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Analysis of in-plane and Transverse Stresses of laminated Composite Plate for FSDT using FEM Approach

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Abstract - *Finite Element Analysis is carried out to perform static analysis on a laminated composite plate based on the First Order Shear Deformation Theory (FSDT). The theory accounts for variation of in-plane displacement, in-plane stress, in-plane shear stress, transverse displacement and transverse shear stress across the thickness of the laminate. The element formulated is a 4-noded isoparametric quadrilateral element having 5 degrees of freedom at each node. In this analysis, the square plate is analyzed for uniformly distributed load under simply supported boundary conditions. A program is written in MATLAB to obtain the finite element solutions for in-plane displacements, in-plane stresses, in-plane shear stress, transverse displacement and transverse shear stress. Solutions are obtained for different number of layers of the laminate with cross-ply orientation for different values of side to thickness ratios. The model is validated with the analytical results. Analysis can be done on thin as well as thick plates satisfactorily by using this model.*

***Key Words*: FSDT, Finite Element Method, Laminated plate, MATLAB software, Stress analysis**.

1. INTRODUCTION

Composite materials have fascinating properties, for example, high quality to weight proportion, simplicity of manufacture, great electrical and thermal properties compared with metals. A laminated composite material comprises of a few layers of a composite mixture comprising of matrix and filaments. Each layer may have comparative or unique material properties with various fiber orientation under differing stacking succession. Configuration design must consider a few options, for example, best stacking succession, ideal fiber points in each layer and in addition number of layers itself.

The Finite Element Method is such an approximate method and powerful numerical technique for the solution of differential and integral equations that arise in various fields of engineering and applied science. FEM is an effective method of obtaining numerical solutions to boundary value, initial value problems.

2. PROPOSED METHODOLOGY

a. The mathematical formulation by means of First Order Deformation Theory [FSDT] is formulated.
b. Finite Element formulation for presented FSDT theory is formulated.
c. Computer program utilizing MATLAB programming is coded, in light of Finite Element formulation.
d. The investigation on displacement parameter is completed using MATLAB programming for different sides to thickness ratio, aspect ratio and different number of laminas for static stacking condition.
e. For validation, these outcomes are compared with standard research papers by Pandya and Kant and J.N. Reddy.
f. Once the validation is done, the parametric investigation is made by changing the properties of the material and geometry

3. FORMULATIONS

3.1 FSDT Formulations

The present first order deformation theory for laminated composite plate has been developed by assuming the displacement field in the following form: -

$$u(x,y,z) = u_0(x,y) + Z\theta x(x,y)$$

$$v(x,y,z) = v_0(x,y) + Z\theta y(x,y)$$

$$w(x,y,z) = w_0(x,y)$$

Where (u_0, v_0, w_0) are the displacement components in the direction of x, y, z respectively of a point on the mid-plane (i.e., z=0)
The strains associated with displacements (u_0, v_0, w_0) are

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \mu_{21}\frac{\sigma_2}{E_2} - \mu_{31}\frac{\sigma_3}{E_3}$$

$$\varepsilon_2 = \frac{\sigma_2}{E_2} - \mu_{32}\frac{\sigma_3}{E_3} - \mu_{12}\frac{\sigma_1}{E_1}$$

$$\varepsilon_3 = \frac{\sigma_3}{E_3} - \mu_{13}\frac{\sigma_1}{E_1} - \mu_{23}\frac{\sigma_2}{E_2}$$

$$(\varepsilon_4, \varepsilon_5, \varepsilon_6) = (\gamma_{12}, \gamma_{23}, \gamma_{13})$$

$$(\sigma_4, \sigma_5, \sigma_6) = (\tau_{12}, \tau_{23}, \tau_{13})$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}};\ \gamma_{23} = \frac{\tau_{23}}{G_{23}};\ \gamma_{13} = \frac{\tau_{13}}{G_{13}}$$

Where,

ε_1, ε_2, ε_3 are strains along x, y, z directions respectively
σ_1, σ_2, σ_3 are stresses along x, y, z directions respectively
E_1, E_2, E_3 are young's modulus of elasticity (τ_{12}, τ_{23}, τ_{13})
, (γ_{12}, γ_{23}, γ_{13}) are shear stress and shear strain respectively
The constitutive relations can be written as

$$\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_3\\\tau_{12}\\\tau_{23}\\\tau_{13}\end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix} * \begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\\\gamma_{12}\\\gamma_{23}\\\gamma_{13}\end{Bmatrix}$$

