Development of Numerical Model for Transients in pipe flow for Water Hammer situation and comparison of different friction equations for transient friction

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Abstract - *Transient flow occurs in pipelines when the pressure and flow rate changes with time. Traditionally, pipe friction during transient events has been modeled using steady friction approximations, such as, the Darcy-Weisbach friction equation. It is then realized that the steady friction approximation produced an insufficient amount of damping as compared to experimental behavior. Numerical model for transients in pipe flow for water hammer situation without the provision of surge tank-for pressure dissipation is developed with MATLAB and different friction equations are compared with the model.*

Key Words: **Keywords: Hydraulic pipe transients, friction factor, water hammer, valve, numerical model, discharge, velocity, pressure head etc.**

1. INTRODUCTION

Reynold did his pioneering study on the transition between laminar to turbulent state of fluid flow in a cylindrical pipe in 1883. Clearly, surface roughness of the conduit can be a factor for this transition. The logic centers around the friction factor, which influences the pressure/energy losses that occur in a pipe due to friction. One-dimensional quasi-steady model of energy dissipation is in common use. Estimation of loss of energy by the Darcy-Weisbach formula holds good only for slow changes of the velocity field in pipe cross-section. In case of fast changes, like fast transients i.e. water hammer, it fails. Friction and consequential damping in unsteady flow can significantly reduce the harmful effects of some pressure transients and can have a strong influence on behavior close to resonance. Friction losses in pipe conduit accounts for huge time and energy. Accurate determination of friction losses helps in accurate designing of the system – leading to overall safety and economy.

2. LITERATURE REVIEW

The basic unsteady flow equations along a pipe due to closing of the valve near the turbine are non-linear and hence its analytical solution is not possible. Allevi L(1902, 34) [1, 2] developed classical solutions through analytical and graphical methods. Bergeron L(1935,36) [3, 4] also offered graphical solution. Before the advent of computer, graphical solutions mentioned above had been widely used in pipe design.

Streeter V.L(1969) [5] developed a numerical model by using a constant value of turbulent friction factor. Pezzinga G(1999) [6] presented both quasi 2-D and 1-D unsteady flow analysis in pipe and pipe networks using finite difference implicit scheme. Head oscillations were solved only for 4 seconds with constant friction. Pezzinga G(2000) [7] also evaluated the unsteady flow resistance by the Method of Characteristics. He used Darcy-Weisbach formula for friction and solved for head oscillations up to 4 seconds only. Damping with constant friction factor is presented but not much pronounced, as the solution time is very small. Watt C.S et al(1980) [8] have solved for rise of pressure by Method of Characteristics (MoC) for only 1.2 seconds and the transient friction values have not been considered. Wiggert D.C and Sundquist M.J(1977) [9] solved the pipeline transients using fixed grids projecting the characteristics from outside the fundamental grid size. They analyzed the effects of interpolation, spacing, and grid size on numerical attenuation and dispersion. Shimada M. and Okushima S. (1984)[10] solved the second order equation of water hammer by a series solution method and the Newton Raphson method. They calculated only maximum water hammer pressure with constant friction factor. The solution was not carried out for sufficiently long time to demonstrate damping of pressure head with increase of time. Choudhury M.H and Hussaini M.Y(1985) [11] solved the water hammer equations by MacCormack, Lambda, and Gabutti explicit FD schemes. Zhao M and Ghidaoui M.S(2003) [12] have solved a quasi-two dimensional model for turbulent flow in water hammer. They have considered turbulent shear stress as resistance instead of friction factor. Bergant A et al(2001) [13] incorporated two unsteady friction models proposed by Zielke W(1968) [14] and Brunone B et al(1991) [15] into MOC water hammer analysis. The numerical results obtained for pressure heads, at valve section and in the midsection up to 1 sec, from the quasi-steady friction model, Zielke model and Brunone model have been compared with the results of measurements of fast valve closure in a laboratory apparatus with laminar flow and low Reynolds numbers turbulent flow condition. Chen NH(1979) [17] proposed his friction factor equation which is valid for Reynolds Number ranging from 4000 to 4x 108. Barr DIH(1981) [18] modified Colebrook C.F(1939) [16] to evaluate friction factor directly. Saikia M.D and Sarma A.K(2006) [19] presented a numerical model using MOC and Barr's explicit friction factor for solution of water hammer situations which clearly illustrates damping of pressure and

