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Fatigue Life Estimation of Machine Components

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Abstract - *The process of progressive localized permanent structural changes occurring in a material subjected to conditions that produce fluctuating stresses or strains at some point or points and that may culminate in cracks or complete fracture after a sufficient number of fluctuations when subjected to Bending, Axial, Torsion or combined equivalent stresses. Hence it is required to determine the life of the machine components to determine the number of cycles to failure and Fatigue factor of safety.*

***Key Words*: Fatigue, Fluctuating Stresses, Bending, Axial, Torsion, Structural, Factor of Safety.**

1. INTRODUCTION

Fatigue Failure: Often machine members subjected to such repeated or cyclic stressing are found to have failed even when the actual maximum stresses were below the ultimate strength of the material, and quite frequently at stress values even below the yield strength. The most distinguishing characteristics are that the failure had occurred only after the stresses have been repeated a very large number of times. Hence the failure is called fatigue failure.

Fatigue Failure Stages

Thus three stages are involved in fatigue failure namely,

- Crack initiation.
- Crack propagation.
- Fracture.

1.1 Fatigue life Evaluation

There are three approaches to fatigue life evaluation:

1. Stress-life (S-N) approach.
2. Strain-life ($\mathcal{E}$-N) approach.
3. Linear Elastic Fracture Mechanics (LEFM) approach.

1.2 Stress life approach

To understand the phenomena of fatigue failure a systematic study has been conducted by a German railway engineer A. Wohler by testing axles to failure in the laboratory under fully reversed loading and his work lead to the existence of a relation between applied stress and the number of cycles to failure. This relation or the S-N diagram became the standard way to characterize the behavior of materials under cyclic stressing, and evaluate the fatigue strength of materials.

S-N diagram

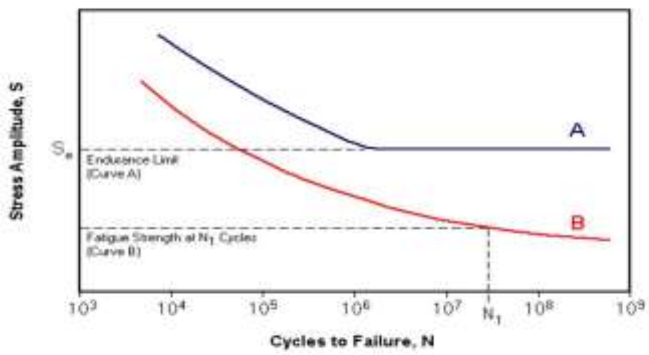


Fig 1: Typical S-N curves.

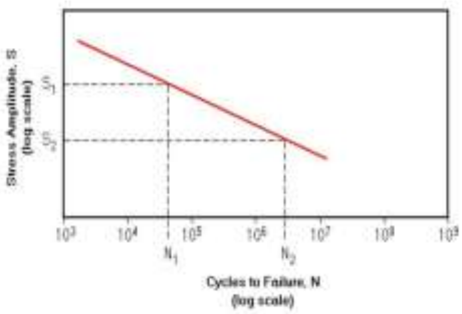


Figure 2: Idealized S-N Curve.

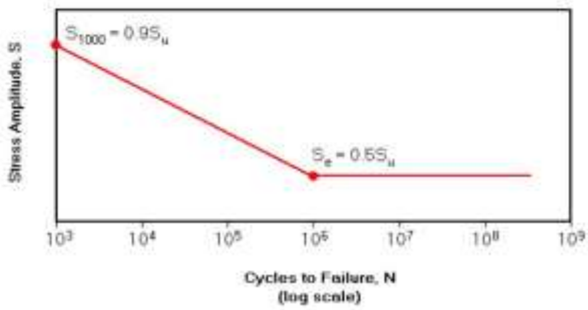


Figure 3: Generalized S-N Curve for Wrought Steels.

