

Optimization Study of Fuzzy Parametric Uncertain System with Observer

Tejal D. Apale¹, Vaibhav H. Jadhav²

¹PG Student Dept. of Electrical Control System Engineering, Walchand College of engineering, Sangli, Maharashtra, India

²RTU SCADA Engineer, SCOPE, Pune

Abstract - This paper deals with the analysis and design of the optimal robust controller for the fuzzy parametric uncertain system. An LTI system in which coefficients depends on parameters described by a fuzzy function is called as fuzzy parametric uncertain system. This paper mainly studies uncertainty in the system. By optimal control design, we get control law and feedback gain matrix which can stabilize the system. The robust controller design is a difficult task so we go for the optimal control approach. The system can be converted into state space controllable canonical form with the α -cut property fuzzy. For optimal control design, we find control law and get the feedback gain matrix which can stabilize the system and optimizes the cost function. Stability analysis is done by using the Kharitonov theorem and Lyapunov-Popov method. The proposed method applied to a response of Continuous Stirred Tank Reactor (CSTR). To this system we applied an observer to control the system response. Linear regulator controller designs result in a state variable feedback law, so that implementing an optimal control requires measurement of all components of the state. In many situations such measurements are not possible and alternative approaches is needed. One such approach was originated by D. G. Luenberger, and is known as Luenberger observers.

Key Words: Fuzzy parametric uncertain system (FPUS), α -cut set, optimal control Kharitonov theorem, Lyapunov-Popov stability, Continuous Stirred Tank Reactor (CSTR), Disturbance Observer based controller (DOB).

1. INTRODUCTION

To design a proper controller for real time nonlinear system have many problems such as uncertainty, disturbance, unknown exact mathematical model etc. Uncertainty is either structure or Unstructured. Mostly the nonlinear system defined in terms of mathematical model doesn't have the exact parameter the calculated parameters are considered. So that type of model consists the parametric uncertainty. And because of that uncertain parameter, we have some information loss, which may be incomplete, unreliable. Also, the uncertainty affects the system response. So, we must design a controller such that it can deal with this type of uncertainty and remove the effects occurred because of that. For that, the fuzzy logic controller gives the best solution.

Many available system contain nonlinearity characteristics, such as microwave oscillation, chemical

process, hydraulic system, etc. It important to study behavior of nonlinear system. The most important part in nonlinear system is Optimization. The dynamic of non-linear system can be strongly depends on either one or more parameter since their operative condition remain stable only if the value of parameters are must be in specific limit. If these parameter gone out of range then the equilibrium point become unstable. Because of this reason, nonlinear controllers like Fuzzy logic controller are used to control such system because they are more robust than other controllers. [2] Fuzzy technique has been widely and effectively used now a days in nonlinear system modelling and control for more than two decades. In many of the model based fuzzy control approaches, the famous t-s fuzzy model is a popular and convenient tool in functional approximation. [3, 4] CSTR is one of the most commonly used non-linear system, which is mostly used in chemical industries, it offers a verity of researches in the area of chemical and control engineering. Due to non-linearity presents in the system, performance of the conventional controller may not be proper. Hence complexity of the system analysis increases. [6] It becomes difficult to have results under certain conditions. So here Type-1 fuzzy controller is used and optimized, for optimization, Jaya algorithm is used, because it is one of the best optimization algorithm which gives good performance and results.

The latter part of the paper is arranged in the following sequence. Section 2 presents type 1 fuzzy controller design and optimization for triangular membership functions and optimization by JAYA algorithm is presented in section 3.

The results of hardware implementation of controllers are presented in section 4. The main conclusions are reached through analysis of results.

