International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 05 Issue: 08 | Aug 2018 www.irjet.net p-ISSN: 2395-0072

Probabilistic Stress Distribution in Thick Cylindrical Pipe using Finite Element Method

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Abstract - *The present work deals with the development of finite element methodology for obtaining the stress distributions in thick cylindrical pipe that carry high temperature fluids. The material properties and loading are assumed to be random variables. Thermal stresses that are generated along radial, axial and tangential directions are computed generally using analytical expressions which are very complex. To circumvent such an issue, the probability theory and mathematical statics have been applied to many engineering problems which allows to determine the safety both quantitatively and objectively based on the concepts of reliability. Monte Carlo simulation methodology is used to study the probabilistic characteristics of thermal stresses which is used for estimating the probabilistic distributions of stresses against the variations arising due to material properties and load. The deterministic solution is compared with ABAQUS solutions. The values of stresses that are obtained from the variation of elastic modulus are found to be low as compared to the case where the load alone is varying. The probability of failure of the pipe structure is predicted against the variations in internal pressure and thermal gradient. These finite element framework developments are useful for the life estimation of piping structures in high temperature applications and subsequently quantifying the uncertainties in loading and material properties.*

Key Words: *P*robabilistic Finite Element Method, Axisymmetric finite elements, Random variables, Material and load variability, Monte Carlo simulation.

1. INTRODUCTION

The axisymmetric pressurized thick cylindrical pipes are widely used in chemical, petroleum, military industries, fluid transmitting plants and power plants as well as in nuclear power plants due to ever-increasing industrial demand. They are usually subjected to high pressures & temperatures which may be constant or cycling. In general, accurately predicting the thermal stresses generating on structural components like thick pipe due to pressure and temperature change is very difficult. So, the probability theory and mathematical statics have been applied which allows us to determine the safety both quantitatively and objectively based on the concepts of reliability. The stress distribution in the NPP piping system remains a main concern and deterministic structural integrity assessment need to be combined with probabilistic approaches to consider uncertainties in material and load properties. The deterministic finite element method for a defined problem can be transformed to a probabilistic approach by considering some of the inputs to be random variables. The uncertainty associated with the strength prediction may be calculated by simulation techniques, such as Monte-Carlo simulation, which allow the values for basic strength variables to be generated based on their statistical distributions (probability density functions).

Relevant strength variable for pipe is elastic modulus and load variables are internal pressure and temperature change. The objective herein is to compile statistical information and data based on literature review on both strength and loads random variables relevant to thick pipe structure for quantifying the probabilistic characteristics of these variables. Quantification of random variables of loads and material properties in terms of their means, standard deviations or COV's and probability of distributions can be achieved in two steps (1) data collection and (2) data analysis. The first step is the task of collecting as many sets of data deemed to appropriate for representing random variables under study. The second is concerned with statistically analysing the data to determine the probabilistic characteristics of these variables.

2. LITERATURE REVIEW

Zhou and Tu [1] carried out the work to estimate the service life of high temperature furnace which is very difficult due variability of creep data. To study the random nature of service life, a new stochastic creep damage model is proposed in his work. A comparison with results calculated by use of the Monte Carlo method verifies the creep damage model. The randomness of the creep damage is demonstrated with a calculation on HK-40 furnace tubes which provides an effective means to assess the reliability of the furnace tubes. In the present work the material parameters of the HK40 are adapted from Zhou and Tu [1].

Chanylew Taye and Alem Bazezew [2] studied the creep analysis of boiler tubes by FEM, in his work an analysis is developed for the determination of creep deformation of an axisymmetric boiler tube subjected to axisymmetric loads.

Holm Altenbach [3] presented the creep model is to reflect the basis features of creep in structures including the evaluation of inelastic deformations, relaxation and redistribution of stresses as well as the local reduction of material strength. The solutions are compared with the finite element solutions of ANSYS and ABAQUS finite element codes with user creep model subroutines. The geometric parameters and loading conditions for the present work are adopted from Holm Altenbach [3].

Oliver C. Ibe [4] presented the study of fundamentals of applied probability and random processes, which are followed in the present work.

