

A REVIEW ON FUNCTIONALLY GRADED PLATE

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Abstract - Functionally graded materials (FGMs) are composite materials, microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to another, and it finds application in various outlets of industry. It is the advanced materials in the field of composites, which can resist high temperatures and are proficient in reducing the thermal stresses. Functionally graded composite materials are fascinating interest among design engineers since structural component properties can be designed and customized into finished parts through processing. The controlled variable is the concentration of reinforcing particles at various points within the component. While locally isotropic, the volume fractions can be made to vary in a controlled manner from point to point. The overall properties of FGM are unique and different from any of the individual material that forms it. In this paper, various methods and theories for modelling and analyses of functionally graded materials had been reviewed. The characteristics, applications and benefits of functionally graded materials are also briefly described.

Key Words: Functionally graded material, Thermal stresses, Volume fractions,

1. INTRODUCTION

Functionally graded materials (FGMs) are known for its tailor-made properties which are achieved through the continuous gradation of material phase from one surface to another. Due to FGMs being involved in the classification of composite materials, the material compositions of FGMs are assumed to vary smoothly and continuously throughout the gradient directions. The earliest FGMs were introduced by Japanese scientists in the mid-1980s as ultra-high temperature resistant materials for aerospace applications. Recently, these materials have found other uses in electrical devices, energy transformation, biomedical engineering, optics, etc.. At the introduction of FGMs, most of the essential concepts and information about the materials were largely unknown outside of Japan. The first book of FGMs written in English was published in London, U.K., and contained comprehensive explanations of fundamentals, manufacturing processes, design and the current applications of FGMs, which were useful and available for general researchers outside of Japan.

The growing mechanism of plants was a source of inspiration for FGMs. In general, the growing mechanism of a tree is governed by the ability of the cells to detect stresses. Thus the outside surface is much stronger in order to protect

the softer part inside the tree which is used for water absorption. The main purpose of FGM development is to produce extreme temperature resistant materials so that ceramics are used as refractories (materials with excellent resistance to heat) to mix with other materials, in order to create FGMs. However, ceramics themselves cannot be used to make engineering structures subjected to high amounts of mechanical loads. This is due to the poor property of Ceramics in toughness, with the result that other materials having a good toughness property, e.g. metals and polymers, are needed to mix ceramics in order to combine the advantages of each material.

An example of FGMs used for a re-entry vehicle is shown in Fig. 1. The FGMs can be used to produce the shuttle structures. The heat source is created by the air friction of high velocity movement. If the structures of the vehicle are made from FGMs, the hot air flow is blocked by the outside surface of ceramics and transfers slightly into the lower surface. Consequently, the temperature at the lower surface is much reduced, which therefore prevents or minimises structural damage due to thermal stresses and thermal shock.



Fig-1 : Analytical model of FGM plate

2. APPLICATIONS

Some of the applications of functionally graded materials are emphasized below:

Aerospace:- Functionally graded materials can withstand very high thermal gradient and due to this it is suitable for use in structures and space plane body, rocket engine component etc. If processing technique is enhanced, FGM are promising and can be used in wider areas of aerospace.

Medicine:- There are many functionally graded material from nature and the examples are living tissues like bones and teeth. To substitute these tissues, a compatible material is needed that will serve the purpose of the novel bio-tissue and the ideal candidate for this application is functionally graded material. In dental and orthopedic claims for teeth and bone replacement, FGM has a wide range of applications.

Defense:- The ability to inhibit crack propagation is one of the most important characteristics of functionally graded material. And due to this ability of FGM it can be used in defense applications, armour plates and bullet proof vests as a penetration resistant material.

Energy:- FGM are used in energy conversion devices. They are used as defensive coating on turbine blades in gas turbine engine and also offer thermal barrier.

Optoelectronics:- FGM also finds its application in optoelectronics as graded refractive index materials and in audio-video discs' magnetic storage media.