2. FUZZY CONTROLLER DESIGN

Most of the available physical dynamical systems in real life, which are not possible to be represented by linear differential equations and have a nonlinear nature. On another side, linear control methods depends on the key assumption of small range of operation for the linear model and, acquired from linearizing the nonlinear system, to be valid. When the required operation range is large, a linear controller is unstable, because the nonlinearities in the plant cannot be properly dealt with the controller. One more assumption of the linear control is that the system model is

definitely linearizable and the linear model is much accurate enough for building up the controller. However, the highly nonlinear and discontinuous nature of many system for example, mechanical and electrical systems does not allow linear approximation practically. As In the process of designing controllers, it is also necessary that the system model is well achievable through a mathematical model and the parameters of the system model are reasonably well-known for controller design.

Significant performance degradation or even instability. The fuzzy model was proposed by Takagi and Sugeno [2] and it is described by fuzzy IF-THEN rules which represent local input-output relations of a nonlinear system. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy "blending" of the linear system models. Almost all nonlinear dynamical systems can be represented by Takagi-Sugeno fuzzy models to high degree of precision. In fact, it is proved that Takagi-Sugeno fuzzy models are universal approximation of any smooth nonlinear system.

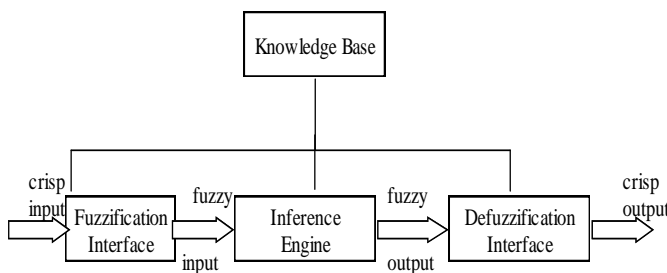


Fig -1: Graphical interpretation of fuzzy

3. OPTIMIZATION FOR TRIANGULAR MEMBERSHIP FUNCTIONS

Here we have to optimize three membership functions one by one using the JAYA optimization technic using MATLAB Code. For that purpose we have to write m files for fitness function of each membership function and decide the bounds of the base for each membership function. This step is important because there are two types of bad membership function. First one type is too redundant and second type is too separated. Due to such membership function we cannot get the desired response or output as we want. Too redundant and too separated membership functions are shown in below Figure,

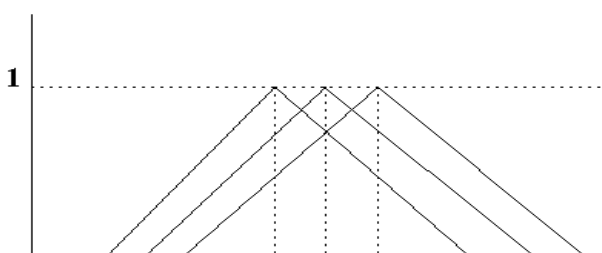


Fig -2: Too redundant membership function

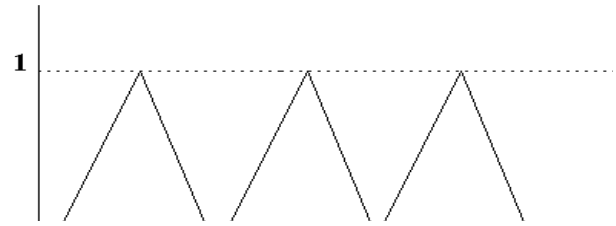


Fig -3: Too separated membership function

For reducing the probability of getting such membership functions the bounds of base are chosen very correctly for each function. Selection of membership function and respective bounds can be done as follow. Triangular membership function can be defined as

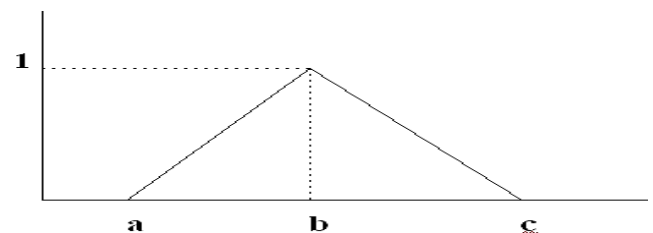


Fig -4: Triangular membership function

$$\begin{aligned}
 f(x; a, b, c) &= 0 & x \leq a \\
 &= \frac{x-a}{b-a} & a \leq x \leq b \\
 &= \frac{c-x}{c-b} & b \leq x \leq c \\
 &= 0 & c \leq x
 \end{aligned}$$

Where a, b, c are the base value of membership function which we are optimizing to get good membership Function.