Where,

$$C_{11} = \frac{E_1(1-\mu_{23}\mu_{32})}{\Delta};\quad C_{22} = \frac{E_2(1-\mu_{13}\mu_{31})}{\Delta};$$
$$C_{12} = \frac{E_1(\mu_{21}+\mu_{31}\mu_{23})}{\Delta};\quad C_{23} = \frac{E_2(\mu_{32}+\mu_{12}\mu_{31})}{\Delta};$$
$$C_{13} = \frac{E_1(\mu_{31}+\mu_{21}\mu_{32})}{\Delta};\quad C_{33} = \frac{E_3(1-\mu_{12}\mu_{21})}{\Delta};$$

Where,

$$\Delta = (1-\mu_{12}\mu_{21} - \mu_{23}\mu_{32} - \mu_{31}\mu_{13} - 2\mu_{12}\mu_{23}\mu_{31})$$

The general transformation matrix 'T' can be written as

$$T = \begin{bmatrix} c^2 & s^2 & 0 & 2sc & 0 & 0\\ s^2 & c^2 & 0 & -2sc & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ -sc & sc & 0 & (c^2-s^2) & 0 & 0\\ 0 & 0 & 0 & 0 & c & -s\\ 0 & 0 & 0 & 0 & s & c\end{bmatrix}$$

Where,

$$c = \cos\alpha, s = \sin\alpha$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 2 & 0 & 0\\ 0 & 0 & 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 0 & 0 & 2\end{bmatrix}$$

Stress strain constitutive relations with references to laminate axes are obtained in the following form;

$$\sigma = T^{-1}GRTR^{-1}\varepsilon$$

$$\sigma = Q\varepsilon$$

Where,

$$Q = T^{1}GT^{-1^t}$$

$$T^{-1^t} = RTR^{-1}$$

$$\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_z\\\tau_{xy}\\\tau_{yz}\\\tau_{xz}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & 0 & 0\\ Q_{21} & Q_{22} & Q_{23} & Q_{24} & 0 & 0\\ Q_{31} & Q_{32} & Q_{33} & Q_{34} & 0 & 0\\ Q_{41} & Q_{42} & Q_{43} & Q_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & Q_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & Q_{66}\end{bmatrix} * \begin{Bmatrix}\varepsilon_x\\\varepsilon_y\\\varepsilon_z\\\gamma_{xy}\\\gamma_{yz}\\\gamma_{xz}\end{Bmatrix}$$

3.2 Finite Element Formulations

In FEM, the physical domain is divided into number of sub-regions, which are named as finite segments.
Geometry of 4-noded element is given by expressions:

$$x = \sum_{i=1}^{n} N_i x_i = N_1 x_1 + N_2 x_2 + \ldots\ldots. + N_n x_n = [N]\{x\}$$

$$y = \sum_{i=1}^{n} N_i y_i = N_1 y_1 + N_2 y_2 + \ldots\ldots. + N_n y_n = [N]\{y\}$$

Where, N_i is the shape functions of 4-noded quadrilateral element and(x_i,y_i) are nodal coordinates of element.

4. NUMERICAL EXAMPLE

Numerical results for various composite plates are presented with different cross-ply lamina schemes under simply supported boundary conditions
The plate having the following characteristics is considered [1]: -

$$E_1 = 25E_2, G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2, v_{12} = 0.25$$

The shear correction co-efficient for the first-order theory is taken to be K=5/6.

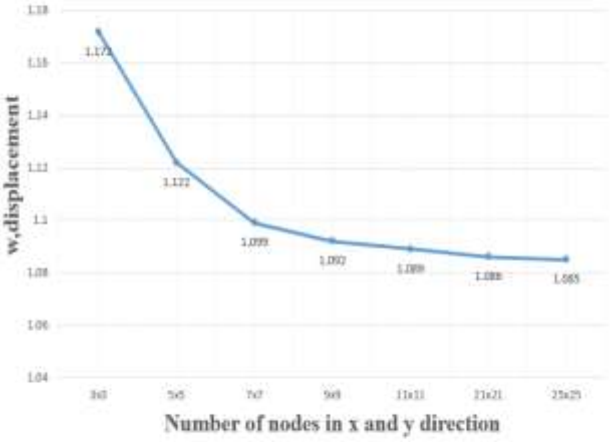

Chart -1: Plot of convergence w

Table-1: Comparison of maximum in-plane stress σ_x with analytical solutions

Number of layers	Fibre orientation	(a/h)	In-plane stress σ_x(present values)	In-plane stress σ_x (Analytical solution[1])	% error
3	0/90/0	10	0.76248	0.7719	1.220
3	0/90/0	100	0.803855	0.8072	0.414
5	0/90/0/90/0	10	0.762385	0.7649	0.328
5	0/90/0/90/0	100	0.821758	0.8264	0.562