The basis of the Stress-Life method is the Wohler S-N diagram, shown schematically for two materials in Figure 1. The S-N diagram plots nominal stress amplitude S versus cycles to failure N. There are numerous testing procedures to generate the required data for a proper S-N diagram. S-N test data are usually displayed on a log-log plot, with the actual S-N line representing the mean of the data from several tests.

S-N curves obtained under torsion or bending load control test conditions often do not have data at the shorter fatigue lives (say less than 103 cycles) due to significant plastic deformation.

2. Power relationship

When plotted on a log-log scale, an S-N curve can be approximated by a straight line as shown in Figure 2. A power law equation can then be used to define the S-N relationship:

$$S_{nf} = A\,(N_f)^{b} \qquad (1)$$

Or $$S_f = Su\,(N_f)^{b} \qquad (2)$$

Or $$S_f = 10^{c}\,(N)^{b} \qquad (3)$$

Where,

$$C = \mathrm{Log}_{10}\left(\frac{S_1^{\,2}}{S_e}\right)$$ and

$$b = \left(-\frac{1}{3}\right)\mathrm{Log}_{10}\left(\frac{S_1}{S_e}\right) \qquad (4)$$

b is the slope of the line sometimes referred to as the Basquin slope.

Given the Basquin slope and any coordinate pair (N, S) on the S-N curve, the power law equation calculates the cycles to failure for a known stress amplitude and is given by,

$$N = 10^{\left(-\frac{c}{b}\right)} \qquad (5)$$

The power relationship is only valid for fatigue lives that are on the design line. For ferrous metals this range is from 10^3 to 10^6cycles. For non-ferrous metals, this range is from $1x10^3$ to $5x10^8$ cycles.

Fatigue Ratio (Relating Fatigue to Tensile Properties)

The ratio of the endurance limit Se to the ultimate strength Su of a material is called the fatigue ratio. It has values that range from 0.25 to 0.60, depending on the material.

For steel, the endurance strength can be approximated by:

$S_e^{1}{}_{,\,Steel} \approx 0.5Su$ for Su <1400 Mpa

$S_e^{1}{}_{,\,Steel} \approx 700$ Mpa for Su >1400 Mpa

In addition to this relationship, for wrought steels the stress level corresponding to 1000

Cycles, S1000, can be approximated by:

$$S_{1000,\,Steel} = 0.9Su \qquad (6)$$

Utilizing these approximations, a generalized S-N curve for wrought steels can be created by connecting the S1000 point with the endurance limit, as shown in Figure 3.

3. Mean Stress and Alternating Stress

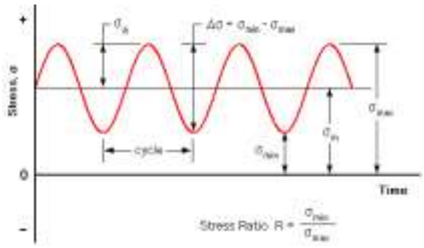


Figure 4: Typical Cyclic Loading Parameters..

Cyclical loading can be thought of as a fully reversed alternating stress superimposed on a constant mean stress level as shown in figure 3.4. The mean stress is the arithmetic mean of the maximum stress and the minimum stress. The alternating stress, also known as the stress amplitude is then the difference between the peak stresses and the mean stress. The equations for calculating the mean stress and alternating stress are as follows:

$$\sigma_{Mean} = \frac{\sigma_{max}+\sigma_{min}}{2} \qquad (7)$$

$$\sigma_{alternating} = \frac{\sigma_{max}-\sigma_{min}}{2} \qquad (8)$$

4. Haigh diagram

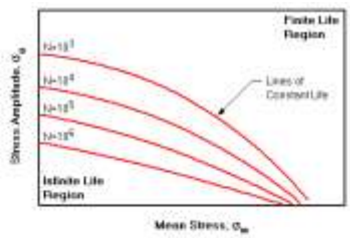


Figure 5: Haigh diagram.