4. OPTIMIZATION BY JAYA ALGORITHM

A simple, easy and powerful optimization algorithm until now is proposed for solving the constrained and unconstrained both optimization problems. And this algorithm is based on the concept that the solution obtained for a given problem should move towards the best solution if it is profit function and should avoid the Worst solution if it is loss function. This algorithm requires only the common control parameters and does not require any algorithm-specific control parameters unlike other Dr.Rao et al.

The results have proved the better effectiveness of the proposed algorithm as compared with other algorithms. Furthermore, the statistical analysis of the experimental work has been carried out by conducting the Friedman's rank unconstrained and constrained test and Holm-Sidak test. The proposed algorithm is found to secure first rank up till now for the 'best' and 'mean' solutions in the Friedman's rank test for all the 24 constrained and unconstrained benchmark problems. In addition to solving the constrained benchmark problems, the algorithm is also investigated on 30 unconstrained benchmark problems taken from the literature and the performance of the algorithm is found better. [4]

Let $f(x)$ is the objective function to be minimized (or maximized). At any iteration i , assume that there are 'm' number of design variables (i.e. $j=1, 2, \dots, m$), 'n' number of candidate solutions (i.e. population size, $k=1, 2, \dots, n$). Let the best candidate best obtains the best value of $f(x)$ (i.e. $f(x)$ best) in the entire candidate solutions and the worst candidate worst obtains the worst value of $f(x)$ (i.e. $f(x)$ worst) in the entire candidate solutions. If, k, i is the value of the variable for the candidate during the iteration, then this value is modified as per the following Eq. (1)

$$X'_{j,k,i} = X_{j,k,i} + r_{1,j,i} (X_{j,best,i} - |X_{j,k,i}|) - r_{2,j,i} (X_{j,worst,i} - |X_{j,k,i}|) \quad (1)$$

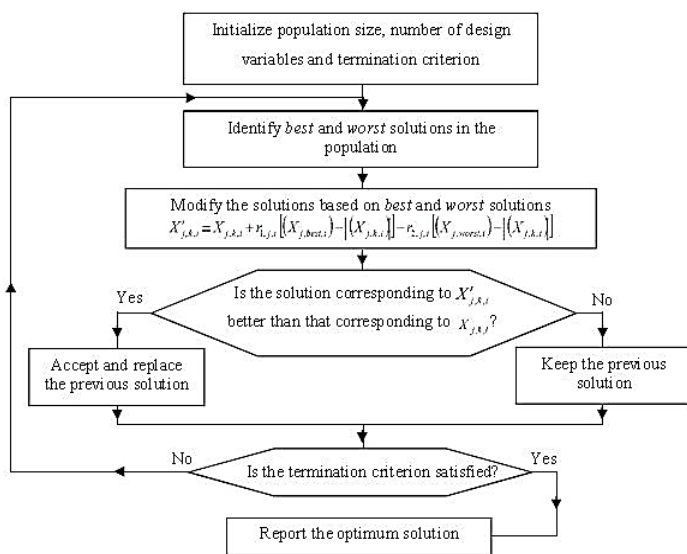


Fig-5: Flowchart of JAYA algorithm

5. FUZZY DISTURBANCE OBSERVER (FDO)

FDO is an effective method in nonlinear control. However, traditional FDO is confined to monitor dynamic disturbance, and the frequency bandwidth of the system is restricted. The basic Idea arises from the fact that a fuzzy logic system can approximate arbitrarily well a highly nonlinear system. Both the external disturbance and the internal parameter variations are combined and treated as a total disturbance and the disturbance is observed by a fuzzy logic system (named a fuzzy disturbance observer).

