EDGE-DOMATIC NUMBER OF A JUMP GRAPH

N. Pratap Babu Rao, sweta. N

[1]*Department of mathematics, S.G. college koopal (Karnataka), India*
[2]*Department of mathematics, verasaiva college, ballari*

ABSTRACT:- With the help of the concept of dominating set E.J. Cockayne and S.T. Heditiniemi [1] have defined the domatic number of a graph. Here we shall introduced the edge analogous of this concept and prove some assertion concerning it.

Let J (G) be undirected jump graph without loops and multiple edges. Two edges $e_{1,} e_2$ of J (G) are called adjacent, if they have an end vertex in common. The degree of an edge e in J (G) is he number of edges of J (G) which is adjacent to e.

An independent set of edges of a jump graph J (G) is a subset of the edge set of J (G) with the property that no two edges of this set are adjacent. A sett 'A' of edges of a jump graph J (G) is said to cover a set of 'B' vertices of J (G) if each vertex of B is an end vertex of at least one edge of A.

An edge dominating set in J (G) is a subset D of edge set E(J(G)) of J(G) with property for each edge e ∈ E(J(G))-D. thereexists atleast one edge x ∈D adjacent to e. An edge domatic partition of J (G) is a partition of E (J (G)), all of whose classes are edge-dominating set in J (G). The maximum number of classes of an edge-domatic partition of J (G) is called the edge-domatic number of J (G) and is denoted by ed (J (G)).

Note that the edge-domatic number of J (G) is equal to the domatic number [1] of the line graph of J (G). First we shall determine edge-domatic complete jump graphs and complete bipartite jump graphs.

Proposition1. Let K_n e the complete jump graph with n vertices n ≥ 2. If n is even then ed (J (K_n) = n-1 if n is odd then ed (J (K_n)) = n

Proof; Let n be even. Then it is well known that K_n can be decomposed into n-1 pairwise edge-disjoint liner factors. The edge set of each of these factors is evidently an edge-domatic partition of K_n with n classes. As the number of edges of K_n is $\frac{1}{2}$ n (n-1). The mean value of the cardinalities of these classes is $\frac{(n-1)}{2}$.This implies that atleast one of these classes has at most is $\frac{(n-1)}{2} = \frac{n}{2}$ - 1 edges. But then this set A of edges covers at most n=n vertices. There are two vertices which are incident to no edge of A and the edge joining these vertices is adjacent to no edge of A which is a contradiction with the assumption that A is an edge-dominating set, we have proved that ed(J(K_n))=n-1 for n even.

Now let n be odd. Denote the vertices of J (K_n) by $u_{1,}$ u_2,……u_n. In sequel aall subscripts will be taken modulo n. for each i= 1, 2,….n. Let E_i be the set6 of all edges u_{i+j}, u_{i-j+1} where j= 1,2,…. (n-1)/2 The reader may verify himself that the sets E_1, E_2,……E_n form a partition of the edge set of J(K_n). each set E_i covers all vertices of J(K_n) except one , each edge of J(K_n) not belonging to E_i is incident with at least one vertex covered by E_i and thus adjacent to atleast one edge of E_i; the set E_1, E_2,……E_n form a domatic partition of J(K_n) and ed(J(K_n)) ≥ n.

Suppose that ed (J (K_n)) ≥ n+1. Then we analogously prove that there exists an edge domatic partition of J (G), one of whose classes has at most$\frac{(n-3)}{2}$. Edges; this set covers at most (n-3) vertices and it is not an edge-dominating set, which is a contradiction. Therefore ed (J (K_n)) = n for n odd..

Proposition2. Let J ($K_{m,n}$) e a complete bipartite jump graph

Then ed (J ($K_{m,n}$)) = max (m, n)

Proof; Without loss of generality let m ≥ n i.e., max (m, n) = m let $K_{m,n}$ be the bipartite graph on the vertex sets A, B such that |A|=m |B|=n. Then for each u is an edge dominating set in J (Km, n) it covers all vertices of B. Therefore the sets E (u) for all u ∈ A form an edge domatic partition of J ($K_{m,n}$) with m classes. We have proved that ed (J ($K_{m,n}$)) ≥ m Now suppose that

ed (J ($K_{m,n}$)) ≥m+1 and consider an edge domatic partition of J ($K_{m,n}$) with m+1 classes. As J ($K_{m,n}$) has mn edges, there exists at least one class C of this partition which contains less than 'n' edges. Then this set C covers neither A nor B. There exists a vertex of A and a vertex of B which are incident with no edge of C and the edge joining then is adjacent to no edge of C. The set C is not edge-dominating which a contradiction is. Hence ed (J ($K_{m, n}$)) =m=max (m, n)

Proposition 3; Let J(C_n) be a circuit of the lengthen, If n is divisible by 3 then ed(J(C_n))=3 otherwise ed(J(C_n))=2.