Cutting tool insert coating, automobile engine components, nuclear reactor constituents, turbine blade, heat exchanger, Tribology, sensors, fire retardant doors, etc are the other areas of application. The list is limitless and more application is springing up as the processing technology, cost of production and properties of FMG improve. Different potential applications require different key issues. For example, in aerospace and nuclear energy applications, the key issue is reliability rather than cost. Hence, FGMs used in these applications could be produced from a high quality of material constituents in order to have the mixtures of incompatible functions such as refractoriness with toughness or chemical inertness with toughness. Oppositely, for applications of cutting and engine components, the main issue is to use FGMs to satisfy the cost/performance ratio reliability. The requirements of FGMs for these applications are wear, heat, and corrosive resistances as well as high strength of the material.

3. MATERIAL PROPERTY GRADATION

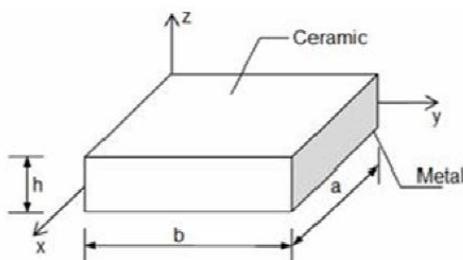


Fig- 2 : FGM Gradation

The material properties gradation in FGM follows power law function, exponential function etc.

Exponential law:- The researches which deals with fracture mechanics problem generally adopt this law. According to this law the material property in P(z) in a specific direction is given by,

$$P(z) = P_t e^{(\frac{1}{h})(\ln \frac{P_b}{P_t})(z + \frac{h}{2})}$$

Suffix 't' and 'b' re-presents the top and bottom surface of the plate respectively, 'h' is the thickness of the plate and 'z' is the specific location along the thickness direction.

Power Law:- This is the most commonly used law behavior in many researches. If FGM plate of uniform thickness 'h' is used for the analysis then according to this law, the material properties P(z) in a specific direction (along 'z') can be determined by

$$P_z = P_b + (P_t - P_b) V_f^p$$

It is noted that material properties are dependent on the volume fraction 'V_f' of FGM which follows the power-law as,

$$V_f = \frac{z}{h} + \frac{1}{2}$$

where 'p' is the volume fraction exponent. Suffix 't' and 'b' are the top and bottom surface of the plate respectively. The power law exponent 'n' can vary from '0' to '∞' it shows the transition of material from fully ceramic to metallic phase, respectively.

Sigmoidal Law:- Power-law function and exponential function are commonly used to describe the gradation of material properties of FGMs. But in both functions, the stress concentrations appear in one of the interfaces in which the material is continuous but changing rapidly. To overcome this, Chung and Chi, suggested the use of another law called sigmoid law in their work which is the mixture of two power-law functions. This law in not independent law, it consists of two symmetric FGM layers having power-law distribution. They also suggested that the stress intensity factors of a cracked body can be reduced to a certain extend by the use of a sigmoid law. According to this law, the two power-law functions are defined by,

$$f_1(z) = 1 - 0.5 \left[\frac{\frac{h}{2} - z}{\frac{h}{2}} \right]^p, 0 \leq z \leq \frac{h}{2}$$

$$f_2(z) = 0.5 \left[\frac{\frac{h}{2} + z}{\frac{h}{2}} \right]^p, -\frac{h}{2} \leq z \leq 0$$

4. EFFECTIVE MATERIAL PROPERTIES (HOMOGENISATION) OF FGM

The effective properties of macroscopic homogeneous composite materials can be derived from the microscopic heterogeneous material structures using homogenization techniques. For determination of the boundaries of the effective properties, several models like rules of mixture (Voigt Scheme), Hashin–Shtrikman type bounds, Mori–Tanaka type models, and self-consistent schemes are available in literature. Generally Voigt scheme and Mori–Tanaka schemes are adopted in analysis of functionally graded material plate and structure by most researchers.