3. Problem Analysis

A thick-walled cylinder pipe carrying high temperature liquid is considered. The fluid inside the pipe is assumed to completely fill the pipe and exerts a constant pressure P. The analysis is carried out in the 2-D plane of the cross section of the pipe see Fig. 1

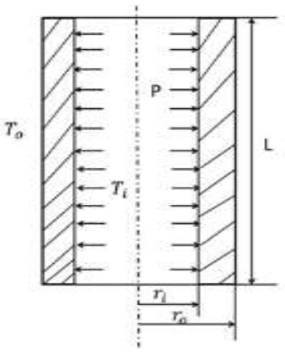

Fig -1: Schematic diagram of the axisymmetric pipe section

The pipe is made up of material HK40 Zhou and Tu [1]. It is stressed to a pressure of 40 Mpa . The temperature of the fluid flowing inside the pipe is T_i is 500^0 C and the outside temperature is T_0 = 420^0C. i.e the pipe is subjected to thermal gradient of 80^0 C. The dimension of the Thick pipe section is taken as L 100 mm, r_i 25 mm, r_0 50 mm respectively. The material properties of HK40 are given as follows: Elastic modulus 1.38 x 10^5 Pa, Poisson's ratio 0.313, Thermal expansion coefficient 1.5 x 10^{-5} (1/°C). The obtained values of the Radial stress, circumferential stress, axial stress and Von Mises stress are shown in the table 1. The graph of various stress vs radius are shown in figure 2 and figure 3.

3.1 Analytical Solution

The stresses for thick walled cylinder pipe under internal pressure (P) and Thermal gradient ΔT

Radial stress $\sigma_r = \sigma_r^P + \sigma_r^T$

$$\sigma_r = \frac{P}{a^2-1}\left[1-\left(\frac{r_0}{r}\right)^2\right] + \frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[-2\ln\left(\frac{r_0}{r}\right) + \frac{\left(\frac{r_0}{r}\right)^2-1}{a^2-1}\ln a\right]$$

Circumferential stress $\sigma_\theta = \sigma_\theta^P + \sigma_\theta^T$

Axial stress $\sigma_z = \sigma_z^P + \sigma_z^T$

Von-Mises stress is given by

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_r-\sigma_\theta)^2 + (\sigma_\theta-\sigma_z)^2 + (\sigma_z-\sigma_\theta)^2}$$

where σ_θ^P is the hoop stress induced by pressure (MPa); σ_z^P,the axial stress induced by pressure (MPa); σ_r^P,the radial stress induced by pressure (MPa); P, the pressure in (MPa);r_0, the outer radius (mm); r_i,the inner radius (mm); a, the ratio of outer to inner radius; a = r_0/r_i ; r, the radius at any position of tube wall(mm) and μ ,the poisson's ratio. σ_θ^T is the hoop stress induced by thermal stress (MPa); σ_z^T,the axial stress induced by thermal stress (MPa); σ_r^T,the radial stress induced by thermal stress (MPa); E, the elastic modulus of material (MPa);α,the thermal expansion coefficient of material ($1/{}^oC$); ΔT ,the Thermal gradient of outer wall and inner wall temperature is $\Delta T = T_i - T_0$

Table -1: Analytical Results

Radius (mm)	Radial stress (σ_r) MPa	Circumferential stress (σ_θ) MPa	Axial stress (σ_z) MPa	Von Mises stress (σ_e) MPa
26.2500	-35.0416	61.7082	8.3467	83.9361
28.7500	-26.9943	53.6610	8.3467	70.0273
31.2500	-20.8000	47.4667	8.3467	59.3306
33.7500	-15.9305	42.5972	8.3467	50.9312
36.2500	-12.0333	38.7000	8.3467	44.2184
38.7500	-8.8658	35.5324	8.3467	38.7720
41.2500	-6.2565	32.9232	8.3467	34.2951
43.7500	-4.0816	30.7483	8.3467	30.5730
46.2500	-2.2498	28.9165	8.3467	27.4476
48.7500	-0.6925	27.3592	8.3467	24.8000

3.2 Axisymmetric Finite Element Analysis Using Abaqus Software

The pipe is made up of material HK40 is considered with pipe length L = 100 mm, Inner radius r_i = 25 mm, outer radius r_0 = 50 mm. The material properties of the material are Elastic Modulus E = 1.38 X 10^5 MPa, Poisson's ratio υ = 0.313, Coefficient of thermal expansion α = 1.5 x 10^{-5}(1/^{0}C). The model is meshed with element type CAX4R, A 4-noded bilinear quadrilateral element and the Mesh grid is 10 x10 elements and is fixed in Axial direction U_2 =0. The loading conditions are Internal Pressure P = 40 MPa, Inside temperature T_i = 500 ℃, Outside temperature T_0 = 420℃ , Thermal gradient $\Delta T = 80$ ℃. Figure 2 shows the model with meshing and applied boundary conditions in Abaqus.

