Application of Type II Kernelized Intuitionistic Fuzzy C Mean in the field of image segmentation of MRI image

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Abstract - Image segmentation has an important role in the field of medical image processing. Abnormal growth of cells in the brain forms brain tumor, which can cause chronic diseases of mankind. In this paper, we are using several type segmentation algorithms for segmenting brain tumor images such as Fuzzy C Mean, Type II Fuzzy C Mean, Kernelized Fuzzy C Mean, Type II Kernelized Fuzzy C Mean, Intuitionistic Fuzzy C Mean and compare them in terms of accuracy, specificity and sensitivity. Finally, our research work shows that our proposed method Type II Kernelized Intuitionistic Fuzzy C Mean has the most accuracy 98.21% and specificity 100%. It also has the least running time 5.051 second compared to other algorithms.

Key Words: Fuzzy C Mean, Type II Fuzzy C Mean, Kernelized Fuzzy C Mean, Type II Kernelized Fuzzy C Mean, Brain Tumor, Image Segmentation.

1. INTRODUCTION

Image segmentation is the process of separating a digital image into multiple regions (sets of pixels). Segmentation has a wild range of application such as in medical imaging, machine vision, object detection, face recognition, fingerprint recognition etc. Segmentation of image has been widely used in medical imaging like as in the area of brain tumor detection, breast cancer detection, vessel segmentation etc. Brain is a very important part of the human life and it is one of the most complicated structures. Brain tumor happens when there is an abnormal increase of cell of the brain that grows by uncontrolled cell division. As the brain is a very complicated part of human body it should be handled very carefully. Brain tumor causes various problems in human body. So it should be removed from brain with the help of surgery. For the purpose of surgery, first we should detect the area of the tumor properly.

Therefore the accurate and efficient detection of image from MRI data by using image segmentation becomes an important research area in medical imaging. The main task is to detect the presence of tumors in MRI images of the brain, and segment the abnormal pixels from the normal pixels. Fuzzy c-means (FCM) is a clustering method which says that one piece of data can belong to two or more clusters simultaneously. It is an extension of crisp clustering [1] having a wide range of functions in the field of pattern recognition. FCM has a wide application in image segmentation recently[2]. To improve the FCM algorithm we applied a better approach called Kernelized FCM algorithm [3]. The main approach of this algorithm is to map the input data into higher dimensions to remove the problem of non linearity in the input space by mapping them higher dimensional space with the help of Mercer function [4]. The direct computation in the higher space takes much time so with the help of Mercer function, we can solve the problem. This method is called as the Kernel method [5]. Clustering is done after the transition of the input data into higher dimensional feature space [6]. An improved version of KFCM can be seen in the Type-II membership values where the Type-II cluster center has more appropriate positions than the FCM cluster centers in a image which is corrupted by noises [8]. Intuitionistic fuzzy C Mean method is a much more generalized concept than fuzzy C mean method. Intuitionistic fuzzy C Mean introduces the degree of membership, the degree of nonmembership along with the degree of hesitation of a data point in the cluster set. Degree of hesitation is usually defined as one minus the degree of membership and degree of nonmembership. In the notion of intuitionistic fuzzy set, degree of non membership not always equal to the one minus degree of membership. Therefore the concept of degree of hesitation has been introduced [10]. But in this paper we have used more generalized method than Intuitionistic fuzzy C mean method. Here we have used Type II Kernelized Intuitionistic Fuzzy C Mean method for segmenting brain tumor images and also we have compared our result with all other existing methods for segmenting brain tumor images. It is seen that our method has the most accuracy 98.21% and specificity 100%. It also has the least running time 5.051 second compared to other existing algorithms.

1.1 Fuzzy Based Image Segmentation

Several approaches have been developed for segmentation. Some of those approaches are Edge Detection Method, Region Based Segmentation Method, thresholding Method, and Clustering Method. Two famous fuzzy based clustering algorithms are Fuzzy c-means (FCM) and kernalized fuzzy c-means (KFCM). Two methods are briefly described below.

