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SYNCHROSQUEEZING TRANSFORM AND ITS APPLICATIONS: A REVIEW

Libya Thomas[1], Anoop K Johnson[2]

[12]ECE; MBCET; Trivandrum; India

Abstract: *Study based on time-frequency representations provides powerful understanding of a multicomponent signal. Synchrosqueezing transform is a promising wavelet based time-frequency representations tool which incorporates reassignment of instantaneous frequency (IF). It sharpens the time-sale representation given by a continuous wavelet transform with an additional advantage of mode retrieval. Synchrosqueezing transform which is used for signal analysis and its various applications are reviewed in this paper and thus it puts forward a overview of the Synchrosqueezing Transform and its various applications.*

Key words—Time-frequency, reassignment, synchrosqueezing, AM/FM, multicomponent signals.

I. INTRODUCTION

SIGNALS shall be modeled as a superposition of amplitude- and frequency-modulated (AM-FM) modes [1]–[3]. Signals like audio signals (music, speech) and medical data such as electrocardiogram, thoracic and abdominal movement are some such signals. And such signals are called multicomponent signals (MCS). There are certain linear techniques such as continuous wavelet transforms (CWT) and short-time Fourier transform (STFT) that are often utilized to characterize such signals in the time-frequency (TF) plane. However, all of these methods share the same drawback, known as the “uncertainty principle”, which says that one cannot localize a signal with arbitrary precision both in time and frequency. Many efforts were presented to cope with this issue and, in particular, a general methodology to improve TF representation, called “reassignment” method (RM) was suggested. This was first introduced in [4], in a limited basis, and then was further developed in [5], as a post-processing method. The major difficulty assosiated with RM is that the reassigned transform is no longer invertible and does not agree for mode reconstruction.

Daubechies and Maes recommended a phase-based technique, called Synchrosqueezing Transform” (SST), in the context of audio signal analysis [6], whose theoretical analysis is followed in [7]. Its purpose is to sharpen the time-scale (TS) representation given by CWT, which is relatively similar to that of RM, with the additional benefit of allowing for mode retrieval. The structure of this paper as follows: Section II explains different transforms and theory of SST, Section III details various applications of SST.

II. THEORY BEHIND THE TRANSFORMS

A transform is a process that converts a raw signal to a processed signal. Some information may not be evident in a domain and may be visible in other. And for extracting those hidden information we perform transformations. Various transformations are used and one such common transform is fourier transform (FT). It can be represented as:

$$F\{g(t)\}=G(f)=\int_{-\infty}^{\infty} g(t)e^{-2\pi ift} \tag{2.1}$$

where g(t) is the original signal.

Fourier transform decomposes a function into several frequencies that makes it up. It gives a frequency domain representation of the original signal. FT is suitable for stationary signal and is not applicable for non stationary since no time information is obtained from FT. So in order to obtain time localization we go for windowing and the window must be short enough so that within the window length a non stationary signal appears to be stationary and this process is called Short time Fourier transform (STFT).

$$STFT\{x(t)\}=\int_{-\infty}^{-} x(t)w(t-\tau)e^{-jwt}\,dt \tag{2.2}$$

where $w(t-\tau)$ represents the window used.

But once when a window size is fixed it remains constant along the entire signal. Thus this limitation is a reason for invention of wavelets.
The wavelet analysis represents the next logic step: a windowing technique with variable sized regions.

$$W_s(a,b) = \frac{1}{|a|^{1/2}} \int_{-\infty}^{\infty} x(t)\bar{\varphi}\left(\frac{t-b}{a}\right) dt \qquad (2.3)$$

where $\bar{\varphi}\left(\frac{t-b}{a}\right)$ is the mother wavelet, a and b are the

scaling and location parameters.
But precision in both time and frequency is not possible in CWT due to uncertainty principle.
In order to circumvent the uncertainty principle a special case of reassignment method called Synchrosqueezing transform (SST) was introduced.

