Attitude Estimation for Satellites using Vector Observations

Qazi Ejaz Ur Rehman[1], Salimuddin Zahir[2], Najam Abbas[3]

[1]Institute of Space Technology, Islamabad
[2, 3]Professor, Dept. of Aeronautics & Astronautics, Institute of Space Technology

---

Abstract - *Attitude estimation is an essential part of a satellite control system. To avoid drift problems due to long operational life, static attitude determination is preferred, which is based on vector information. Commonly used vectors include magnetic field, sun direction, star direction etc. Information of these vectors is required in two different reference frames. In static attitude determination, a constrained weighted least-square problem is solved which gives a transformation matrix between the two reference frames. This work requires to study and model different vector information for satellites and study existing algorithms for static attitude determination. Computational efficiency may be explored in these algorithms from implementation point of view. The obtained attitude will then be used in some dynamic filter (such as unscented, Kalman, predictive filter etc.) and performance will be studied. Two major issues of dynamic filtering will be analyzed in detail. Firstly, due to dynamic nonlinearities, filters may face divergence problems. Secondly, the performance may degrade in the presence of noise and uncertainties. Performance improvement in both of these cases will be the main challenging task of the work.*

***Key Words*: Attitude Estimation, Vector Observations, Dynamic Filters, Satellite Control System, Weighted least-square problem, Transformation Matrix, Frame of References, Computational Efficiency**

1. INTRODUCTION

Satellites are man-made devices placed intentionally into the orbit, and these are often regarded as artificial satellite to distinguish from natural satellites e.g. Earth's Moon.

In 1957, Soviet Union launched the world's first satellite, named as Sputnik1. About 8,100 satellites have been launched since then by almost 40 countries. As per the recent statistics, round about 4,900 remain in the designated orbits, of these about 1900 were operational; while the rest are space debris and have served their purposes. Furthermore, approximately low-Earth orbit (LEO) has 500 operational satellites occupied, similarly, 50 satellites are functioning in medium-Earth orbit (MEO) (at 20,000 km), while the remaining are in geostationary orbit (GEO) (at 36,000 km). There are few large satellites that have been launched in parts and have been assembled in the orbit. Besides that over a dozen space probes have been launched and placed into orbits around other heavenly bodies such as artificial satellites to the Moon, Mercury, Venus, Mars, Jupiter, Saturn, a few asteroids, a comet and the Sun.

Satellites are designed to serve numerous purposes. Among several other application, satellites can be used to draw star maps as well as planetary surfaces, and also help us in studying the planets by taking images of planetary surfaces for which they are put in action. Besides that, they are also used for communication across the globe, navigation, weather forecasting, human recreational purposes, space telescopes. The orbits of satellites are designed as per the nature of functionality, and vary in great number of ways. Well-known (overlapping) classes of orbits include low Earth orbit (LEO), polar orbit, and geostationary orbit. The launch vehicles are rockets that places satellites into the designated orbit, such that, rockets lift off from the launch pads from land, from a submarine at sea, a mobile maritime platform, or aboard a plane depending upon the orbit and economic factor.

Satellites are highly sophisticated & partially-independent computer-controlled systems, and its subsystems attend many tasks, such as thermal control, orbit control, telemetry, orbit control and attitude control.

However, satellite deteriorates in performance with the passage of time, thus, compromising accuracy in attitude and orbit control. Therefore, this paper will focus on increasing the attitude performance of satellite through vector observation (e.g. Sun Vector, Manometer), as attitude determination and control subsystem plays vital role to maintain the direction of the spacecraft and its instrument.

2. METHODOLOGY

There are numerous methods to determine the attitude of the satellite but the most common sensors are Sun sensors and magnetometers due to their cost-effectiveness, sophistication and readily availability in different versions of sizes and masses. Satellite needs to be slanted towards a specific direction, and two or more vectors are required as reference direction. Unit vectors in the direction of Sun and Earth's Magnetic field are the commonly used vectors, and measurement results obtained through attitude sensor can provide the orientation of these vectors in the reference frame of reference. Nonetheless, a major drawback lies in utilization of such sensors due to their intrinsic limitation and unavailability of measurements of Sun sensor when satellite reaches in the regime of eclipse.