The results of a fatigue test using a nonzero mean stress are often presented in a Haigh diagram, shown in Figure 3.5. A Haigh diagram plots the mean stress, usually tensile, along the x-axis and the oscillatory stress amplitude along the y-axis. Lines of constant life are drawn through the data points. If the applied stress level is below the endurance limit, the component is said to have infinite life. If the applied stress level is above the endurance limit, the component is said to have finite life. The infinite life region is the region under the curve and the finite life region is the region above the curve. For finite life calculations the endurance limit in any of the models can be replaced with a fully reversed (R = -1) alternating stress level corresponding to the finite life value i.e. S_{nf} can be replaced to S_f.

A very substantial amount of testing is required to generate a Haigh diagram, and it is usually impractical to develop curves for all combinations of mean and alternating stresses. Several empirical relationships that relate alternating stress to mean stress have been developed to address this difficulty. These methods define various curves to connect the endurance limit on the alternating stress axis to either the yield strength, Sy, ultimate strength Su, or true fracture stress S_f on the mean stress axis. The following relations are available in the Stress-Life module:

Goodman (England, 1899) : $\frac{S_a}{S_f} + \frac{S_m}{S_u} = 1$ (9)

Gerber (Germany, 1874) : $\frac{S_a}{S_f} + \left(\frac{S_m}{S_u}\right)^2 = 1$ (10)

Soderberg (USA, 1930) : $\frac{S_a}{S_f} + \frac{S_m}{S_y} = 1$ (11)

Morrow (USA, 1960s) : $\frac{S_a}{S_f} + \frac{S_m}{\sigma_f} = 1$ (12)

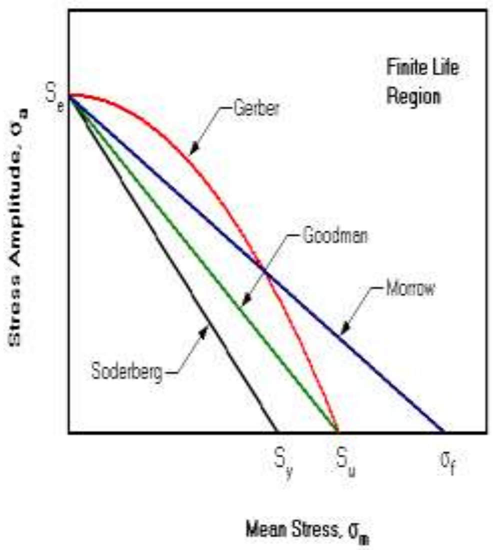


Figure 6: Comparison of Mean Stress Equations.

A graphical comparison of these equations is shown in Figure 6. The Goodman equation was the first attempt at coming up with a relationship between fatigue and material properties, using the tensile strength. This was followed by a slightly more accurate Gerber equation, also using the tensile strength, and the slightly less accurate Soderberg equation, using the yield strength. The Morrow equation improves the accuracy more by using the true fracture stress instead of the tensile strength. The two most widely accepted methods are those of Goodman and Gerber. Experience has shown that test data tends to fall between the Goodman and Gerber curves. Goodman is often used due to mathematical simplicity and slightly conservative values.

5. Constant fatigue life diagram

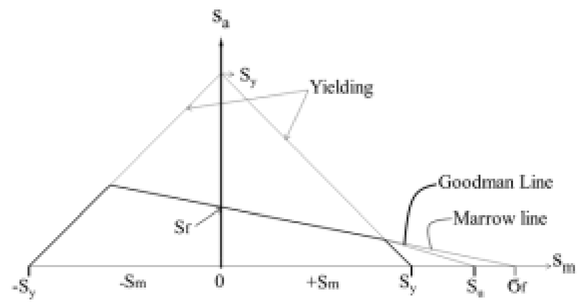


Figure 7: Fatigue and yielding criteria for constant life of unnotched parts.