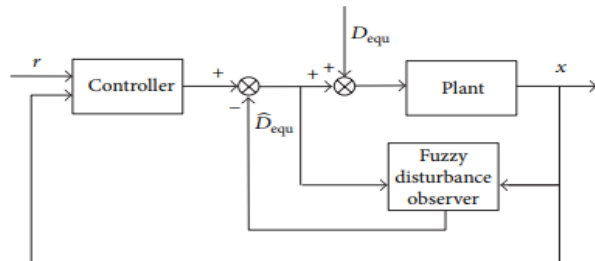


Fig-6: Structure of the control system with fuzzy disturbance observer.

5.1 LUENBERGER OBSERVER

In applications feedback control or diagnosis systems the entire state vector of the system must be controlled. If the state vector cannot be measured, what is a typical case in most complex systems, one suitable approximation to the state vector is needed. That can be substituted into a control law. A system which produces such an approximation, is called an Observer or Luenberger Observer for continuous time linear system

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2)$$

$$y(t) = Cx(t) \text{ where } x \in R^n, u \in R^m, y \in R^r,$$

a linear system observer equation is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L[y(t) - \hat{y}(t)] \quad (3)$$

$$\hat{y}(t) = C\hat{x}(t).$$

Observer error is defined as

$$e = x - \hat{x}. \quad (4)$$

Differentiating equation (4)

$$\dot{e} = \dot{x} - \dot{\hat{x}} \quad (5)$$

And substituting equations (2), (3) into (5) we get

$$\dot{e} = Ax + Bu - [A\hat{x} + Bu + L(y - \hat{y})] \quad (6)$$

$$\text{Or } \dot{e} = A(x - \hat{x}) - L(y - \hat{y}) \quad (7)$$

$$\text{And with (7) } \dot{e} = A(x - \hat{x}) - LC(x - \hat{x}) \quad (8)$$

$$\dot{e} = (A - LC)e. \quad (9)$$

Solution of eqn. (14) is given by

$$e(t) = e^{-LCt} e(0) \quad (10)$$

The eigenvalues of the matrix $A - LC$ can be made arbitrary by appropriate choice of the observer gain L , when the pair $[A, C]$ is observable (i.e. observability condition holds). So the observer error $e \rightarrow 0$ when $t \rightarrow \infty$. The given system is observable if and only if the $n \times n$ matrix, $[C^T, A^T C^T, \dots, (A^T)^{(n-1)} C^T]$ is of rank n . This matrix is called the observability matrix

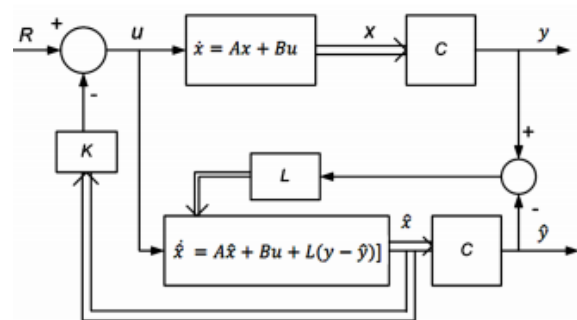


Fig-7: Block diagram of Luenberger observer

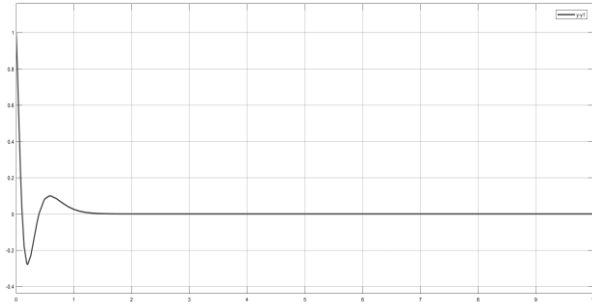


Fig-8: simulation result of Luenberger observer

In fig.8 to make system converge to zero Luenberger Observer is used. That means all states starting from any condition going to zero hence this is the controlling of the system. Same approach is applied for Practical nonlinear system that is continuous stirrer tank reactor (CSTR).