Numerous researches carried out and several algorithms have been proposed which intend to improve the attitude estimation of satellite through the measurements by magnetometer and sun sensors. Such that weighted least square problem is solved to minimize Whaba's loss function in order to get the state vectors (quaternion estimations), which are passed to satellite equation of motions to determine the attitude of the satellite at any instance. Furthermore, an elaborative approach has also been depicted by examining SVD-aided UKF (SaUKF) algorithm for satellite attitude estimation. In this approach, at the first stage, Whaba's problem is solved by SVD method and estimate quaternion measurements are obtained for the satellite's attitude. While at the second phase these obtained values are used as measurements results for an UKF. The SaUKF provides improved attitude knowledge and attitude rate estimates.

These approaches stand out from other methods due to their easy nature of implementation and accuracy in attitude estimation, especially when satellite is in eclipse and also have robust nature against estimation deteriorations.

2.1 SATELLITE' EQUATIONS OF MOTION

The derived equation of motion for satellites can be simplified to the following algebraic expression by using the quaternion representations. [1]:

$$\dot{\boldsymbol{q}}(t) = \frac{1}{2}\Omega\left(\boldsymbol{\omega}_{BR}(t)\right)\boldsymbol{q}(t) \quad 1$$

In equation (1), q is made of four attitude parameters, $q = [q_1 \ q_2 \ q_3 \ q_4]^T$ such that first three quantities are vectors while the fourth one is a scalar quantity. Hence, the quaternion takes the form of $q = [g^T \ q_4]^T$, $g = [q_1 \ q_2 \ q_3]^T$. Furthermore, $\Omega(\omega_{BR})$ is the skew symmetric matrix i.e.:

$$\Omega(\omega_{BR}) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \quad 2$$

Here $\boldsymbol{\omega}_{BR}$ is composed of vectors ω_1, ω_2 and ω_3; it depicts angular velocity in body frame in reference to the Orbit frame. Due to the use of sensors, the angular rate vector shall be defined, thus, $\omega_{BI} = [\omega_x \ \omega_y \ \omega_z]$ can be used to define rate vector in body frame of reference with respect to the inertial coordinate system. Following is the established relationship between ω_{BI} and ω_{BR}

$$\omega_{BR} = \omega_{BI} - A[0 \ -\omega_0 \ 0]^T \quad 3$$

Where ω_0 is angular velocity in its orbit with respect to the inertial frame of reference, and has value of $\omega_0 = (\mu/r^3)^{0.5}$, and μ is constant which is product of further two constants (GM_E), here M_E is the mass of the earth and G is the gravitation constant, and r is the radius from the centre of earth to the satellite. Lastly, A is the quaternion related rotational matrix:

$$A = (q_4^2 - \boldsymbol{g}^2)I_{3\times3} + 2\boldsymbol{g}\boldsymbol{g}^T - 2q_4[\boldsymbol{g}\times]. \quad 4$$

Here $\boldsymbol{I}_{3\times3}$ is an identity matrix, and $[g\times]$ is a skew-symmetric matrix given below

$$[g\times] = \begin{bmatrix} 0 & -g_3 & g_2 \\ g_3 & 0 & -g_1 \\ -g_2 & g_1 & 0 \end{bmatrix} \quad 5$$

Attitude and attitude rates are necessary to be calculated in order to determine the full static attitude using satellites equations of motions, which can further be found using following resultant equation from Euler's equations keeping the perturbation in notice

$$J\frac{\boldsymbol{\omega}_{BI}}{dt} = \boldsymbol{N}_d - \boldsymbol{\omega}_{BI} \times (J\boldsymbol{\omega}_{BI}) \quad 6$$

Here J =*diag* (J_x, J_y, J_z)) is the inertial matrix of principal moments of inertia, $\mathbf{N_d}$ is the resultant torques of external disturbances affecting the satellite.

$$\boldsymbol{N}_d = \boldsymbol{N}_{ad} + \boldsymbol{N}_{gg} + \boldsymbol{N}_{sp} + \boldsymbol{N}_{md} \quad 7$$

Where $\mathbf{N_{ad}}$ is aerodynamic disturbance torque, $\mathbf{N_{gg}}$ gravity-gradient torque, $\mathbf{N_{sp}}$ is the disturbance due to solar pressure torque, and $\mathbf{N_{md}}$ is the residual magnetic torque disturbance which occurs due to the Earth's magnetic field and satellite's dipole.

2.2 SENSOR MODELS

In this section derived mathematical models of sensors will be shown.