Various methods to determine the effective properties of the plate are:

Mori-Tanaka Scheme: - This method is suitable for composites with areas of the graded microstructure having a clearly defined continuous matrix and a discontinuous particulate phase. The matrix phase is assumed to be reinforced by spherical particles of a particulate phase. The subscript ‘e’ denotes the effective value of a particular material property where as ‘c’ and ‘m’ denoted that of ceramic and metallic constituents respectively. The Bulk modulus (K) and Shear modulus (μ) is calculated as given below:

$$K_e = K_b + \frac{V_c \times K_{diff}}{1 + \frac{V_m \times K_{diff}}{K_b + 1.33\mu_b}}$$

$$\mu_e = \mu_b + \frac{V_c \times K_{diff}}{1 + \frac{K_m \times K_{diff}}{K_b + f_1}}$$

Where,

$$K_{diff} = K_t - K_b$$

$$\mu_{diff} = \mu_t - \mu_b$$

$$f_1 = \mu_b \times \frac{9K_b + 8\mu_b}{6(K_b + 2\mu_b)}$$

Young’s modulus (E), Poisson’s ratio (ν) can be calculated from them as given below.

$$E = \frac{9K_e \times \mu_e}{3K_e + \mu_e}$$

$$\mu = \frac{3K_e - 2\mu_e}{2 \times (3K_e + \mu_e)}$$

Voigt Scheme : - Voigt model has been adopted in most analyses of FGM structures due to the advantage that it is easy to calculate and can be considered as the upper and

lower bounds for the effective elastic properties of a heterogeneous material. The effective material properties P_f , like Young's modulus E_f , Poisson' ratio ν_f , thermal expansion coefficient α_f , and thermal conductivity K_f may be expressed as,

$$P_f = P_t \times V_c + P_b \times V_m$$

where P_t and P_b denoted the temperature-dependent properties of the top and bottom surfaces of the plate, respectively. V_m and V_c and are the metal and ceramic volume fractions which can be expressed by,

$$(V_c + V_m) = 1$$

If volume fraction V_c is assumed to follow a simple power law as

$$V_c = ((z/h) + 0.5)^n$$

where ‘n’ is the volume fraction index and takes only positive values

Sigmoid Function: - The use of power law distribution results in sharp gradients in material properties when the power law index moves away from unity. Smooth distribution of properties can be ensured by using two power law functions to describe the distribution. Therefore, to ensure smooth distribution of stresses among all the interfaces, Chung and Chi (2001) defined the volume fraction using two power-law functions. The two power law functions are defined by

$$f_1(z) = 1 - \frac{1}{2} \left[1 - \frac{2z}{h} \right]^p \text{ for } 0 \leq z \leq \frac{h}{2} \text{ and}$$

$$f_2(z) = \frac{1}{2} \left[1 - \frac{2z}{h} \right]^p \text{ for } 0 \leq z \leq \frac{h}{2}$$

By using rule of mixtures, the young’s modulus of the FGM can be written as,

$$E(Z) = f_1(z) \times E_1 + (1 - f_1(z)) \times E_2 \text{ for } 0 \leq z \leq \frac{h}{2}$$

$$E(Z) = f_2(z) \times E_1 + (1 - f_2(z)) \times E_2 \text{ for } -\frac{h}{2} \leq z \leq 0$$

Exponential Function: -Generally researches which deals with fracture mechanics problems adopt this law.

According to this law the material property in $P(z)$ in a specific direction is given by,

$$P(z) = P_t e^{\left(\frac{1}{h}\right) \left(\ln \frac{P_b}{P_t}\right) \left(z + \frac{h}{2}\right)}$$

5. PLATE KINEMATICS

Plate structures are major load carrying elements in structural mechanics, both in aeronautics, on land and in naval engineering. Such plates are often subjected to significant in plane compression forces and/or shear loading. Various plate theories are available to describe the static and dynamic behavior of such plates. Understanding the differences between the theories, and the application of them, is of interest both to engineers working in the field of plate structures, as well as researchers working with the development of new knowledge on plates.