Constant fatigue life diagrams relating Sa and Sm are often modelled is as shown schematically in figure 7. In fig 7, the intercepts, Sf, at Sm = 0 for a given line found from the fully reversed, R = -1, modelled S-N curves. In Fatigue design with constant amplitude loading and unnotched parts, if the coordinates of the applied alternating and mean stresses fall within the Modified Goodman and Marrow lines shown in the figure 7, then fatigue failure should not occur prior to the given life. Note that the difference between the Modified Goodman and Marrow equations is often rather small; thus either model often provides similar results. If yielding is not to occur, then the applied alternating and mean stresses must fall within the two yield lines connecting ±Sy. If both fatigue failure and yielding are not to occur, then neither criterion, as indicated by the three bold lines in the figure 7, should be exceeded.

6. Modified Goodman Diagram

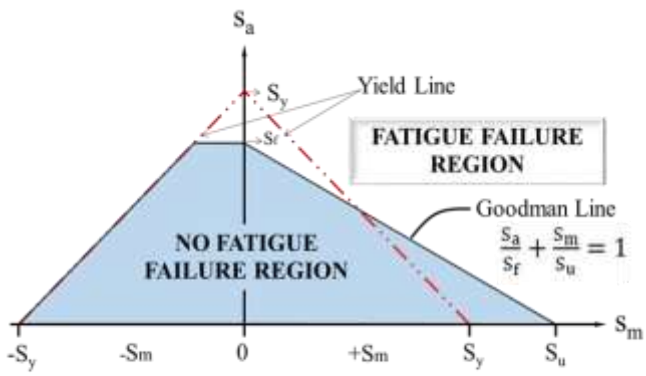


Figure 8: Modified Goodman chart.

The Modified Goodman diagram shown in Figure .8, account for yielding as a failure mode. With alternating stress on the y axis and mean stress on the x axis, we draw a straight line from the positive fatigue strength to the coordinate equivalent to the tensile strength on both axes. The Goodman equation is given as,

$$\frac{S_a}{S_f} + \frac{S_m}{S_u} = 1$$

7. Procedure to find the life of the component

1) If the component is circular get the diameter of the component.
2) Calculate either of the below from preliminary calculations.
   i) Torsional moment (Tmax &Tmin, N-mm).
   ii) Bending moment (Mmax &Mmin, N-mm).
   iii) Axial force (Fmax &Fmin, N).
3) Calculate the stresses:

(a) Torsional Stress: $Tmax = \frac{16Tmax}{\pi * d^3}$ (13)

$Tmin = \frac{16Tmin}{\pi * d^3}$ (14)

(b) Bending Stress: $Mmax = \frac{32Mmax}{\pi * d^3}$ (15)

$Mmin = \frac{32Mmin}{\pi * d^3}$ (16)

(c) Axial Stress: $Fmax = \frac{Fmax}{\frac{\pi * d^2}{4}}$ (17)

$Fmin = \frac{Fmin}{\frac{\pi * d^2}{4}}$ (18)

4) Calculate the alternating stress and mean stress:

(a) Torsion: $Ta = \frac{T_{max} - T_{min}}{2}$ (19)

$Tm = \frac{T_{max} + T_{min}}{2}$ (20)

(b) Bending: $Ma = \frac{M_{max} - M_{min}}{2}$ (21)

$Mm = \frac{M_{max} + M_{min}}{2}$ (22)

(c) Axial: $Sa = \frac{S_{max} - S_{min}}{2}$ (23)

$Sm = \frac{S_{max} + S_{min}}{2}$ (24)

5) Calculate the equivalent stress for the following combined loading

(a) For Bending and Axial:
Alternating stress (Bending + Axial)

$S = Ma + Sa$ (25)

Mean stress (Bending + Axial)

$S = Ma + Sa$ (26)

Principal stress $S_{1,2} = \frac{S}{2} \pm \sqrt{\left(\frac{S}{2}\right)^2 + \tau^2}$ (27)

Calculate the principle stresses $S_{1,2}$ and find equivalent stress. By von-mises stress equation

$S_{eq\,alt} = \sqrt{S_1^2 + S_2^2 - S_1 S_2}$ (28)