2.2.1 MAGNETOMETER

As aforementioned that magnetometers sensor would be used to estimate attitude such that these need to be calibrated, therefore, Earth's magnetic field measurement is given in below equation

$$\begin{bmatrix} B_x(q,t) \\ B_y(q,t) \\ B_z(q,t) \end{bmatrix} = A \begin{bmatrix} B_1(t) \\ B_2(t) \\ B_3(t) \end{bmatrix} + \eta_1 \quad 8$$

$B_x(q,t)$, $B_y(q,t)$ and $B_z(q,t)$ are utilized in body frame of reference as these magnetic vectors could be measured from the on-board magnetometer sensors. B_1 (t), B_2 (t) & B_3 (t) are the values of Earth's Magnetic Field in the orbital co-ordinate frame of reference which can be accurately calculated through common model, *International Geomagnetic Reference Field (IGRF)*[2]. Furthermore, $\boldsymbol{\eta_1}$ **is** zero-mean Gaussian white noise.

$$E[\eta_{1k}\,\eta_{1j}^T] = I_{3\times3}\,\sigma_m^2\delta_{kj} \quad 9$$

2.2.2 SUN SENSOR

The Sun direction depends solely upon Julian day (T_{TDB}) with respect to inertial co-ordinates regarding the Earth center, and can be derived using exact time and satellite's reference epoch [3].

$$M_{sun} = 357.5277233^0 + 35999.05034T_{TDB}, \quad 10a$$

$$\lambda_{ecliptic} = \lambda_{M_{Sun}} + 1.914666471^0 sin(M_{sun}) \quad 10b$$

$$+0.019994643sin(2M_{sun}),$$

$$\lambda_{M_{sun}} = 280.4606184^0 + 36000.77005361T_{TDB}, \quad 10c$$

$$\varepsilon = 23.439291^0 - 0.0130042T_{TDB} \quad 10d$$

These equations could be used to find the ecliptic longitude of the sun ($\lambda_{ecliptic}$) and its linear model **ε**; where $\mathbf{M_{sun}}$ and $\lambda_{M_{sun}}$ are the main variable depicting Mean anomaly and mean longitude of the sun. Furthermore, sun direction in inertial frame of reference can be d from equations 10.

$$S_{ECI} = \begin{bmatrix} cos\lambda_{ecliptic} \\ sin\lambda_{ecliptic}\ Cos\varepsilon \\ sin\lambda_{ecliptic}\ sin\varepsilon \end{bmatrix} \quad 11$$

There is need to transform the unit sun direction vector orbital frame as the satellite is also rotating along propagation of its trajectory. Hence, sensor measurement vector ($\mathbf{S_b}$) in the body frame of reference and direction model vector ($\mathbf{S_o}$) in the orbit frame of reference of the sun could be shown as

$$S_b = AS_o + \eta_2 \quad 12$$

Here η_2 is zero-mean Gaussian white noise with characteristic equation as:

$$E[\eta_{2k}\ \eta_{2j}^T] = I_{3\times3}\ \sigma_s^2 \delta_{kj} \quad 13$$

Where $\boldsymbol{\sigma}_s$ is the standard deviation of errors of Sun sensor. Furthermore, the satellite's position and orbit parameters should be known to model Sun vectors and earth's Magnetic field in the orbit frame of reference.

2.3 FILTER MODELS FOR ATTITUDE ESTIMATION

In order to determine attitude and angular rates of satellites accurately, the estimation stages has been categorized into two cascading stages: SVD and UKF. In the first stage, SVD will find the variance values for each axis and attitude angles with the utilization of two vectors, thus, minimizing the Wahba's loss function. After that, the obtained values will be passed through the UKF to get higher accuracy.