Proof; A circuit is isomorphic to its own line graph therefore its edge-domatic number is equal to its domatic number and for it this assertion was proved in [1].

Now we shall prove two theorems.

Theorem 1; For each finite undirected jump graph J (G) we have

$$\delta (J (G)) \leq ed (J (G)) \leq \delta_e (J (G)) + 1$$

where ed(J(G)) is the edge-domatic number of J(G), δ(J(G)) is the minimum degree of an edge of J(G) and δ_e(J(G)) is minimum degree of an edge of J(G).These bounds cannot be improved.

Proof; The number δ_e (J (G)) is equal to the minimum degree of a vertex of the line graph of J (G). According to [1], the domatic number of this line graph cannot be greater than δ_e(J(G)) + 1 this domatic number is equal to the edge-domatic number of J(G), Hence ed(J(G)) ≤ δ(J(G)) + 1.

Now, we shall prove that δ (J (G)) ≤ ed (J (G)). By induction we shall prove the following assertion. If the degree of each vertex of G is greater than or equal to k (where k is an arbitrary positive integer) then there exists an edge –domatic partition of J (G) with k-classes. For k=1 theassrtion is true; the required partition consisting of one class equal to the whoke E(J(G)) which is evidently an edge-dominating set in J(G). Now let $k_0 \geq 2$ and suppose that the assertion is true;

for k= k_0-1..

Consider a graph J (G) in which the degree of each vertex is at least k_0. Let E_0 be a maximal (with respect to the set inclusion) independent set of edges of J (G). This set is edge dominating ; otherwise an edge could be added to it without violating the independence, which would be contradiction with the maximality of E_0. Let J(G_0) be the jump graph obtained from jump graph J(G0 by deleting all edges of E_0 each vertex of J(G) is incident at most with one edge of E_0,therefore each vertex of J(G_0) has the degree at least k_0-1. According to the induction hypothesis, there exists an edge domatic partition ℘ of J (G_0) with k_0-1 classes then ℘ ∪ {E_0} is an edge domatic partition of J (G) with k_0 classes, which was to be proved. The proved assertion implies ed (J (G)) ≥ δ (J (G)). If J (G) is a circuit C_n and n is dividible by 3 then ed (J (G)) = δ_e (J (G)) +1. If g is a circuit C_n and n is not divisible by 3 then ed (J (G)) = δ (J (G)) (by proposition1)

Theorem2; Let J(G) be a tree, let δ_e(J(T)) be the minimal degree of an edge of J(T) then ed(J(T)) = δ_e(J(T)) + 1.

Proof; Let us have the colours 1, 2,……. δ_e (J (T)) + 1.. we shall colour the edges of J(T) by them First we choose terminal edge e_0 of J(T) and colour it by the colour 1 Now let us have an edge e of J(T) with the end vertices u,v; suppose that all edges incident with v are already coloured. Moreover, if the number of these edges is less than δ_e (J (T)) + 1.

We suppose that they are coloured by pair wise different colours in the opposite case we suppose that all colours 1, 2,….. δ_e (J (T)) + 1.

Occur among the colours of these edges .Now we shall colour the edges incident with n and distinct from edge e. We colour them in the following way. If there are colour by which no edge incident with v is coloured, we use all of them (This must be always possible according to the assumption). If the number of edges to be coloured is less than δ_e (J (T)) + 1. (Some of them may be repeat) The result is colouring of edges of J (T) by the colour 1, 2, …… δ_e (J (T)) + 1.

With the property that each edge is adjacent to edges of all different from its own one. If $C_{i= \text{for } 1, 2, \ldots}$ δ_e(J(T)) + 1.is the set of all edges of J(T) coloured by the colour I, then the sets C_1, c_2,………..$C_{\delta e\ J(T))+1}$ form an edge domatic partition of T with δ_e(J(T)) + 1. Classes and ed (J (T)) ≥ δ_e (J (T)) + 1.

According to theorem 1 it cannot be greater.

$$\therefore \; ed (J (T)) = \delta e (J (T)) + 1.$$

REFERENCES

[1] Cockayne E.J, Hedetniemi S.T. ; Towards theory of domination in graphs Network 7 (1977) 247-261.

[2] Mulder H.M, The interval Function of a Graph Amsterdan1980.

[3] Ore. O Theory of Graphs .Providence 1962

[4] Zelinka ,B. Odd graphs Arch. Math. Brno (to appear)

[5] Zelinka, B. Edge- domtic number of a graph; Czechoslovak Mathematical Journal,33(108)1983,praha

.