II.a. Synchrosqueezing Transform (SST)

In synchrosqueezing it moves and reassign the coefficients resulting from a CWT based frequency information to get more concentrated picture of time-frequency(TF) plane. The entire process provide a sharpened and focused representation of multicomponent signal in TF plane and a decomposition method that enables to separate and demodulate the different modes. The process is as represented in figure 1.

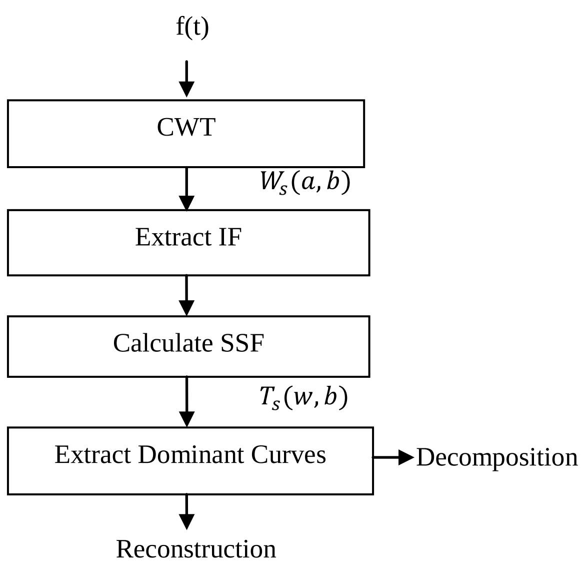

Figure 1. Flow chart of SST

First step is the CWT. And it is applied on to the signal as per the given equation (2.3). Once the CWT is computed the coefficients obtained has to be reallocated to a new point ie. The points are shifted to a TF plane.
Let the Fourier representation of the mother wavelet be $\varphi(\varepsilon)$ and if it is concentrated at $\varepsilon = w_0$, then the CWT W_s(a,b) will be concentrated around the line $a = \frac{w_0}{w}$.
where a is the scaling parameter and w the original frequency of the mode.
While concentrating on point b the oscillating properties of b points to the original frequency w. A new reallocated wavelet is obtained and the instantaneous frequency(IF) is obtained by taking the derivative of the wavelet coefficients of the new map. This leads to the suggestion to compute, for any (a,b) for which W_s(a,b) $\neq$ 0, a

candidate IF $w_s(a,b)$ for a signal s is given by,

$$w_s(a,b) = -i\left(W_s(a,b)\right)^{-1} \frac{\partial W_s(a,b)}{\partial b} \qquad (2.4)$$

The information from time-scale is transferred to the T-F plane, according to the map (b,a) $\rightarrow (b, W_s(a,b))$, the frequency variables w and scale variable a are binned and the transform occurs at every discrete values of a_k. Hence the SST $T_s(w,b)$ is likely to occur at the center of each bin and summing all these the synchrosqueezing function (SSF) can be obtained as

$$T_s(w_l,b) = (\Delta w)^{-1} \sum_{a_k:\, |w(a_k,b)-w_l| \le \frac{\Delta w}{2}} W_s(a_k,b) a_k^{-3/2} (\Delta a)_k \qquad (2.5)$$

The reconstruction of the modes can be done by integrating the coefficients with the SSF or can be done by summing the vertical coefficients along the ridge of the TF plane.
The CWT used here can be replaced by STFT which results in a method called Finite STFT based Synchrosqueezing transform (FSST). A suitable way to choose between CWT and STFT is the range of the signal.
The FSST handle a wide range of modulations at low frequency and cwt based SST works satisfactorily for high frequency in most cases. The FSST has a matrix representation which is more convenient to work rather than other methods. A more sharpened TF representation than ordinary SST is obtained.
Normalized energy is one of the measure to check the sharpness of the representation. This method gives a zero order IF estimation and thus works well on signals with slow time varying IFs. Hence we go for High Order

FSST in which the amplitude and phase of a given signal is represented using a Taylor series. And this method can further enhance the TF representation.

III. VARIOUS APPLICATIONS OF SST

The SST can be extended to various applications and some applications are discussed in the following section.