Classical plate theory: -A number of theories which are used for the analysis of isotropic plates and composite plate can be extended for the analysis of the FGM plates. The simplest theory is the classical plate theory. In this theory it is assumed that there is no deformation in the mid plane of the plate. The middle plane remains neutral. The displacement gradient are small. The straight lines normal to the mid surface remains straight and normal after deformation. The length of the normal remain unchanged. And the normal stress is small and can be neglected.

This theory is applicable only in the case of thin plates where the effect of transverse shear stress is neglected. The displacement field used in this theory is as given below:

$$\begin{aligned}
 U(X_0, Y_0, Z) &= U_0(X_0, Y_0) - Z \left(\frac{\partial W_0}{\partial X} \right) \\
 V(X_0, Y_0, Z) &= V_0(X_0, Y_0) - Z \left(\frac{\partial W_0}{\partial Y} \right) \\
 Z(X_0, Y_0, Z) &= Z_0(X_0, Y_0)
 \end{aligned}$$

First order shear deformation theory: - The Mindlin plate theory (or moderately thick plate theory or shear deformation theory) was developed in the mid-1900s to allow for possible transverse shear strains. In this theory, there is the added hurdle that vertical line elements before deformation do not have to remain perpendicular to the mid-surface after deformation, although they do remain straight.

Thus shear strains ϵ_{yz} and ϵ_{zx} are generated, constant through the thickness of the plate. The displacement field associated with this theory is given below:

$$\begin{aligned}
 U(X_0, Y_0, Z) &= U_0(X_0, Y_0) - Z\theta_x \\
 V(X_0, Y_0, Z) &= V_0(X_0, Y_0) - Z\theta_y \\
 W(X_0, Y_0, Z) &= W_0(X_0, Y_0)
 \end{aligned}$$

Third order shear deformation theory:- Many number of higher order shear deformation plate theories were developed to properly imprecise the nonlinear distribution of transverse shear strains along the plate thickness. Such

HSDTs have confirmed to be highly applicable to laminated composite plates. Reddy's plate theory is known for being the most popular simple HSDT used for composite plate analysis. Higher-order theories can symbolize the kinematics better, may not require shear correction factors, and can yield more accurate inter-laminar stress distributions. However, they involve higher-order stress resultants that are difficult to interpret physically and require considerably more computational effort. Therefore, such theories should be used only when necessary.

The third-order plate theory of Reddy is based on the displacement field,

$$\begin{aligned}
 U &= U_0 + Z\phi_x - Z^2 \left(\frac{1}{2} \frac{\partial \phi_z}{\partial x} \right) - Z^3 \left[C_1 \left(\frac{\partial W_0}{\partial x} + \phi_x \right) + \frac{1}{3} \frac{\partial \phi_z}{\partial x} \right] \\
 V &= V_0 + Z\phi_y - Z^2 \left(\frac{1}{2} \frac{\partial \phi_z}{\partial y} \right) - Z^3 \left[C_1 \left(\frac{\partial W_0}{\partial y} + \phi_y \right) + \frac{1}{3} \frac{\partial \phi_z}{\partial y} \right] \\
 W &= W_0 + Z\phi_z - Z^2 \phi_z
 \end{aligned}$$

where, u,v,w denotes the displacement variables. U_0, V_0, W_0 are the in-plane displacements with respect to a reference plane. W_0 is the out-of-plane displacements with respect to the reference plane. ϕ_x, ϕ_y, ϕ_z are the rotation of normal with respect to mid-surface of the plate.