1.2 Fuzzy C Mean:

The whole data set \( X = \{ x_1, x_2, \cdots, x_n \} \) is classified into \( V = \{ v_1, v_2, \cdots, v_c \} \) homogeneous cluster groups. It is based on minimization of the following Lagrangian multiplier method: \( J_m = \sum_{i=1}^{c} \sum_{k=1}^{n} \left( \mu_{ik} \right)^m \left\| x_k - v_i \right\|^2 \) (1)

where \( 1 \leq m < \infty, \mu_{ik} \in [0, 1] \) is denoted as the degree of membership of \( x_k \) belongs to the \( i \)th cluster \( v_i \). \( x_k \) is the \( d \)-dimensional \( k \)th measured data, \( v_i \) is the \( d \)-dimensional center of the cluster, and \( \| \cdot \| \) is any norm expressing the similarity between any calculated data and the center.

Fuzzy partitioning is carried out through the membership function which an iterative optimization of the objective function (1), therefore it need to update of membership \( \mu_{ik} \) with the help of following equations:

\[
\mu_{ik} = \frac{1}{\sum_{j=1}^{c} \left( \left\| x_k - v_i \right\| / \left\| x_k - v_j \right\| \right)^{m-1}} \tag{2}
\]

subject to

\[
\sum_{i=1}^{c} \mu_{ik} = 1, \quad \forall \, k \quad \text{and} \quad \sum_{k=1}^{n} \mu_{ik} < n, \quad \forall \, i.
\]

After the calculation of each membership value of each data, updation of each cluster center \( v_j \) needs to be done by considering all the data belonging to each cluster. The required formula given by

\[
v_i = \frac{1}{n_i} \sum_{k=1}^{n} \left( \mu_{ik} \right)^m x_k, \quad \text{where}
\]

\[
n_i = \sum_{k=1}^{n} \left( \mu_{ik} \right)^m \tag{3}
\]

We have to continue this process until

\[
\max_{ik} \left\{ \left( \mu_{ik}^{(t+1)} - \mu_{ik}^{(t)} \right) \right\} < \varepsilon, \quad \text{where} \quad \varepsilon \quad \text{is a termination criterion between 0 and 1, whereas} \quad \left( t + 1 \right) \quad \text{and} \quad t \quad \text{are the successive iteration steps.}
\]

1.3 Kernelized Fuzzy C Means Algorithm:

Another way to deal with these problems is to use kernel functions to project the data into a higher dimensional space so that the data could be separated more easily. An advantage of the latter method is that a so-called kernel trick can be used to transform linear algorithm into nonlinear one using inner product. Define a non-linear map by \( \psi : x \rightarrow \psi(x) \in F \), where \( x \in X \), \( X \) represents the input data space and \( F \) is the transformed feature space. In kernel trick, we replace the Euclidean distance \( \| x_k - v_i \| \) with \( \| \psi(x_k) - \psi(v_i) \|^2 \).

Accordingly, the objective function in equation (1) in the feature space can be rewritten as:

\[
J_m = \sum_{i=1}^{c} \sum_{k=1}^{n} \left( \mu_{ik} \right)^m \| \psi(x_k) - \psi(v_i) \|^2 \tag{4}
\]

Here \( \| \psi(x_k) - \psi(v_i) \|^2 \) can be calculated using the kernel function in the input space as follows:

\[
\| \psi(x_k) - \psi(v_i) \|^2 = GK(x_k, x_k) + GK(v_i, v_i) - 2 GK(x_k, v_i) \tag{5}
\]

where

\[
GK(x, y) = \langle \psi(x), \psi(y) \rangle = \psi(x)^T \cdot \psi(y)
\]

is an inner product kernel function. If we adopt the Gaussian function as a kernel function[8, i.e.,
\[ G(K(x, y)) = \exp\left(-\frac{\|x - y\|^2}{\sigma^2}\right), \] 
then

\[ G(K(x, x)) = 1. \] 
Then equation (5) becomes

\[ \left\| \psi(x_k) - \psi(v_i) \right\|^2 = 2 \left(1 - G(K(x_k, v_i))\right) \] 
(6)