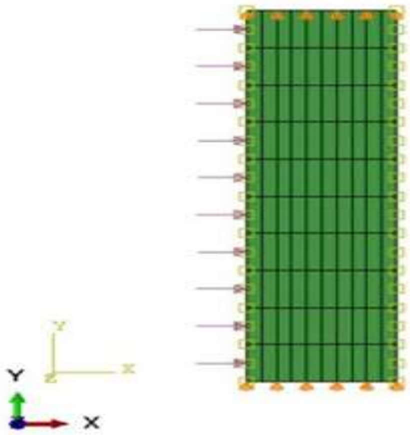

Fig -2: Meshing, Boundary conditions, internal pressure and Thermal gradient of Axisymmetric Thick pipe

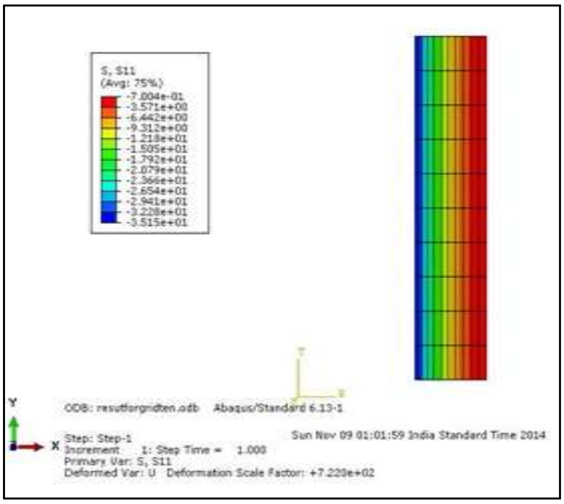

Fig -3: Radial stress (Transverse view)

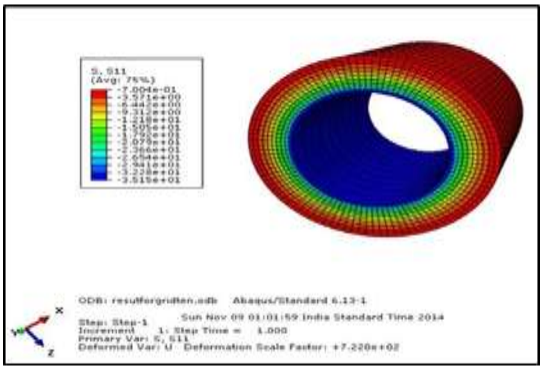

Fig -4: Radial stress (Circumferential view)

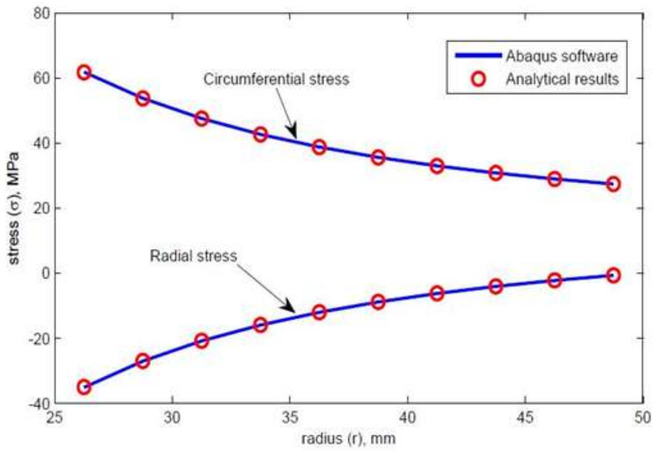

Chart -1: Comparison of Analytical and FEA using Abaqus

3.3 Probabilistic Study - Monte-Carlo Simulation

The Monte-Carlo method involves randomly sampling the distributed input variables many times to build a statistical picture of the output quantities. The method has a wide range of applicability, engineering applications being only one. The method is particularly appropriate when there are many independent variables which can influence the outcome. The Monte-Carlo method is being used increasingly in structural integrity applications.

The probabilistic simulation uses a Monte-Carlo method with Latin Hypercube sampling. This is an efficient technique which permits many distributed variables to be addressed. Each variable takes a finite number of values each of which represents a range of values (a 'bin'). All bins are of equal probability. The Latin Hypercube algorithm ensures that all bins of variables are sampled in the minimum number of trails (though not, of course, in all possible combinations). Moreover, because all bins are of equal probability it follows that all trails of equal probability, thus ensuring that all trails are of equal weight in the simulation.