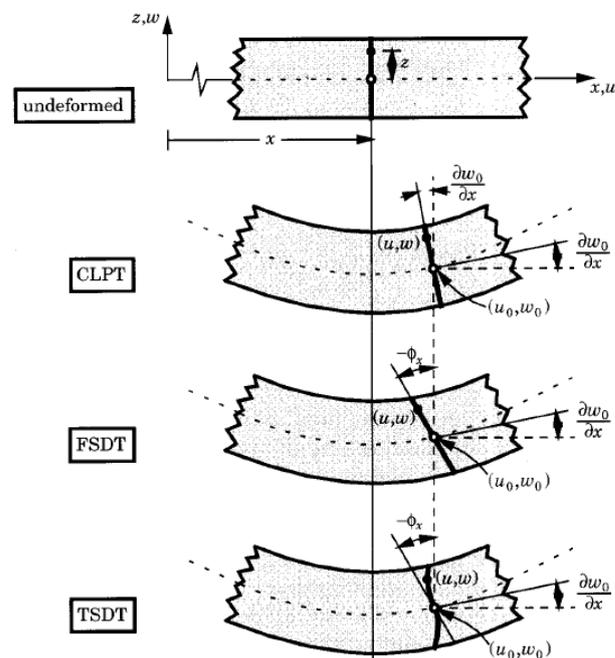


Fig-3: Transverse shear deformation of a plane according to various plate theories

6. LITERATURE REVIEW

Functionally graded materials were developed for making engineering components which are generally subjected to mechanical loads under high temperature environment. It is important to understand the behavior of composite

structures which are subjected to static and dynamic loads, for structural designs. Therefore, these effects on material properties and structural behavior have been taken into consideration in some previous investigations. An overview of existing techniques used for manufacturing functionally graded materials and their applications are also reviewed and presented in this chapter. The review contents which are relevant and useful for further investigation of this research can be expressed as follows.

N. Tejaswini et al. presented an overview on Functionally Graded Material. The Manufacturing processes of different types of FGMs are discussed. Also, the areas of applications of FGM, advantages and difficulties of FGMs are presented. The different manufacturing methods are Vapour Deposition Technique, Powder Metallurgy (PM), Centrifugal Method, and Solid Freeform (SFF) Fabrication Method.

Binol Varghese et al. (2014), presented an overview of fabrication processes, areas of application, the various analytical approaches available in the literature for FGM modeling. And illustrated the vast application of this original material using a case study in which behavior of a latest material referred to as Functionally Graded Shape Memory Alloy composites (FG-SMA) has been analysed. Among various fabrication methods highlighted solid free form fabrication (SFF) technique gives better results because of the manufacturing flexibility it offers. To improve the overall performance and to bring down the fabrication cost more researches need to be conducted on improving SFF process.

J. N. Reddy & C. D. Chin (1998), done the thermo-mechanical analysis of functionally graded cylinders and plates. The dynamic thermo-elastic response of functionally graded cylinders and plates is studied. Thermo-mechanical coupling is included in the formulation, and a finite element model of the formulation is developed. For a functionally graded axisymmetric cylinder subjected to thermal loading, the heat conduction and the thermo-elastic equations are solved.

G.M.S. Bernardo et al. (2015), performed a study on the structural behavior of FGM plates static and free vibrations analyses by considering a continuous variation of their phases and thus of their properties, and by considering a discrete stacking of a sufficient number of layers, in order to ensure a less abrupt variation profile of their properties based either on a meshless method or on different approaches based on the finite element method.

K Swaminathan et al. (2016), studied the developments, applications, various mathematical idealizations of materials, temperature profiles, modeling techniques and solutions methods that are adopted for the thermal analysis of FGM plates. An attempt has been made to classify the various analytical and numerical methods used for the stress, vibration and buckling analyses of FGM plates under one dimensional or three-dimensional variation of temperature with constant /linear / nonlinear temperatures profiles across the thickness. An effort has been made to focus the

discussion on the various research studies carried out till recently for the thermal analysis of FGM plates. Three dimensional elasticity solutions were found to be the most accurate solutions for thermal analysis of FGM plates. Due to the mathematical complexity involved, the studies are restricted to analytical methods with only simply supported edges or clamped edges.