Using (6), equation (4) can be rewritten as

\[ J_m = 2 \sum_{i=1}^{c} \sum_{k=1}^{n} (\mu_{ik})^m (1 - G(K(x_k, v_i))) \] 
(7)

Following the standard FCM algorithm, for minimizing the objective function \( J_m \), we have to consider first order derivatives of \( J_m \) with respect to \( \mu_{ik} \) and \( v_i \) equal to zero and accordingly, for its local extrema, we obtain

\[ \mu_{ik} = \frac{1}{\sum_{j=1}^{c} (1 - G(K(x_k, v_j)))^{-1} (m-1)} \] 
\[ \forall \; i = 1, 2, \ldots, c, \; k = 1, 2, \ldots, n \] 
(8)

\[ \nu_i = \frac{\sum_{k=1}^{n} (\mu_{ik})^m G(K(x_k, v_i)) x_k}{\sum_{k=1}^{n} (\mu_{ik})^m G(K(x_k, v_i))} \] 
\[ \forall \; i = 1, 2, \ldots, c \] 
(9)

We have to continue this process until

\[ \left| \mu_{ik}^{(t+1)} - \mu_{ik}^{(t)} \right| < \varepsilon \]
where \( \varepsilon \) is a termination criterion between 0 and 1, whereas \( t + 1 \) and \( t \) are the successive iteration steps.

1.4 Intuitionistic Fuzzy Set:

An Intuitionistic fuzzy set (IFS) in \( X \) can be defined as:

\[ A = \{(x, \mu_{IFS}(x), \nu_{IFS}(x), \Pi_{IFS}(x)) : x \in X\}, \]

where \( \mu_{IFS}(x) + \nu_{IFS}(x) + \Pi_{IFS}(x) = 1 \) with the condition

\[ 0 \leq \mu_{IFS}(x) + \nu_{IFS}(x) \leq 1 \]
for each \( x \in X \). Here, \( \mu_{IFS}(x) \) denotes the degree of membership, \( \nu_{IFS}(x) \) denotes the degree of nonmembership and \( \Pi_{IFS}(x) \) denotes the degree of hesitation of each \( x \in X \). For each \( x \in X \), \( \mu_{IFS}(x) \) will be calculated using formula given in the equation (8).

The non-membership function is computed by using the Sugeno type intuitionistic fuzzy generator as follows:

\[ \nu_{IFS}(x) = \frac{1 - \mu_{IFS}(x)}{1 + \rho \mu_{IFS}(x)}, \; \rho > 0 \] 
(10)

The hesitation degree is calculated as follows:

\[ \Pi_{IFS}(x) = 1 - \mu_{IFS}(x) - \frac{1 - \mu_{IFS}(x)}{1 + \rho \mu_{IFS}(x)} \] 
(11)

It is seen that the non membership term \( \frac{1 - \mu_{IFS}(x)}{1 + \rho \mu_{IFS}(x)} \) is less than \( 1 - \mu_{IFS}(x) \) due to the denominator greater than 1.

In order to incorporate intuitionistic fuzzy property in conventional fuzzy clustering algorithm, cluster centers are updated in the following manner.

Hesitation degree is initially calculated using equation (11) and the intuitionistic fuzzy membership values (incorporating hesitation degree) are obtained as follows:

\[ \mu_{ik}^* = \mu_{ik} + \Pi_{ik} \] 
(12)

where \( \mu_{ik} \) \( (\mu_{ik}) \) denotes the intuitionistic (conventional) fuzzy membership of \( k^{th} \) data in \( i^{th} \) class. Now, substituting eq. (12) in eq. (3), that is, conventional fuzzy c means method, the modified cluster center is obtained as:

\[ \nu_i^* = \frac{1}{n_i^*} \sum_{k=1}^{n} (\mu_{ik}^*)^m x_k, \] 
where

\[ n_i^* = \sum_{k=1}^{n} (\mu_{ik}^*)^m \] 
(13)
We have to continue this process until
\[
\max_{i,k} \left\{ \left| \mu_{ik}^{*(t+1)} - \mu_{ik}^{*(t)} \right| \right\} < \varepsilon, \text{ where } \varepsilon \text{ is a termination criterion between 0 and 1, whereas } (t+1) \text{ and } t \text{ are the successive iteration steps.}
\]