2.3.1 SINGULAR VALUE DECOMPOSITION

SVD aims to solve the wahba's problem [4], and will estimate the attitude by using model vector and results. Reference to loss function (Eq. 14), $\mathbf{b_i}$ and $\mathbf{r_i}$ are unit vectors of sun direction and Earth's magnetic field in body and orbit co-ordinate systems respectively, provides values for every single time interval. Therefore, by obtaining optimum value for the orthogonal matrix A, the attitude states can be found [5]

$$L(A) = \frac{1}{2}\Sigma_i a_i |b_i - Ar_i|^2 \quad 14$$

Here, a_i is always a non-negative weight. Furthermore, the loss function could be simplified into

$$L(A) = \lambda_0 - tr(AB^T) \quad 15$$

Where;

$$\lambda_0 = \Sigma a_i, \quad 16a$$

$$B = \Sigma a_i b_i r_i^T \quad 16b$$

As mentioned earlier SVD will be used to optimize the loss function and maximize the trace function Eq: 15. Therefore, B matrix has singular value Decomposition (SVD) [5]:

$$B = U\Sigma^T V^T = Udiag[\Sigma_{11}\ \ \Sigma_{22}\ \ \Sigma_{33}]V^T \quad 17$$

Where U and V are orthogonal matrices, and singular values fulfil criteria $\Sigma_{11} \geq \Sigma_{22} \geq \Sigma_{33} \geq 0$, so, the optimum attitude matrix can be given as

$$U^T A_{opt} V = diag[1\ \ 1\ det(U)\ det(V)] \quad 18$$

$$A_{opt} = Udiag[1\ \ 1\ det(U)\ det(V)]V^T \quad 19$$

To analyze covariance (P_{svd}) which hold important position while dealing with integrated filter technique, following model is given.

$$P_{svd} = Udiag[(s_2 + s_3)^{-1}\ \ (s_3 + s_1)^{-1}\ \ (s_1 + s_2)^{-1}]U^T \quad \mathbf{20}$$

Where, $s_1 = \Sigma_{11},\ s_2 = \Sigma_{22} and\ s_3 = \Sigma\, det(U)\, det(V)\, \Sigma_{33}$ are secondary singular values.

This methodology provides instantaneously attitude displacement, however, a drawback lies as this method fails, whenever the satellite is in eclipse and vectors are parallel.

2.3.2 UNSCENTED KALMAN FILTER (UKF)

UKF, is a subtle approach of accurate approximation using Unscented Transform for solving the multi-dimensional integrals rather than linear approximation of the non-linear equations as per the procedure of implementation of Extended Kalman Filter (EKF) [6]. On the top of that it has advantage of easier nature to approximate non-linear distribution. The conventional algorithm for UKF can be referred to [7] for perusal.

UKF cannot be used in standard format especially when a quaternion in the kinematic modeling of satellite's motion is used, and it is due to the limiting factor of quaternion unity $\mathbf{q}^T\mathbf{q} = 1$. There is no surety that predicted quaternion mean of Unscented Kalman Filter will satisfy this constrain, if eq 1 is applied directly to the filter. However, this problem is solved in paper [7] and the same methodology will be used. Moreover, reference to $q = [g^T \; q_4]^T$, quaternion with vector and scalar parts. The local error-quaternion can be written as $\boldsymbol{\delta q} = [\boldsymbol{\delta g}^T \; \delta_4]^T$, the vector GRP can be given as

$$\delta \boldsymbol{p} = f[\delta \boldsymbol{g}/(a + \delta q_4)] \qquad 21$$

Here; 0<a<1 and f is a scalar quantity. If a=0 & f=1, Eq. 21 becomes Gibbs Vector, similarly, if a=1 & f =1 then Eq. 21 becomes modified Rodrigues parameters. However, here f is chosen as f=2(a+1) in contrast to the paper [7]. Inverse transformation from $\boldsymbol{\delta p}$ to $\boldsymbol{\delta q}$ is given by:

$$\delta q_4 = \frac{-a \parallel \delta \boldsymbol{p} \parallel^2 + f\sqrt{f^2 + (1-a^2) \parallel \delta \boldsymbol{p} \parallel^2}}{f^2 + \parallel \delta \boldsymbol{p} \parallel} \qquad 22a$$

$$\delta \boldsymbol{g} = f^{-1}(a + \delta q_4)\delta \boldsymbol{p} \qquad 22b$$

2.3.3 SVD AIDED UKF FILTER (SaUKF)

Two method are combined to give better estimation of satellites rather than implementation of SVD or UKF, because in case of failure of SVD (e.g. in solar eclipse)and uncertain perturbations, UKF being robust provides rectification for the measurements, hence, providing better attitude, displacement and rates estimation.