Gutierrez-Miravete et al. (2015), presented a modeling work in functionally graded materials to determine the temperature distributions and the associated thermal stresses. From the values of the elementary constituents using the rule of mixtures and the Mori-Tanaka method, the material properties are determined. The thermal stress module in COMSOL is used to calculate the resulting temperature and stress fields for plates subjected to various boundary conditions. The specific system investigated is nickel-zirconia. Since this system is the basis of components widely used in the high temperature propulsion industry, it was selected. The Thermal Stress module in COMSOL Multi-physics was used to build the model. To mesh the plate a mapped mesh strategy was used with second order brick elements.

Serge Abrate (2007), presented a study that showed that functionally graded plates behave like homogeneous plates. He showed that no special tool is required for handling the continuous variation in material properties through the thickness of the plate and extensive results are presented that proves that FGM plates behaves very much like homogeneous plates. This simple result is developed using the classical plate theory (CPT) and is shown to hold true when higher order plate theories or the three-dimensional elasticity theory is used.

B.A. Samsam Shariat et al. (2005), studied the buckling behavior of rectangular functionally graded plates with geometrical imperfections. Using the classical plate theory the equilibrium, stability, and compatibility equations of an imperfect functionally graded plate is derived. It was assumed that the anisotropy of the plate, across the thickness, is described by a power function of the thickness variable. The critical buckling load of a sample plate is obtained and compared for different geometrical ratios. The results are reduced and compared with the results of perfect functionally graded and imperfect isotropic plates.

D. Saji et al. (2008), developed the Finite element formulation for the thermal buckling of moderately thick rectangular functionally graded material (FGM) plates based on the first order shear deformation theory (FSDT). One dimensional heat conduction equation is employed to represent the temperature distribution across thickness of the FGM plate. Material properties of the plate are considered to be function of temperature. It is assumed that the material properties of the FGM plate vary as a power function along the plate thickness. Finite element code is developed. And also computation of critical thermal buckling temperature of the FGM plates is carried out.

Vishesh R Kar et al. (2015), examined the stability of functionally graded spherical panel under thermal environment. The effective material properties are evaluated through Voigt's micromechanical model and continuous gradation is achieved using power-law distribution of the volume fraction of constituents. In addition, material properties are taken as temperature dependent. Finite element solutions are obtained through commercially available tool ANSYS using an eight node serendipity element.

Bhavani V. Sankar (2002), solved the thermo-elastic equilibrium equations for a functionally graded beam in closed-form to obtain the axial stress distribution. The thermo-elastic constants of the beam and the temperature were assumed to vary exponentially through the thickness. The Poisson ratio was held constant. The exponential variation of the elastic constants and the temperature allow exact solution for the plane thermo elasticity equations. A simple Euler-Bernoulli-type beam theory is also developed based on the assumption that plane sections remain plane and normal to the beam axis. The stresses were calculated for cases for which the elastic constants vary in the same manner as the temperature and vice versa. The residual thermal stresses are greatly reduced, when the variation of thermo-elastic constants are opposite to that of the temperature distribution. When both elastic constants and temperature increase through the thickness in the same direction, they cause a significant raise in thermal stresses. Beam theory is adequate in predicting the thermal residual stresses for the case of nearly uniform temperature along the length of the beam. An elasticity solution is obtained for FG beams subjected to temperature gradients. Poisson's ratio is assumed to be a constant, and Young's modulus is assumed to vary in an exponential fashion through the thickness. The thermo-elastic coupling coefficient and also the temperature were assumed to vary exponentially through the thickness. The results indicate that the thermo-elastic properties of the beam can be tailored to reduce the thermal residual stresses for a given temperature distribution.