5.2 Calculation of TF:

For calculating Transfer function of CSTR cooling process the step response is taken into consideration. The transfer function is calculated by using process reaction curve. The process has very large dead time and is highly damped. Therefore the step response can be fitted into a simple first-order model with dead-time.

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1}$$

Where, K=Process gain, θ =Dead-time, τ =Time constant
Therefore, the transfer function of the process is given by

$$G(s) = \frac{0.12e^{-2s}}{3s + 1}$$

The Transfer Function of Valve is

$$G_v(s) = \frac{0.112}{0.8s + 1}$$

By using Pade’s approximation the second order transfer function is calculated as

$$G(s) = \frac{-0.12s + 0.12}{3s^2 + 4s + 1}$$

The state space matrices are given as

$$A = \begin{bmatrix} -1.33 & -0.667 \\ 0.5 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix}$$

$$C = [-0.16 \quad 0.32] \quad D = [0]$$

Consider two uncertain parameters which represent in fuzzy no. as first $a=tri(0.03 \ 0.05 \ 0.07)$ and second is $b=tri(1.2 \ 1.5 \ 1.7)$ and $c=tri(0.2 \ 0.4 \ 0.6)$. The α cut for this is $a=[(0.02\alpha+0.03), (0.07-0.02\alpha)]$ $b=[(0.2\alpha+1.2), (1.7-0.2\alpha)]$, and $c=[(0.2\alpha+0.2), (0.6-0.2\alpha)]$.

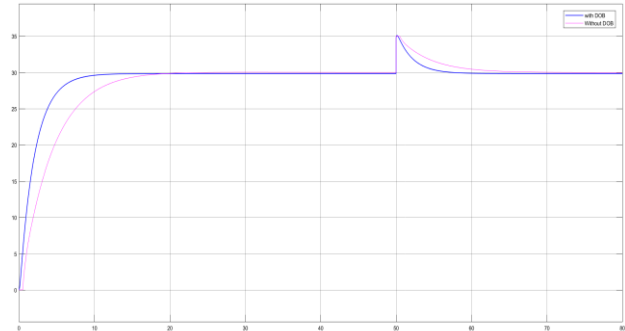


Fig-9: simulation result of FPUS with LQR with and without DOB

Disturbance observer based controller is additionally attached to the system. Fig. 9 shows the comparative results between two system setup, one is without observer and another is with connecting luenberger observer. From simulation result is has been proved that system when connected with observer gives better response rather when without observer. Performance specification for above result is illustrated in below table 1.

Table-1: Performance specification for system with and without DOB

| Name | Rise Time | Settling Time | Overshoot | Peak |
|-------------|-----------|---------------|-----------|---------|
| With DOB | 75.6701 | 299.8469 | 17.6448 | 35.1019 |
| Without DOB | 77.6890 | 311.9480 | 16.6932 | 35.0036 |

Table 1 contains Performance specification for system with and without DOB. From table 1 it is clear that settling time of the system when connected to the observer is less as compared to the system when not connected to DOB.

6. CONCLUSION

The controller is designed for FPUS using the α -cut property of fuzzy set. Controller designed for a critical condition that is for maximum uncertainty interval and that will stabilize the other interval of uncertainty.

This technique is applied to continuous stirred tank reactor and study the responses. Also, the proposed technique is applied with a fuzzy optimized membership function using Jaya algorithm. The stability of the system is checked by Kharitonov polynomials and Popov-Lyapunov stability theorem.

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