U$$

$$(\because S^{2n} = U^*T^{2n}U)$$

$$= U^*(T^*T^n)^2U$$

Since $$T^{*2}T^{2n} = (T^*T^n)^2$$

$$S^{*2}S^{2n} = (S^*S^n)^2.$$

Thus $S \in\ n\ power\ class\ (Q)$.

(iii) By [ii] we have

$$(T/_M)^{*2}(T/_M)^{2n} = \left(T^{*2}/_M\right)\left(T^{2n}/_M\right)$$

$$= (T^*T^n)^2/_M$$

$$= \left[(T/_M)^*(T/_M)^n\right]^2$$

Hence

$$T/_M \in n\ power\ class\ (Q)$$

Example 3.2

Consider the two operator $T = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ and $X = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ acting on the two dimensional Hilbert space then $T \in 2 \ \ power \ \ class\,(Q)$. But $S \notin 2 \ \ power \ \ class\,(Q)$

Proof: Given $T = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

And also given $X = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

We know that, $X^{-1} = \frac{1}{|X|} adj(X)$

$$X^{-1} = \frac{1}{|1-0|} adj \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$= \frac{1}{1} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}.$$

Next prove that, $S = XTX^{-1}$

$$= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+0 & 0+1 \\ 0+0 & 0+1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+0 & -2+1 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$$

$$\therefore S = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} = XTX^{-1}. \text{(say)}$$

Now again by direct decomposition

$$(S^*S^2)^2 \neq (S^*)^2(S^2)^2$$

Now $S^* = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$

$$(S^*)^2 = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4+0 & 0+0 \\ -2-1 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ -3 & 1 \end{pmatrix}$$

$$S^2 = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4+0 & -2-1 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -3 \\ 0 & 1 \end{pmatrix}$$

$$(S^2)^2 = \begin{pmatrix} 4 & -3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 16+0 & -12-3 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & -15 \\ 0 & 1 \end{pmatrix}$$

Now to find

$$S^*S^2 = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8+0 & -6+0 \\ -4+0 & 3+1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & -6 \\ -4 & 4 \end{pmatrix}$$

$$(S^*S^2)^2 = \begin{pmatrix} 8 & -6 \\ -4 & 4 \end{pmatrix} \begin{pmatrix} 8 & -6 \\ -4 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 64+24 & -48-24 \\ -32-16 & 24+16 \end{pmatrix}$$

$$(S^*S^2)^2 = \begin{pmatrix} 88 & -72 \\ -48 & 40 \end{pmatrix} \text{-------(1)}$$

Next to find

$$(S^*)^2(S^2)^2 = \begin{pmatrix} 4 & 0 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 16 & -15 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 64+0 & -60-0 \\ -48+0 & 45+1 \end{pmatrix}$$

$$= \begin{pmatrix} 64 & -60 \\ -48 & 46 \end{pmatrix} \text{------------------(2)}$$

From (1) & (2) we have $(S^*S^2)^2 \neq (S^*)^2(S^2)^2$

Hence S is not 2-normal

Next to T is 2 normal

$$(T^*T^2)^2 = (T^*)^2(T^2)^2$$

Given $T = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

Now $T^* = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$$(T^*)^2 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$$

$$T^2 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(T^2)^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 16+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 0 \\ 0 & 1 \end{pmatrix}$$

Now to find

$$T^*T^2 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(T^*T^2)^2 = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 64+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}$$

$$(T^*T^2)^2 = \begin{pmatrix} 64 & 0 \\ 0 & 1 \end{pmatrix} \text{------------(3)}$$

Next to find

$$(T^*)^2(T^2)^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 16 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 64+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 64 & 0 \\ 0 & 1 \end{pmatrix} \text{-------------------(4)}$$

From (3) & (4) we have

$$(T^*T^2)^2 = (T^*)^2(T^2)^2$$

Hence T is 2- normal

but $S \notin 2$ *power class* (Q)

Theorem 3.3

If $T \in L(H)$ is *n-normal* then $T \in n$ *power class*(Q)

Proof

Let T is *n-normal*

Then $T^*T^n = T^nT^*$

Pre multiply by T^* and post multiply by T^n on both sides

$$T^*T^*T^nT^n = T^*T^nT^*T^n$$

$$T^{*2}T^{2n} = (T^*T^n)^2$$

Hence $T \in n$ *power class*(Q) .

Example 3.4

If $T = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $T^* = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ shows that an operator of 2 *power class*(Q) need not be 2 *-normal*.