III.a. Atmospheric Noise Suppression

Low radio waves, extremely low frequency and very low frequency waves has very effective long range propagation in the so called earths ionosphere.
Atmospheric noise has impulsive nature and thus cannot be shown as Gaussian noise.
The SST based on CWT compresses the TF plot of wavelet transform in the frequency domain and boosts the localization properties. In order to extract a signal of interest from among atmospheric noise we can use an adaptive soft limiter to suppress impulsive noise and followed by SST denoising.
SST can offer high precision TF representation for various signals, it is restricted by the assumption that the amplitude varies slowly. But the atmospheric noise exhibits intensive impulsiveness. So the received signal will not satisfy the slow varying amplitude. Hence we adopt an adaptive soft limiter to utilize SST. And the impulsive component is limited by the limiter thus the atmospheric noise will turn to be asymptotically Gaussian and can be suppressed using SST.

III.b. Seismic Data Analysis

Time-frequency representations are widely used in seismic data analysis since a lot of information are hidden in seismic amplitude profiles.
A widely used method for TF decomposition of the seismic data was STFT but its predefined window length limits is use. SST is an adaptive and invertible transform that improves the readability of a wavelet based TF map. Here the application of SST is extended to hydrocarbon detection, ground roll suppression and random noise attenuation. The high frequency components of seismic waves decays rapidly as seismic wave propagate due to the presence of oil and gas and it results in a low frequency in the seismic section. This low frequency is used to detect hydrocarbon reservoirs, where the accuracy of TF decomposition is important. SST provides good TH representation and can be used to find the hydrocarbon reservoirs; by detecting the low shadows caused by them on the seismic waves.
For attenuating the ground roll TF filters are widely used. From the data obtained by applying SST on the seismic data a filter can be designed as

$$\mathrm{Y(t)}= SST^{-1}\{SST(f(t)H(t,f)\} \tag{3.1}$$

where SST and SST^{-1} are the SST and inverse SST and H(t,f) is the TF filter designed based on the difference between seismic signal and ground roll in TF map.

$$\mathrm{H(t,f)}=\begin{cases} 1\,; f\in[f_i(t)-\frac{B(t)}{2}, f_i(t)+B(t)/2 \\ 0\,; \quad otherwise \end{cases} \tag{3.2}$$

where $f_i(t)$ is the IF of seismic signal and B(t) is the bandwidth of TF filter.

Coming into the denoising a wavelet threshold algorithm is adapted as a successful method. it has got various merits like convenient and efficient calculation as well as better performance for components with low SNR. The threshold function combined with SST to attenuate random noise can be defined as:

$$\widehat{S_{t,f}} = \begin{cases} sgn(S_{t,f})(|S_{t,f}|-\alpha\lambda); |S_{t,f}| \geq \lambda \\ 0; \quad |S_{t,f}|<\lambda \end{cases} \tag{3.3}$$

where α determines the type of threshold. When α=0 then it is hard threshold and when α= 1 it becomes soft threshold.
ƛ denotes the threshold and it can be defined as:

$$\lambda=\sqrt{(2\log n)}\,\sigma_n \tag{3.4}$$

where n is the number of samples and σ_n the standard deviation of noise. The standard deviation can be obtained from the wavelet coefficients by using the equation:

$$\sigma_n = 1.4826\, median\{|W_f(a_{1:n_v}, b) - median[W_f(a_{1:n_v}, b)]|\} \tag{3.5}$$

where $W_f(a_{1:n_v}, b)$ are the wavelet coefficients and 1.4826 is the normalizing factor used.

III.c. Fault Diagnosis Of Wind Turbine

As the generating capacity of the wind turbine increased wind turbine fault analysis and diagnosis has became a significant issue. Wind turbines are driven by natural

wind and thus the signal at the gear box which is used for the fault diagnosis varies and is influenced by nature of non-stationary and noisy. A TF representation can be used to analyze variable speed signal. SST since gives a concentrated TF representation is employed for this application.
Firstly the input signal is decomposed into its components and then a TF analysis is performed based on the calculated SNR. The components are then compressed using SST.
And at last all the TFR components are added up to obtain a TF display from which the detection result is obtained.
The decomposition at the initial step can be done using LMD (local mean decomposition) method. the function of signal and time cannot be obtained due to the complex structure of gearbox. Hence LMD is a ideal method to denoise which is sensitive to noise.