The parameters which are required to calculate thermal stresses, and which are taken as distributed in simulations are: elastic modulus, internal pressure and temperature change. Normal distributions are used for stresses and lognormal distribution is used for material properties.

Case 1: Lognormal distribution for Young's modulus of elasticity (E):

$$f_X(E;\mu,\sigma) = \frac{1}{x\sigma_x\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu_x}{\sigma_x}\right)^2\right]$$

Lognormal distribution for Youngs Modulus

Monte-Carlo simulations (MCS) N = 1000 runs are carried out to estimate the stress distribution for number of elements in radial direction

Mean E , Variance = σ^2

Coefficient of variance = 0.2 = $\frac{\sigma}{\mu} = \frac{\sigma}{E}$

Mean = $(\mu_x) = \log\frac{\mu^2}{\sqrt{\sigma^2+\mu^2}}$

Standard deviation = $(\sigma_x) = \sqrt{\log\frac{\sigma^2}{\mu^2+1}}$

Elastic modulus is lognormal distributed with mean and standard deviation respectively.

Case 2: Due to load variability

This distribution is basis for many statistical methods. The normal density function for a random variable X is given by:

f_X (x)= 1/(σ√2π) exp[-(x-μ)^2/(2σ^2)], -∞<x<∞.

It is common to use the notation X~N(μ,σ^2) to provide an abbreviated description of a normal distribution. The notation states that X is normally distributed with mean μ and varianceσ^2. In this study, P and ΔT are random variables in r-radial and z-axial direction. Normal distribution of the form

〖f_X (x;μ,σ)= f〗_X (P,ΔT;μ,σ)= 1/(σ√2π) exp[-(x-μ)^2/(2σ^2)], -∞<x<∞.

Normal distribution for pressure (P):

Mean = P Variance = σ^2

Coefficient of variance = 0.1 = σ/μ=σ/P

Standard deviation = (σ_x) = √V

Pressure (P) = x*σ_x+μ

Pressure is normal distributed with mean and standard deviation of 40 MPa and 4 MPa respectively.

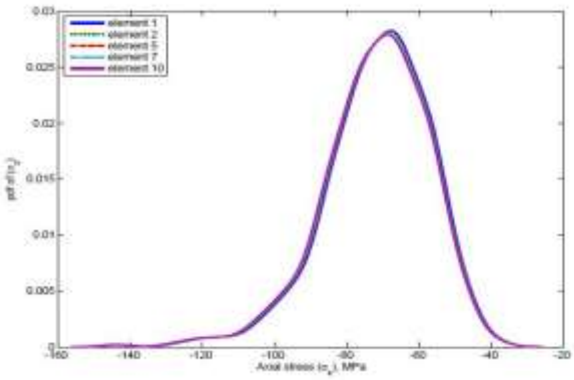

Chart 2: Comparison of pdf of axial stress for different elements when E is a random variable

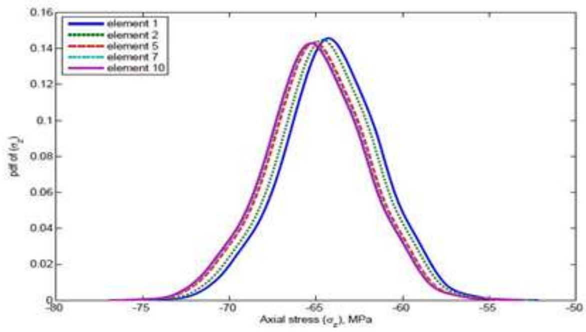

Chart 3: comparison of pdf of axial stress for different elements when $E, P\ and\ \Delta T$ are random variables

4. CONCLUSION

Analytical results are validated with the ABAQUS software along with FEM and a very good match is obtained among all these findings. Finite element frame work is developed to investigate the effects of uncertainties in pipe structure due to material properties and loading. Random variable models are used to model the variabilities in material properties and load using Monte Carlo simulations. Monte Carlo simulations are used to study the probabilistic characteristics of stress distribution of pipe structure. The values of thermal stresses that are obtained from the variation in material properties like modulus of elasticity are found to be low as compared to the case where the load alone is varying. The present methodology is used for estimating the probabilistic distributions of thermal stresses against the variations arising due to material properties and as well as variations due to thermal loading. The probability of failure of the pipe structure is predicted against the variations in internal pressure and thermal gradient. The developed methodology can be useful for the life estimation of piping structures that can be used for high temperature applications against creep, fatigue failures for further studies.

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