1.5 Incorporation of Kernel function:
For updating cluster centre using Kernel function, we first calculate \( \mu_{ik} \) using equation (8), then we go to find \( \mu_{ik}^{*} \) using equation (12). Thereafter replacing \( \mu_{ik} \) by \( \mu_{ik}^{*} \) in equation (9), \( v_{i}^{*} \), the modified cluster center will be obtained. Again, this process will be continued until
\[
\max_{i,k} \left\{ \left| \mu_{ik}^{*(t+1)} - \mu_{ik}^{*(t)} \right| \right\} < \varepsilon, \text{ where } \varepsilon \text{ is a termination criterion between 0 and 1, whereas } (t+1) \text{ and } t \text{ are the successive iteration steps.}
\]

1.6 Type-2 Fuzzy C-Means (T2FCM):
Calculating \( \mu_{ik} \) by equation (2), we will modify \( \mu_{ik} \) by the following equation:
\[
\alpha_{ik} = \mu_{ik} - \frac{1 - \mu_{ik}}{2} \quad (14)
\]
where \( \alpha_{ik} \) is considered as Type-II fuzzy membership of \( k \)-th data in \( i \)-th class. Then cluster centers \( \lambda_{i}^{*} \)'s are obtained by updating in each iteration as follows:
\[
\lambda_{i}^{*} = \frac{1}{\eta_{i}} \sum_{k=1}^{n} \left( \alpha_{ik} \right)^{m} x_{k}, \quad \text{where}
\]
\[
\eta_{i} = \sum_{k=1}^{n} \left( \alpha_{ik} \right)^{m} \quad (15)
\]

1.7 Incorporation of Kernel function in T2IFCM:
Here we use the kernelized type II intutionistic fuzzy Cmean where it is found that it is more effective in medical image segmentation and it works well for noisy image as well. There are several algorithms to segment the MRI images .We can finally see that with the help of our proposed method we can have most accurate clustering methods to segment the brain images better than other clustering methods.

Step 1. Calculate \( \mu_{ik} \) using equation (8).
Step 2. Find \( \alpha_{ik} \) using equation (14).
Step 3. Calculate \( \Pi_{ik} \) using following equation
\[
\Pi_{ik} = 1 - \alpha_{ik} - \frac{1 - \alpha_{ik}}{1 + \rho \alpha_{ik}} \quad (16)
\]
Step 4. Calculate \( \alpha_{ik}^{*} \) with the help of the following equation
\[
\alpha_{ik}^{*} = \alpha_{ik} + \Pi_{ik} \quad (17)
\]
Step 5. Obtain modified cluster centers \( \lambda_{i}^{*} \) using following equation
\[
\lambda_{i}^{*} = \frac{\sum_{k=1}^{n} \left( \alpha_{ik}^{*} \right)^{m} GK(x_{k}, \nu_{i}) x_{k}}{\sum_{k=1}^{n} \left( \alpha_{ik}^{*} \right)^{m} GK(x_{k}, \nu_{i})}.
\]
Step 6. This process will be continued until
\[
\max_{i,k} \left\{ \left| \alpha_{ik}^{*(t+1)} - \alpha_{ik}^{*(t)} \right| \right\} < \varepsilon, \text{ where } \varepsilon \text{ is a termination criterion between 0 and 1, whereas } (t+1) \text{ and } t \text{ are the successive iteration steps.}
\]

2. Performance Evaluation:
For performance evaluation first we have calculated true positive, true negative, false negative and false positive. From this sensitivity, specificity, accuracy are being calculated.