As attitude determination: SVD uses quaternions, however, in SVD aided UKF (SaUKF), the attitude errors regarding GRP are acquired:

$$\delta \boldsymbol{q}_{obs} = \boldsymbol{q}_{mes} \otimes [\hat{q}_0(K+1|\boldsymbol{K})]^{-1} \qquad 23$$

Here $\mathbf{q}_{mes}$ is obtained from SVD method, are predicted mean quaternion multiplied with quaternion. Hence, regarding $\delta \boldsymbol{q}_{obs} = [\delta \boldsymbol{g}_{obs}^T \; \delta q_{4,obs}]^T$, the attitude error as a resultant is calculated as

$$\delta \boldsymbol{p}_{obs} = f[\delta \boldsymbol{g}_{obs}/(a + \delta q_{4,\, obs})] \qquad 24$$

A scheme for the methodology implementation for estimation of attitude and attitude rates is given below

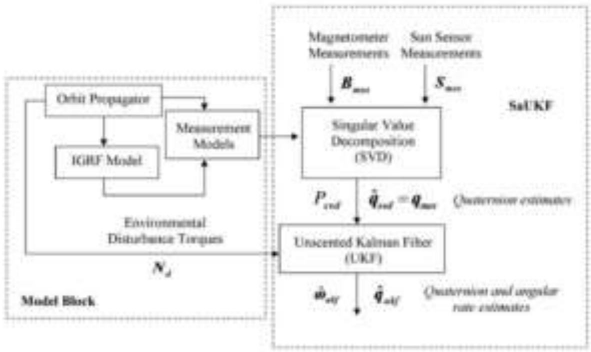

Figure 1: Flow Diagram for attitude and rates estimation using the SaUKF

3. SIMULATION RESULTS

Recurring simulations were performed to analyze the satellite attitude. The parameters for satellite include mass of 6.2668 Kg and inertial matrix: J=diag(0.0204 0.0602 0.0664) kg.m^2. The orbit under observation is considered nearly circle with an eccentricity of e=6.4 x 10^{-5} and I =74^0 inclination at 615 km. Sensors are assumed to be calibrated, a graphical demonstration is shown in below figure

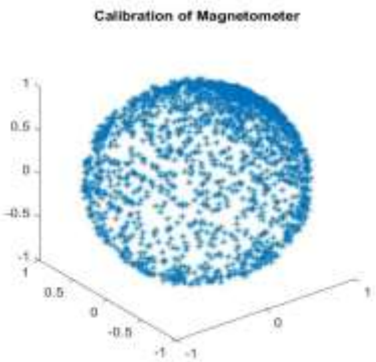

Figure 2: Magnetic Calibration

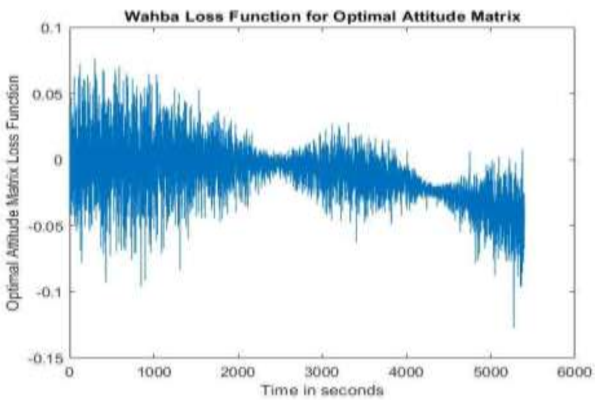

Figure 3: Wahba Loss Function for Optimal Attitude Matrix

Furthermore, Wahba Loss function (Fig: 3) has been optimized for optimal attitude matrix and Euler angles (Fig: 4) have been derived as well.