$S_{eq\,Mean} = \sqrt{S_1^2 + S_2^2 - S_1 S_2}$ (29)

Now calculate Smax and Smin by

$S_{max} = Seq_{Mean} + S_{eqalt}$ (30)

$S_{min} = S_{eq\,Mean} - S_{eqalt}$ (31)

(b) For Bending and Torsion:
Find the equivalent stress by using,

$M_{eq\,max} = \frac{1}{2}\left[M_{Max} + \sqrt{M_{Max}^2 + T_{Max}^2}\right]$ (32)

$T_{eq\,max} = \sqrt{M_{Max}^2 + T_{Max}^2}$ (33)

$M_{eqmin} \frac{1}{2}\left[M_{Min} + \sqrt{M_{Min}^2 + T_{Min}^2}\right]$ (34)

$T_{eq\,min} = \sqrt{M_{Min}^2 + T_{Min}^2}$ (35)

From von-mises stress equation,

$S_{Max} = \sqrt{M_{eq\,max}^2 + 3\tau_{eq\,max}^2}$ (36)

$S_{Min} = \sqrt{M_{eq\,min}^2 + 3\tau_{eq\,min}^2}$ (37)

6) From Goodman equation we have,

$\frac{S_a}{S_f} + \frac{S_m}{S_u} = 1$ (38)

7) Calculate fully reversed fatigue limit S_f:

$S_f = \frac{S_a}{1 - \frac{S_m}{S_u}}$ (39)

8) Find the number of cycles to failure[6]:
By power relationship we have,

$S_f = 10^c (N)^b$

Where,

$C = Log_{10}\left(\frac{S_1^2}{S_e}\right)$ (40)

$b = \left(-\frac{1}{3}\right) Log_{10}\left(\frac{S_1}{S_e}\right)$ (41)

$\mathbf{N = 10^{\left(-\frac{c}{b}\right)} * S_f^{\left(\frac{1}{b}\right)}}$ (42)

8. Procedural steps followed to find the number of cycles to failure

1. The component is circular and the diameter of the component is,
   Diameter = 30 mm.
2. Calculate either of the below from preliminary calculations:
   i. Torsional moment
      Tmax = 1590500 N-mm
      Tmin = 530143 N-mm
   ii. Bending moment
      Mmax = 530143 N-mm
      Mmin = 397607 N-mm
   iii. Axial force

Fmax = 282743 N
Fmin = 353429 N

3. Calculate the stresses:
   i. Torsional Stress:
   From equation (13) & (14)
   Tmax = 300 Mpa
   Tmin = 100 Mpa
   ii. Bending Stress:
   From equation (15) & (16)
   Mmax = 200 Mpa
   Mmin = 150 Mpa
   iii. Axial Stress:
   From equation (17) & (18)
   Smax = 400 Mpa
   Smin = 500 Mpa

4. Calculate the alternating stress and mean stresses:
   i. Torsion:
   From equation (19) & (20)
   Ta = 100 Mpa
   Tm = 200 Mpa
   ii. Bending:
   From equation (20) & (21)
   Ma = 25 Mpa
   Mm = 175 Mpa
   iii. Axial:
   From equation (21) & (22)
   Sa = -50 Mpa
   Sm = 450 Mpa