Let say,
\[ P = \text{Original MRI image of brain tumor and } S = \text{Segmented MRI image of brain tumor.} \]
\[ m \times n \text{ is the size of the image, where } m= \text{number of rows and } n= \text{number of Column.} \]
True Positive, True Negative, False Positive and False Negative
Let, $T_p$, $T_n$, $F_p$, $F_n$ are the true positive, true negative, false positive and false negative values respectively.

Step 1: $T_p = 0$.

Step 2: for $i = 0$ to $(m - 1)$ step of 1.

Step 3: for $k = 0$ to $(n - 1)$ step of 1.

Step 4: If $P(i, k) = 1$ and $S(i, k) = 1$ then $T_p = T_p + 1$.

Else if $P(i, k) = 0$ and $S(i, k) = 0$ then $T_n = T_n + 1$.

Else if $P(i, k) = 0$ and $S(i, k) = 1$ then $F_p = F_p + 1$.

Else if $P(i, k) = 1$ and $S(i, k) = 0$ then $F_n = F_n + 1$.

Step 5: print $T_p$, $T_n$, $F_p$, $F_n$.

Sensitivity, specificity, accuracy can be calculated by using $T_p$, $T_n$, $F_p$, $F_n$.

Sensitivity = \[
\frac{T_p}{T_p + F_n}
\]  \hspace{1cm} (18)

Specificity = \[
\frac{T_n}{T_n + F_p}
\]  \hspace{1cm} (19)

Accuracy = \[
\frac{T_p + T_n}{T_p + T_n + F_p + F_n}
\]  \hspace{1cm} (20)

3. Experimental Results and Simulation Study:

At first we have read the image then we have run program on the images. After running the program we have written the matrix in the text file. Then we convert matrix to images using irfanview. The fig.1. Shows the input MRI brain tumor images and segmented output images. TABLE. I show the results of performance evaluation and feature extraction.

Performance Evaluation Parameters 1-TP, TN, FP, FN

<table>
<thead>
<tr>
<th>ALGORITHM</th>
<th>SENSITIVITY(%)</th>
<th>SPECIFICITY(%)</th>
<th>ACCURACY(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCM</td>
<td>100.00</td>
<td>87.002</td>
<td>87.48</td>
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<tr>
<td>Type-II</td>
<td>44.95</td>
<td>100.00</td>
<td>98.14</td>
</tr>
<tr>
<td>KFCM</td>
<td>50.78</td>
<td>100.00</td>
<td>98.17</td>
</tr>
<tr>
<td>TYPE-II-KFCM</td>
<td>51.27</td>
<td>100.00</td>
<td>98.19</td>
</tr>
<tr>
<td>TYPE-II_KFCM-INS</td>
<td>51.69</td>
<td>100.00</td>
<td>98.21</td>
</tr>
</tbody>
</table>

FIGURE 1: It is given at the end.

GRAPH 1:

Performance Evaluation Parameters 1-TP, TN, FP, FN
### Table 2: Required running time and iterations for each algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Running Time</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy-C-Mean</td>
<td>6.003 Sec</td>
<td>4</td>
</tr>
<tr>
<td>Type II Fuzzy C Mean</td>
<td>5.471 Sec</td>
<td>12</td>
</tr>
<tr>
<td>Kernelized Fuzzy C Mean</td>
<td>9.248 Sec</td>
<td>10</td>
</tr>
<tr>
<td>Type II-Kernelized Fuzzy C Mean</td>
<td>7.953 Sec</td>
<td>8</td>
</tr>
<tr>
<td>Type II-Kernelized-Intuitionistic-Fuzzy-C-Mean</td>
<td>5.051 Sec</td>
<td>14</td>
</tr>
</tbody>
</table>

3. CONCLUSIONS

In this paper a new algorithm for segmentation of brain tumor MRI image has been proposed. At first we have done several segmentation methods and after that we have used Type II Kernelized Intuitionistic Fuzzy C mean for clustering purpose and compare them in terms of performance parameters. We have run this proposed segmentation algorithm on brain tumor MRI images and calculated sensitivity, specificity and accuracy. In conclusion, accuracy, specificity, sensitivity are found out to be 98.21%, 100% and 51.69% respectively.

REFERENCES