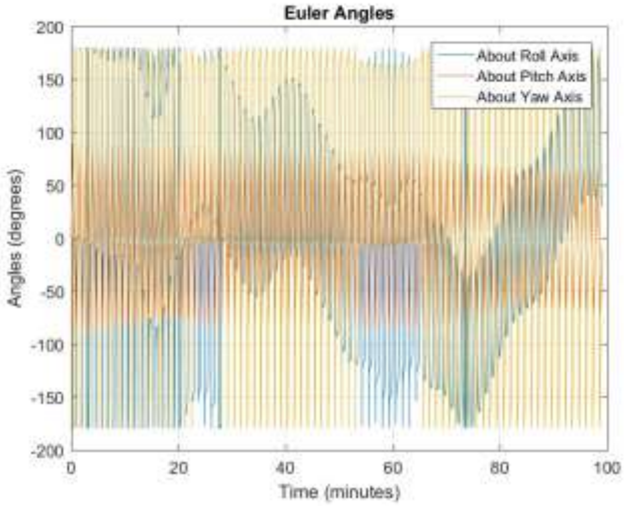

Figure 4: Euler Angles

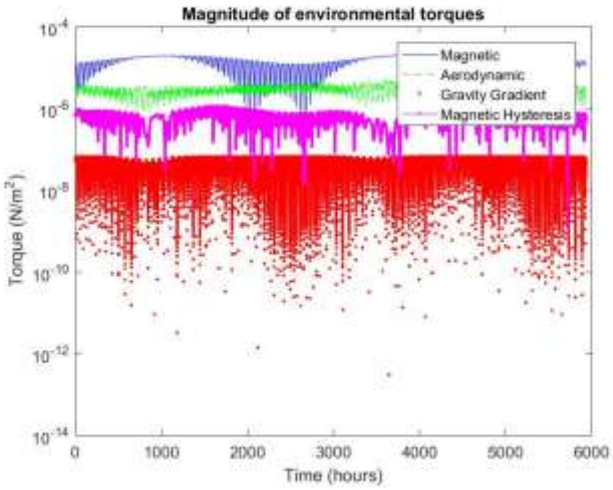

Figure 5: Magnitude of External Torques

Figure 4, depicts the simulation for external torques which serve as uncertainties to prevent accurate measurement of attitude.

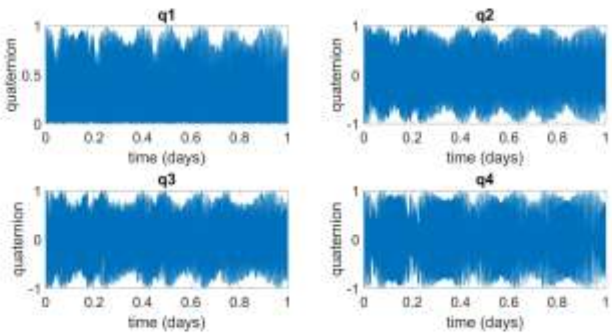

Figure 6: Quaternions Estimation using SVD

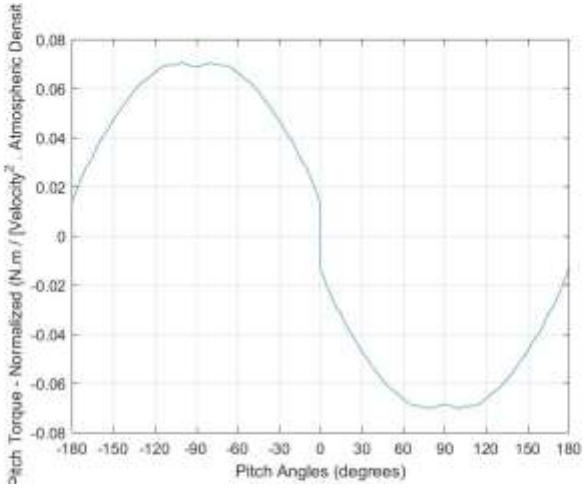

Figure 7: Variation of Pitch Torque w.r.t Pitch Angles

3.1 QUATERNION ERROR ESTIMATION

An attempt was also made to study the error estimation for quaternion which has an implication on the linear rates of the satellite.

