GLOBAL ACCURATE DOMINATION IN JUMP GRAPH

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ABSTRACT: A dominating set D of a jump graph is an accurate dominating set if |V-D| has no dominating set of cardinality |D|. An accurate dominating set D of a graph G is also an accurate dominating set of Ḡ. The global accurate dominating number \( \sqrt{ga}(J(G)) \) are obtained and exact values of \( \sqrt{ga}(J(G)) \) for some standard graphs are found. Also a Nordhaus-Gaddum type results established.

Key words: accurate dominating set, global accurate dominating set, global accurate domination number.

Mathematics subject-Classification:05C

I Introduction

All graphs considered here are finite, undirected without loops and multiple edges. Any undefined terms in this paper may found in kulli [1]

A set D of vertices in a jump graph is a dominating set of J(G), if every vertex not in D is adjacent to a vertex in D. The domination number of a jump graph is denoted by \( \sqrt{j(G)} \) is the minimum cardinality of a dominating set in J(G).

A dominating set D of a jump graph J(G) is accurate dominating set. If V(J(G)) - D has no domination set of cardinality |D|. The accurate domination number \( \sqrt{a}(J(G)) \) of J(G) is the minimum cardinality of an accurate dominating set. This concept was introduced by kulli and kattimani in [2]

A dominating set D of ajump graph J(G) is a global dominating set. If D is also a dominating set of J(Ḡ). The global domination number \( \sqrt{g}(J(G)) \) of J(G) is the minimum cardinality of a global dominating set [5].

In [4] kulli and kattimani introduced the concept of global accurate domination as follows.

An accurate dominating set D of a graph G is a global accurate dominating set, if D is also an accurate dominating set of Ḡ. The global accurate domination number \( \sqrt{ga}(G) \) of G is the minimum cardinality of a global accurate dominating set. Analogously, a set d of a jump graph J(G) is a global accurate dominating set if D is also an accurate dominating set of J(Ḡ). The global accurate domination number \( \sqrt{ga}(J(G)) \) of J(G) is minimum cardinality of a global accurate dominating set.

Let \( \lfloor x \rfloor \) denote the greatest integer less than or equal to x. A \( \sqrt{a} \)-set is minimum accurate dominating set.

2. Results

We characterize accurate dominating set which are global accurate dominating sets.

Theorem 1: An accurate dominating set D of a jump graph J(G) is a global accurate dominating set if and only if the following condition holds.

For each vertex v\( \in V(J(G))-D \), there exists vertex u \( \in D \) such that u is not adjacent to v. There exists a vertex w \( \in D \) such that w is adjacent to all vertices in V(J(G))-D.

Theorem2. Let J(G) be a jump graph such that neither J(G) nor J(Ḡ) has an isolated vertex, Then

\( \sqrt{ga}(J(G)) = \sqrt{ga}(J(Ḡ)) \)
\(\sqrt{a}(J(G)) + \sqrt{a}(J(\bar{G})) \leq \sqrt{a}(J(G)) \leq \sqrt{a}(J(\bar{G}))\)

**Theorem 3.** Let \(J(G)\) be a jump graph such that neither \(J(G)\) nor \(J(\bar{G})\) have an isolated vertex then

\[ \sqrt{a}(J(G)) \leq \sqrt{ga}(J(G)) \]

**Proof:** Every global accurate dominating set is an accurate dominating set then above inequality holds.

**Theorem 4:** Let \(j(G)\) be a jump graph such that neither \(j(G)\) nor \(j(\bar{G})\) have an isolated vertex then

\[ \sqrt{a}(j(G)) \leq \sqrt{ja}(j(G)) \]

**Proof:** Every global accurate dominating set is an accurate dominating set then above inequality holds.

Exact values of \(\sqrt{ga}(j(G))\) for some standard graphs are given in Theorem 5.