Solution:

Given $T = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $T^* = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

To prove that T is 2 *power class*(Q)

i.e.,) $T^4 = (T^*T^2)^2$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+1+0 & 0+0+0 & 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T^{*2} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(T^2)^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now to find

$$T^{*2}(T^2)^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{--------(5)}$$

Now

$$(T^* T^2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+1 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Next to find

$$(T^* T^2)^2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{-----------(6)}$$

From (5) & (6) we have

$$T^{*2}T^4 = (T^*T^2)^2$$

Hence $T \in 2\ power\ class(Q)$

Next to prove T is not *2 normal.*

i.e.,) $T^*T^2 \neq T^2T^*$

Now find

$$(T^* T^2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+1 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{--------------- (7)}$$

$$(T^2T^*) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{----------- (8)}$$

From (7) & (8) we have

$$T^*T^2 \neq T^2T^*$$

T is not *2 normal.*

Hence T is *2 power class (Q)* but it is not *2 normal.*

Example : **3.5** Consider the operator $T = \begin{pmatrix} i & 2 \\ 0 & -i \end{pmatrix}$ then $T^* = \begin{pmatrix} i & 0 \\ 2 & -i \end{pmatrix}$ show that T *is 2 power class(Q) but not 3 power class(Q).*

Solution:

Given $T = \begin{pmatrix} i & 2 \\ 0 & -i \end{pmatrix}$ and $T^* = \begin{pmatrix} i & 0 \\ 2 & -i \end{pmatrix}$

To prove that T *is 2 power class(Q).*

i.e.,) $T^{*2}T^4 = (T^*T^2)^2$

Now $T^{*2} = \begin{pmatrix} i & 0 \\ 2 & -i \end{pmatrix}\begin{pmatrix} i & 0 \\ 2 & -i \end{pmatrix}$

$$= \begin{pmatrix} i^2+0 & 0+0 \\ 2i-2i & 0+i^2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

and $T^2 = \begin{pmatrix} i & 2 \\ 0 & -i \end{pmatrix}\begin{pmatrix} i & 2 \\ 0 & -i \end{pmatrix}$

$$= \begin{pmatrix} i^2+0 & 2i-2i \\ 0+0 & 0+i^2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$T^4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

First to find

$$T^{*2}T^4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1+0 & 0+0 \\ 0+0 & 0-1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \text{---------}(9)$$

Next to find

$$T^*T^2 = \begin{pmatrix} i & 0 \\ 2 & -i \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -i & 0 \\ -2 & i \end{pmatrix}$$

Since

$$(T^*T^2)^2 = \begin{pmatrix} -i & 0 \\ -2 & i \end{pmatrix}\begin{pmatrix} -i & 0 \\ -2 & i \end{pmatrix}$$

$$= \begin{pmatrix} i^2+0 & 0 \\ 2i-2i & 0+i^2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \text{----------------}(10)$$

From (9) & (10) we have

$$T^{*2}T^4 = (T^*T^2)^2$$

T *is 2 power class(Q)*

To prove that T *is not 3 power class(Q)*

i.e.,) $T^{*2}T^6 \neq (T^*T^2)^3$

now $T^{*2} = \begin{pmatrix} i & 0 \\ 2 & -i \end{pmatrix}\begin{pmatrix} i & 0 \\ 2 & -i \end{pmatrix}$

$$= \begin{pmatrix} i^2+0 & 0+0 \\ 2i-2i & 0+i^2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

And $T^2 = \begin{pmatrix} i & 2 \\ 0 & -i \end{pmatrix}\begin{pmatrix} i & 2 \\ 0 & -i \end{pmatrix}$

$$= \begin{pmatrix} i^2+0 & 2i-2i \\ 0+0 & 0+i^2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$T^6 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1+0 & 0 \\ 0 & 0-1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

First to find

$$T^{*2}T^6 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{---------}(11)$$

Next to find

$$T^*T^2 = \begin{pmatrix} i & 0 \\ 2 & -i \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -i+0 & 0+0 \\ -2+0 & 0+i \end{pmatrix}$$

$$= \begin{pmatrix} -i & 0 \\ -2 & i \end{pmatrix}$$

Since

$$(T^*T^2)^3 = \begin{pmatrix} -i & 0 \\ -2 & i \end{pmatrix}\begin{pmatrix} -i & 0 \\ -2 & i \end{pmatrix}\begin{pmatrix} -i & 0 \\ -2 & i \end{pmatrix}$$

$$= \begin{pmatrix} i^2+0 & 0 \\ 2i-2i & 0+i^2 \end{pmatrix}\begin{pmatrix} -i & 0 \\ -2 & i \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -i & 0 \\ -2 & i \end{pmatrix}$$

$$= \begin{pmatrix} i+0 & 0+0 \\ 0+2 & 0-i \end{pmatrix}$$

$$= \begin{pmatrix} i & 0 \\ 2 & -i \end{pmatrix} \text{------------------}(12)$$

From (11) &(12) we have

$$T^{*2}T^6 \neq (T^*T^2)^3$$

T is not 3 power class(Q)

Theorem 3.6

If T is *n power class (Q)* and T is *quasi n normal* then T is *n+1 power class (Q).*

Proof.

If T is n *power class(Q)*

Then $T^{*2}T^{2n} = (T^*T^n)^2$

Post multiply by T^2 on both sides

$$T^{*2}T^{2n}T^2 = (T^*T^n)^2T^2$$

$$T^{*2}T^{2n+2} = (T^*T^n)(T^*T^n)TT$$

Since T is *quasi n normal* we have

$$T^{*2}T^{2(n+1)} = (T^*T^n)T(T^*T^n)T = (T^*T^{n+1})^2$$

Hence $T \in n+1$ *power class*(Q).

CONCLUSION

In this paper, basic definitions, related theorems and examples of $n-$ *power class* (Q) operators are investigated.

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