Table-2: Comparison of maximum in-plane stress σ_y with analytical solutions

Number of layers	Fibre orientation	(a/h)	In-plane stress σ_y(present values)	In-plane stress σ_y (Analytical solution[1])	% error
2	0/90	10	1.076902	1.0715	0.504
2	0/90	100	1.075344	1.0761	0.070
3	0/90/0	10	0.327484	0.3072	6.602
3	0/90/0	100	0.194110	0.1925	0.836
5	0/90/0/90/0	10	0.539213	0.5525	2.404
5	0/90/0/90/0	100	0.456816	0.4559	0.200

Table-3: Comparison of maximum shear stress τ_{xy} with analytical solutions

Number of layers	Fibre orientation	(a/h)	Shear stress τ_{xy} (present values)	Shear stress τ_{xy} (Analytical solution[1])	% error
2	0/90	10	0.094822	0.0960	1.227
2	0/90	100	0.092993	0.0933	0.329
3	0/90/0	10	0.05206	0.0514	1.284
3	0/90/0	100	0.042365	0.0426	0.5516
5	0/90/0/90/0	10	0.041523	0.0436	4.763
5	0/90/0/90/0	100	0.038405	0.0386	0.505

Table-4: Comparison of maximum shear stress τ_{yz} with analytical solutions

Number of layers	Fibre orientation	(a/h)	Shear stress τ_{yz} (present values)	Shear stress τ_{yz} (Analytical solution[1])	% error
3	0/90/0	10	0.324043	0.3107	4.294
3	0/90/0	100	0.316273	0.2842	11.285
5	0/90/0/90/0	10	0.398840	0.4410	9.561
5	0/90/0/90/0	100	0.383358	0.4108	6.682

Table-5: Comparison of maximum shear stress τ_{xz} with analytical solutions

Number of layers	Fibre orientation	(a/h)	Shear stress τ_{xz} (present values)	Shear stress τ_{xz} (Analytical solution[1])	% error
2	0/90	10	0.620121	0.5772	7.436
2	0/90	100	0.623433	0.5813	7.248
3	0/90/0	10	0.799168	0.7548	5.877
3	0/90/0	100	0.372234	0.4247	12.353
5	0/90/0/90/0	10	0.799865	0.6901	15.905
5	0/90/0/90/0	100	0.429941	0.3746	14.773

5. CONCLUSIONS

In the present work, analysis of laminated composite plate is carried out using finite element method which is based on FSDT formulation. In view of this, MATLAB program is coded and analysis is completed. Results are validated with standard values from reference book [1]. And parametric study is done for stress values and non-dimensional displacements.

a. From the convergence study, it is seen that if higher noded element is used, then it will give better results. However, the displacement values are converging at mesh size of 11 x 11 nodes. The change is 0.27% from previous value.

b. When non dimensional maximum In-plane stress σ_y (for 2 layers), values from present work are compared with analytical solutions of J.N. Reddy reference book, error for thin plate (a/h=100) and thick plate is within 1%(Here we can see that error for shear stress is very less i.e. only 0.070%).

c. For two layers, errors of shear stresses τ_{xy} are less (i.e. within 1.5%) whereas errors of shear stresses τ_{xz} are relatively high for thick and thin plates.

d. When non dimensional maximum In-plane stress σ_x (for 3layers), values from present work are compared with analytical solutions of J.N. Reddy reference book, error for thin plate (a/h=100) and thick plate(a/h=10) is within 3.00%. Similarly, In-plane stress σ_x (for 5 layers), values from present values are compared with analytical solutions of J.N. Reddy reference book, error for thin plate (a/h=100) and thick plate(a/h=10) is within 1.00%.

e. Maximum In-plane stress σ_y (for 3layers), values from present work and analytical solutions [1]. i.e. error is less (i.e. within 7%)

f. When non dimensional max In-plane stress σ_y (for 5 layers), values present work are compared with analytical solutions of J.N. Reddy reference book, error for thin plate (a/h=100) and thick plate(a/h=10) is within 3.00%.

g. When non dimensional max shear stress τ_{xy} (for 3layers), values present work are compared with analytical solutions of J.N. Reddy reference book, error for thick plate (a/h=10) and thin plate (a/h=100) is within 1.5%.

h. When non dimensional max In-plane stress τ_{xy} (for 5 layers), values of present work are compared with analytical solutions of J.N. Reddy reference book, errors for thin plate (a/h=100) and thick plate are within 5.00%(Here we can see that error for shear stress of thin plate is very less i.e. only 0.5%).

i. For three layers, errors of shear stresses τ_{xz} and τ_{yz} are within 6% in case of thick plates whereas the error is high in case of thin plates.

j. For five layers, errors of shear stresses τ_{xz} and τ_{yz} are relatively high for thick and thin plates.

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