The LMD method for a given signal x(t) is as follows:

1. Prime the parameters, set the index of PF i=1 and the residue

u_0=x(t). (3.6)

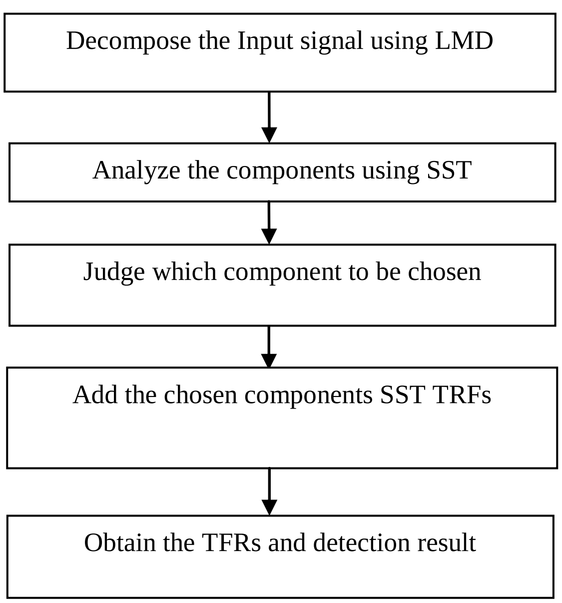

Figure 2. Processing flowchart of the method used.

where PF called the product function is the product of the modulated signal and the corresponding envelope signal.

2. Extract the i_{th} PF.

a. $h_{11}(t) = u_0(t) - m_{11}(t);$ (3.7)

where m_i=$(n_i + n_{i+1})/2$, and n_i is the local extreme of the signal x.

b. The envelope function is derived by smoothing the local envelope estimates same way as the local means.

$$S_{11}(t) = h_{11}(t)/a_{11}(t) \quad (3.8)$$

c. Until

$$|S_{1n}(t)| \leq 1, a_1(t) = a_{11}(t)a_{12}(t)\ldots\prod_{q=1}^{n} a_{1q}(t) \quad (3.9)$$

where a_i is the i_{th} envelope estimate,

$a_i = |n_i - n_{i+1}|/2.$

Thus PF can be updated as:

$$PF_1(t) = a_1(t)S_{1n}(t) \quad (3.10)$$

3. A new signal is obtained by subtracting PF from the original signal.

$$u_1(t) = u_0(t) - PF_1(t) \quad (3.11)$$

4. The process is repeated until $u_k(t)$ is a constant or monotonic.

After the process a series of decomposition is obtained and the final residue can be obtained as:

$$x(t) = \sum_{p=1}^{k} PF_p(t) + u_k(t) \quad (3.12)$$

A TF representation of all PF components namely displaying instantaneous amplitude and instantaneous frequency can be obtained by using LMD.
Once the decomposition is over SST is used to analyze the components and then chose the components required.

III.d. Detection for Earthquake-Damaged Structure

An application of SST based on CWT is the detection of frequency shift caused on earthquake damaged structures. Every structure has got a natural frequency. In structural health monitoring estimation of the natural frequency of the structures are important for detecting the damage. Damage caused by earthquakes decreases the natural frequency of the structure and this change in frequency is directly proportional to the change in stiffness of the structure. This can be related as:

$$f=(1/2\pi)(\frac{k}{m})^{1/2} \tag{3.13}$$

These variations can be found out by using TF representations. The SST is computed as explained in section II.