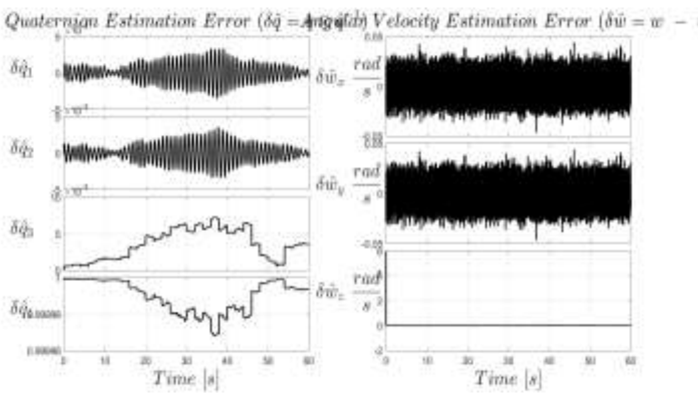

Figure 8: Quaternion and Estimation Error

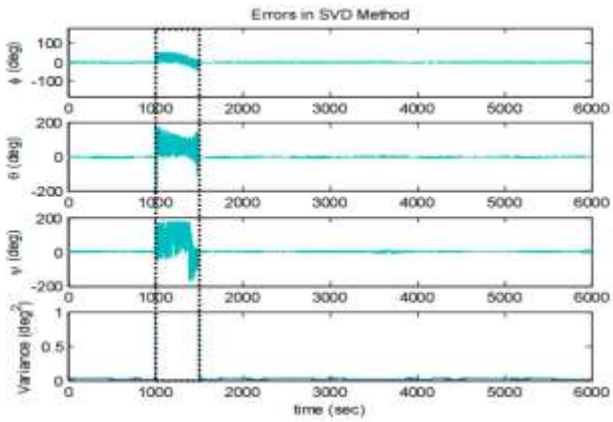

Figure 9: SVD ERROR

As SVD does not provide values when satellite is in eclipse due to the lack of sun sensor measurement, and magnetometer is the only sensor available due to which unreliable data is provided to the satellite model. In reference to Figure 9, the satellite is in eclipse from 1000 to 2000 Sec. Therefore, SaUKF provide more accurate measurements in contrast to the desired profile as compare to both filters which is shown in below figure

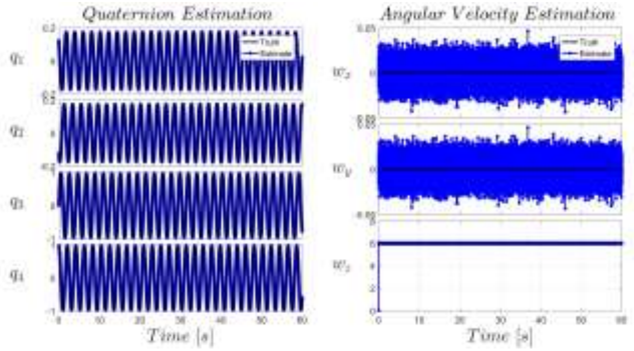

Figure 10: Estimation with the Cascaded Filter

3.2 SATELLITE ATTITUDE SIMULATIONS

Following graphs depicts the satellites profile in orbit with reference to perturbations as well as behavior during mission profile under the regulation of SVD Aided Unscented Kalman Filter. It should also be kept in mind that filter will switch whenever the satellite will exit from eclipse which might incur some perturbation, and this needs to be sure that this switching occurs after the stability of satellite in order to avoid unnecessary drifts.