**Theorem 5:**

- \(\sqrt{ga}(J(K_p)) = p\)
- \(\sqrt{ga}(J(C_p)) = \left\lceil \frac{p}{2} \right\rceil + 1\) if \(p \geq 3\)
- \(\sqrt{ga}(J(p_p)) = \left\lceil \frac{p}{2} \right\rceil + 1\) if \(p \geq 2\)
- \(\sqrt{ga}(J(K_{m,n})) = m+1\) if \(m \leq n\)
- \(\sqrt{ga}(J(W_p)) = \left\lceil \frac{p}{2} \right\rceil + 1\) if \(p \geq 5\)

For any regular jump graph \(J(G) = \left\lceil \frac{p}{2} \right\rceil + 1\) if \(p \geq 2\)

Now we obtain an upper bound for \(\sqrt{ga}(J(G))\)

**Theorem 6.** Let \(J(G)\) has two non adjacent vertices \(u\) and \(v\) such that \(u\) is adjacent to some vertex in \(V(J(G)) - u\) this implies that \(V(J(G)) - \{u\}\) is a global accurate dominating set of \(G\). Thus

\[ \sqrt{ga}(J(G)) \leq |V(J(G)) - \{u\}| \] or

**Proof:** Suppose result holds. Assume that \(J(G) \neq K_p\). Then \(J(G)\) has at least three vertices \(u, v, \) and \(w\) such that \(u\) and \(v\) are adjacent and \(w\) is not \(u\). Then this implies that \(V(J(G)) - \{u\}\) is a global accurate dominating set of \(J(G)\). This proves necessity.

Converse is obvious.

**Theorem 8.** Let \(D\) be an accurate dominating set of \(J(G)\) if there exists two vertices \(u \in V(J(G)) - D\) and \(v \in D\) such that \(u\) is adjacent only to the vertices of \(D\) and \(v\) is adjacent to the vertices of \(V(J(G)) - D\). Then

\[ \sqrt{ga}(J(G)) \leq \sqrt{a}(J(G)) + \]

**Proof:** Let \(D\) be a \(\sqrt{a}\)-set of \(J(G)\) if there exists a vertex \(u \in V(J(G)) - D\), such that \(u\) is adjacent only to the vertices of \(D\) then \(D \cup \{u\}\) is a global accurate dominating set of \(G\), thus

\[ \sqrt{ga}(J(G)) \leq |D \cup \{u\}| \]

\[ \leq |D| + 1 \]
Or \[ \sqrt{g_a}(J(G)) \leq \alpha_0(J(G)) + 1 \]

In jump graph \( J(G) \), a vertex and an edge incident with it are said to cover each other. A set of vertices that cover all the edges of \( J(G) \) is a vertex cover of \( J(G) \). The vertex covering number \( \alpha_0(J(G)) \) of jump graph \( J(G) \) is the minimum number of vertices in a vertex cover. A set \( S \) of vertices in \( J(G) \) is independent if no two vertices in \( S \) are adjacent. The independence number \( \alpha_0(J(G)) \) of \( J(G) \) is the maximum cardinality of an independent set of vertices. The Clique number \( \beta_0(J(G)) \) of \( J(G) \) is the maximum order among the complete sub graph of \( J(G) \).

**Theorem 9:** Let \( J(G) \) be a jump graph without isolated vertices then

\[ \sqrt{g_a}(J(G)) \leq \alpha_0(J(G)) + 1 \]

**Proof:** Let \( s \) be a maximum independent set of vertices in \( J(G) \). Then for any vertex \( v \in S \), \{ \( V(J(G)) - S \) \} \cup \{v\} is a global accurate dominating set of \( J(G) \) thus

\[ \sqrt{g_a}(J(G)) \leq | \{ V(J(G)) - S \} \cup \{v\}| \]

\[ \leq | V - S | + 1 \]

\[ \leq p - \beta_0(J(G)) + 1 \]

Or \[ \sqrt{g_a}(J(G)) \leq \alpha_0(J(G)) + 1 \]

We obtain a Nordhus - gaddum type result

**Theorem 10:** Let \( J(G) \) be a jump graph such that neither \( J(G) \) nor \( \overset{\sim}{J(G)} \) have an isolated vertex Then,

\[ \sqrt{g_a}(J(G)) + \sqrt{g_a}(\overset{\sim}{J(G)}) \leq p + \sqrt{\alpha}(J(G)) - w(J(G)) + 2 \]

**Proof:** By theorem 9 \[ \sqrt{g_a}(J(G)) \leq \alpha_0(J(G)) + 1 \]

Therefore \[ \sqrt{g_a}(J(G)) \leq \alpha_0(J(G)) + 1 \]

\[ \leq p - \beta_0(J(G)) + 1 \]

\[ \leq p - w(J(G)) + 1 \]

Hence \[ \sqrt{g_a}(J(G)) + \sqrt{g_a}(\overset{\sim}{J(G)}) \leq p + \sqrt{\alpha}(J(G)) - w(J(G)) + 2 \]

**REFERENCES**