The steps involves are:

1.Uniformly sample the signal.
2.Find CWT of the samples signal.
3.Estimate IFs using the coefficients of the wavelet transform.
4.Combine the CWT and IF followed by reassignment of the coefficients, called synchrosqueezing

And the frequency shift can be obtained from the TF plot and equation 3.13 from which the damage can be estimated.

III.e. Monogenic Synchrosqueezing Transform(MSST)

Synchrosqueezing can decompose 1D function as a superposition of various fundamental modes which has to be parted well both in time and frequency. Based on unidimentional wavelet transform a multicomponent signal can be well represented using synchrosqueezing transform. But here a bidimensional case of synchrosqueezing transform is defined. The wavelet transform is used both to decompose an analytic signal into several constituents, and also to evaluate the amplitude and the instantaneous frequency of each component. The main benefit of using the wavelet analysis is to evade from the use of the discrete Hilbert transform, since there is a link between the wavelet coefficients of the analytic signal related to a real function f, and its analytic wavelet coefficients (i.e. the wavelet coefficients of f among an analytic wavelet). By substituting analytic signal with monogenic signal it can be extended to bidimensional cases too. By viewing monogenic signal as a 3D vector field, then wavelet transform can be used to examine it component wise. In order to understand a continuous monogenic transform we need to consider a real admissible wavelet φ and define

$$\varphi^{(M)} = M\varphi = \begin{pmatrix} \varphi \\ R_1\varphi \\ R_2\varphi \end{pmatrix} \tag{3.14}$$

For discrete monogenic synchrosqueezing transform the first step to be performed is variable discretization. Therefore a discrete set A,K O for scale a with normalized frequencies (k_1,k_2) and orientation θ is build respectively. Then the discrete monogenic wavelet transform c_F (a, b), and the estimations of the instantaneous frequencies $\wedge_1$(a, b) and $\wedge_2$(a, b) are computed. And it can be defined by:

$$\partial_{bi} c_F(a,b) = \int_{R^2}^{0} f(x)\partial_{bi}\varphi_{a,\varphi b}(x)dx \tag{3.15}$$

MSST also consist of a partial reallocation of the coefficients obtained depending on the frequency,space as well as orientation parameters. When considering a threshold γ for all a we can simply define

$$S_{F,r}(b,k_1,k_2,n_\theta) = \frac{\log(2)}{n_v} \sum_{a\epsilon A\ S.t \left\{ \begin{array}{c} |c_F(a,b)| \geq \gamma \\ |k_1 - Re(\wedge_1(a,b)\overline{n_\theta})| \leq \frac{\Delta k_1}{2} \\ \left|k_2 - Re(\wedge_2(a,b)\overline{n_\theta}\right| \leq \frac{\Delta k_2}{2} \end{array} \right\}} \frac{c_F(a,b)}{a} \tag{3.16}$$

A partial reallocation of the monogenic wavelet transform according to space, frequency and orientation parameters is performed in monogenic synchrosqueezing transform.
Once MSST is computed the modes has to be separated and reconstructed in order to obtain the instantaneous frequency at a point say b.
The l^{th} instantaneous frequency (IMMF) can the be obtained as

$$\widehat{F_l(b)} = \sum\nolimits_{\widehat{k_l(b)}-\hat{k} \leq k \leq \widehat{k_l(b)}+\hat{k}} S_{F,\gamma}(b,k) \tag{3.17}$$

where $\widehat{k_l(b)}$ is the approximate estimate of $|\nabla\varphi_l(b)|$. The approximated l^{th} IMMF can be obtained by summing the coefficients of synchrosqueezing transform with the vicinity of this instantaneous frequency.

IV. CONCLUSION

In this paper, a study on Synchrosqueezing transform and its various applications were presented. Applications of Synchrosqueezing transform like atmospheric noise suppression where an effective method to cancel atmospheric noise from a low frequency communication which suffers from strong atmospheric noise and then seismic data analysis using SST were reviewed. Also the application of SST for fault analysis of a wind turbine which highly depends on the frequency of the signal on its gear box was also discussed. Where while applying SST various frequency components can be extracted by obtaining a concentrated time-frequency representation.

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