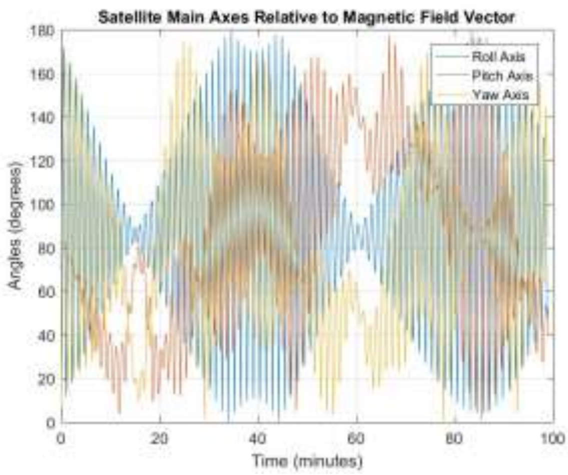

Figure 11: Satellite Main Axes Relative to Magnetic Field Vector

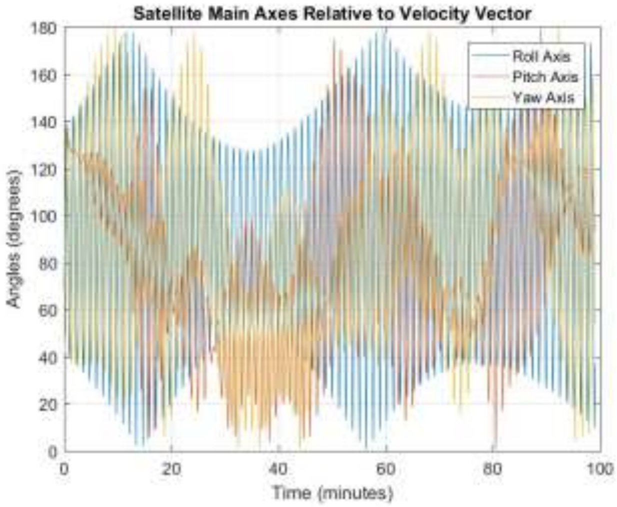

Figure 12: Satellite Main Axes Relative to Velocity Vector

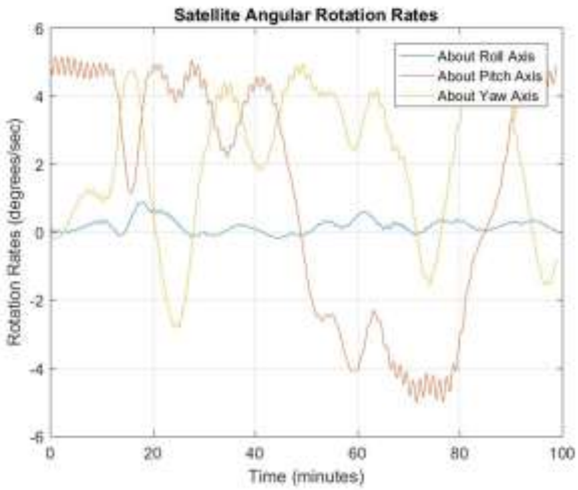

Figure 13: Satellite Angular Rotation Rates

4. CONCLUSION

In this paper an attempt was made to approximate the attitude and angular rates through the cascading of two high performance filter, SVD followed by Unscented Kalman Filter in order to get more accuracy in attitude estimation. Computational power utilized was also explored (given in Table 1), and can be seen that UKF has less computation time in contrast to SVD, while, SVD Aided-UKF (SaUKF) has the highest computation time but more close to SVD. Therefore, SaUKF is more suitable for satellite estimation.

Table 1: Computation Time for Each Approach

	SVD	**UKF**	**SaUKF**
Computation time (sec) for 10 runs	13.25	10.35	14.56

REFERENCES

[1] Wertz, J.R. (1988). Spacecraft Attitude Determination and Control. Dordrecht, Holland: Kluwer Academic Publishers.

[2] Finlay, C., Maus, S., Beggan, C.D., Bondar, T.N., et al. (2010). International Geomagnetic Reference Field: the eleventh generation. Geophysical Journal International, 183, 1216-1230

[3] Vallado, D.A. (2007). Fundamentals of Astrodynamics and Applications. Space Technology Library. USA: Microcosm Press/Springer, 21

[4] Wahba, G. (1965). Problem 65-1: A Least Squares Estimate of Satellite Attitude. Society for Industrial and Applied Mathematics Review, 7(3), 409

[5] Markley, F.L., Mortari, D. (2000). Quaternion attitude estimation using vector observations. Journal of the Astronautical Sciences, 48(2-3), 359-380.

[6] Julier, S.J., Uhlmann, J.K., Durrant-Whyte, H.F. (1995). A new approach for filtering nonlinear systems. American Control Conference, Seattle, USA. 1628-1632.

[7] Crassidis, J.L., Markley, F.L. (2003). Unscented filtering for spacecraft attitude estimation. Journal of Guidance Control and Dynamics, 26(4), 536-542

[8] J. Crassidis and J. Junkins, Optimal estimation of dynamic systems. Boca Raton, Fla.: Chapman and Hall/CRC, 2012.

[9] R. Murray, Optimal-Based Control. Pasadena: California Institute of Technology, 2017.

[10] B. Wie, H. Weiss and A. Arapostathis, "Quaternion feedback regulator for spacecraft eigenaxis rotation", Journal of Guidance Control and Dynamics, 1989.

[11] R. Paielli and R. Bach, "Attitude control with realization of linear error dynamics", Journal of Guidance Control and Dynamics, 1993.

[12] M. Blanke and M. Larsen, "Satellite Dynamics and Control in a Quaternion Formulation (2nd edition)", Technical University of Denmark, 2010.

[13] L. Markley, "Attitude Error Representations for Kalman FIltering", NASA Goddard Spaceflight Center, 2000. [7] J. Kuipers, Quaternions and rotation sequences. Princeton, NJ [u.a.]: Princeton University Press, 2007.

[14] C. Pong, "High-Precision Pointing and Attitude Estimation and Control Algorithms for Hardware-Constrained Spacecraft", PhD. MIT, 2014.