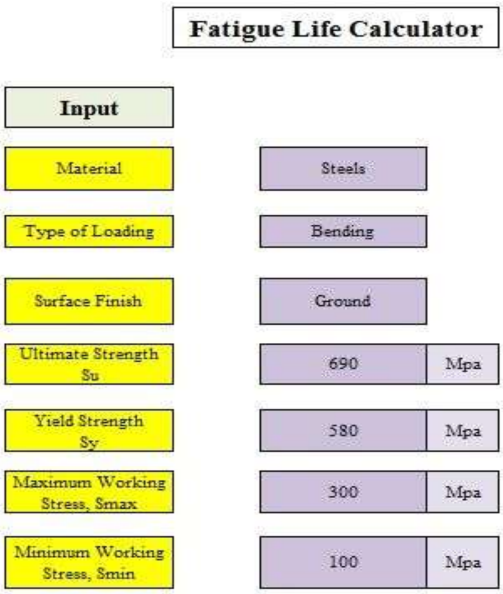
5. Calculate the equivalent stress for the following combined loading
   i. For bending and axial:
   From equation (25)
   Alternating stress (Bending + Axial)
   Sa = -25 Mpa
   From equation (26)
   Mean stress (Bending + Axial)
   Sm = 625 Mpa
   From equation (27)
   Principal stress
   Sa_1 = 88.27 Mpa
   Sa_2 = -113.2 Mpa
   Sm_1 = 683.5 Mpa
   Sm_2 = -58.52 Mpa
   By von-mises stress equation (28) & (29)
   $S_{eq\ alt}$ = 174.9 Mpa
   $S_{eq\ Mean}$ = 714.4 Mpa
   Now calculate Smax and Smin from equation (30) & (31)
   S_{max} = 889.4 Mpa
   S_{min} = 539.5 Mpa
   ii. For bending and torsion:
   From equation (32)
   $M_{eq\ max}$ = 280.2 Mpa
   From equation (33)
   $T_{eq\ max}$ = 360.5 Mpa
   From equation (34)
   $M_{eq\ min}$ = 165.13 Mpa
   From equation (35)
   $T_{eq\ min}$ = 180.27 Mpa
   From von-mises stress equation, (36) & (37)
   S_{Max} = 684.44 Mpa
   S_{Min} = 353.18 Mpa

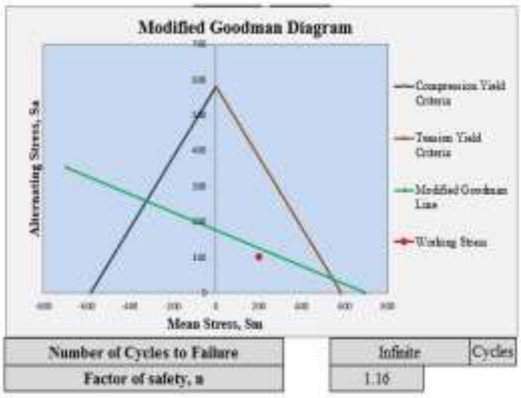
6. Get the following inputs from any of the above calculations to find the number of cycles to failure for example consider the given data below.
   i. Maximum Stress
   Smax = 300 Mpa
   ii. Minimum stress
   Smin = 100 Mpa
   iii. Ultimate strength
   Su = 690 Mpa
   iv. Yield strength
   Sy = 580 Mpa
   v. Endurance strength
   Se = 175.6 Mpa
   From equation (23) & (24)
   Sa = 100 Mpa
   Sm = 200 Mpa

7. Calculate fully reversed fatigue limit S_f from equation (39)
   Sf = 140.81 Mpa

8. Find the number of cycles to failure from equation (42):
   From equation (40) & (41)
   c = 3.341
   b = -0.1828
   Therefore number of cycles to failure from equation (42)
   N = 3.33*10^6 cycles

9. Results and Discussions





The material used may be either steel, cast iron, cast steel, wrought aluminium alloy or cast aluminium alloy and the surface finish of the material may be either Ground, Machined /cold drawn, hot rolled or As-forged. The component used is subjected to Bending, Axial or Torsional loads. The maximum and minimum working stress is found by preliminary calculation or stress analysis. From the above inputs the following details can be obtained,

1. The Number of cycles to failure, N in cycles.
2. If the working stress falls below the Modified Goodman line, the component is said to have infinite number of cycles.
3. If the working stress falls above the Modified Goodman line, the component is said to have finite number of cycles.

Finally from the above calculation we can conclude that the number of cycles to failure for the steel component is infinite and the fatigue factor of safety n, required is 1.16.

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