R. Javaheri et al. (2002), derived the equilibrium and stability equations based on the classical plate theory of a simply supported rectangular plate made of functionally graded material under thermal loads. When it is assumed that the material properties vary as a power form of thickness coordinate variable z and when the variational method is used, the system of fundamental differential equations is established. The derived equilibrium and stability equations for functionally graded plates are identical with the equations for homogeneous plates. Buckling analysis of functionally graded plates under four types of thermal loads is carried out resulting inclosed-form solutions.

7. BENEFITS OF FGM

Some of the advantages of functionally graded materials are highlighted below:

1. FGM as an interface layer to connect two incompatible materials can greatly enhance the bond strength.
2. FGM coating and interface can be used to reduce the residual stress and thermal stress.
3. FGM coating can be used to connect the materials to eliminate the stress at the interface and end point stress singularity.
4. FGM coating not only enhances the strength of the connections but can also reduce the crack driving force.
5. FGM has the ability to control deformation, dynamic response, wear, corrosion etc.
6. FGM also provides the opportunities to take the benefits of different material systems e.g., ceramics and metals.
7. Ceramic part has good thermal resistance, wear and oxidation (rust) resistance whereas metallic part has superior fracture toughness, high strength and bonding capability.
8. FGM has wide range of applications in dental and orthopedic applications for teeth and bone replacement.
9. FGM are used in energy conversion devices. They also provide thermal barrier and used as protective coating on turbine blades in gas turbine engine.

8. DIFFICULTIES OF FGM

There are some issues that need further study and to be resolved, mainly in the following aspects:

1. A proper database of gradient material (including material system, parameters, material preparation and performance evaluation) is to be developed.
2. Still need further research and examination on the physical properties of the material model. Microscopic structure and the quantitative relationship between preparation conditions to be established in order to accurately and reliably predict the physical properties of graded materials.
3. Research should focus on variation of gradient material with respect to thermal stress relaxation of the material as well as keep the road open to variety of engineering applications.
4. Still need to improve the continuum theory, quantum (discrete) theory, percolation theory and micro-structure model, and rely on computer

simulation of the material properties for theoretical prediction particular.

5. Functionally gradient materials prepared are samples of small size, simple structure. More practical valued materials still need to be developed.
6. The total preparation costs are high.

8. CONCLUSION

In modern engineered structures pure metals or alloys are less frequently used as a result of higher requirements and complexity of the applied materials. In order to fulfill the expectations of constructors, new material combinations are designed based on metals, ceramic materials and plastics. Composites belong to a new class of advanced materials allowing to join different components and obtain unprecedented properties. Materials with a composition gradient belong to a particularly interesting group based on composites, offering the change of properties when altering one of the dimensions. FGM provides a relative change in terms of the properties of the materials over the distance and direction. Thus, the FGM has a wide range of engineering applications and it is expected to increase as the cost of material processing and fabrication processes are reduced by improving processing techniques. In the case of functionally graded materials disadvantageous sharp interfaces existing in a composite material are eliminated. These sharp interfaces are replaced with a gradient interface, which guarantees smooth transition from one material to another. The overview presented here shows only a few selected potential applications of FGMs to industrial practice. The proposed solutions can be treated as a contribution to knowledge about graded materials and its dissemination. However, the subject area still requires further advanced research into both characterization of their properties and their behavior under conditions resembling operational conditions. It has to be highlighted that at present the production cost of graded materials is relatively high but due to a tremendous interest in the subject matter expressed worldwide as well as higher level of knowledge about these materials, it should be reduced in the long term. As a result, graded materials, until now referred to as materials of the future, are expected to become 21st century materials. Although 3D analytical solutions are considered benchmark results, but since it involves tedious calculations most of the work has been done using approximated 2D theories and compared with already published solutions. The most important or critical factor involved in the analysis of FGM is the proper gradation as it influences the static, dynamic as well as its buckling behavior. And among various methods used to describe material gradation Voigt's scheme and Mori Tanaka scheme is commonly used.

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