discharge with time. Ghanbari A. et al(2011) [20] developed a new friction factor correlation with Reynolds Number and relative roughness by means of simple logarithmic and exponential functions. Jinping LI et al (2010)[21] calculated the water hammer by 3D flow simulation in water pipeline system. Urbanowicz K. et al (2010) [22] presented a new weighting function as a sum of exponential components in order to enable efficient calculation of the unsteady component wall shear stress. Vitkovsky J.P. et al (2006)[23] tested unsteady friction models and found convolution-based model successful. Urbanowicz K and Zarzycki Z. (2012)[24] modeled unsteady component of wall shear stress as a convolution of local fluid acceleration and as weighing function. Adamkowski A. and Lewandowski M. (2006)[25] analysed unsteady friction models of Zielke, Trikha, Vardy and Brown, Brunone et al and their validation based on own experiment in order to test transient pipe flows in a wide range of Reynold's number. Pothof I. (2008)[26] proposed and validated a new formulation for the unsteady shear stress against eight transient scenarios in four different systems, conveying water with steady state Reynold's number varying between 1940 to 1.5 million. Vitkovsky J. et al (2006)[27] presented quantification of the numerical error that occur when using weighting function-based models for the simulation of unsteady friction in pipe transients. Vardy A. et al (2009)[28] presented test results of unsteady skin friction experiments including acceleration, deceleration and acoustic resonance on a large- scale pipeline apparatus for higher Reynold's number upto 400,000.

3. GOVERNING EQUATION

The flow inside the pipe in case of water hammer situation is unsteady and to analyse the unsteady situation inside the pipe constant friction model cannot be used. Different researchers has proposed different friction models for unsteady analysis. In this study we have considered mainly two friction equations, one is proposed by Barr and the other one is proposed by Chen.

The basic equations of continuity and momentum in unsteady flow along pipe due to closing of the valve near the turbine may be written as:

Continuity:

$$\frac{\delta H}{\delta t} + \frac{a^2}{gA}\frac{\delta Q}{\delta x} = 0 \qquad \ldots\ldots\ldots(1)$$

Momentum:

$$\frac{\delta H}{\delta x} + \frac{1}{gA}\frac{\delta Q}{\delta t} + \frac{f}{2gDA^2}Q|Q| = 0 \qquad \ldots\ldots\ldots(2)$$

Where, H= pressure head, A = area of pipe or conduit, a=velocity of pressure wave, Q= discharge, g= acceleration due to gravity, t = time, f =friction factor, D= diameter of pipe or conduit x = distance along the pipe.

4. METHOD OF CHARACTERISTICS (MOC)

In the MoC the partial differential equations transforms into ordinary differential equations along characteristics line. Equations (1) and (2) are presented as the following finite difference equations for pressure head H and discharge Q,

$$H_k^{\,j+1} = \frac{1}{2}\left(H_{k-1}^{\,j} + H_{k+1}^{\,j}\right) + \frac{a}{2gA}\left(Q_{k-1}^{\,j} - Q_{k+1}^{\,j}\right) + \frac{af\Delta t}{4gDA^2}\left(Q_{k+1}^{\,j}\left|Q_{k+1}^{\,j}\right| - Q_{k-1}^{\,j}\left|Q_{k-1}^{\,j}\right|\right)$$

$$Q_k^{\,j+1} = \frac{1}{2}\left(Q_{k-1}^{\,j} - Q_{k+1}^{\,j}\right) - \frac{gA}{2a}\left(H_{k-1}^{\,j} + H_{k+1}^{\,j}\right) - \frac{af\Delta t}{4gDA^2}\left(Q_{k-1}^{\,j}\left|Q_{k-1}^{\,j}\right| - Q_{k+1}^{\,j}\left|Q_{k+1}^{\,j}\right|\right)$$

5. BARR'S FRICTION EQUATION

The friction factor f in the above equations is replaced by the following Barr's explicit approximations which covers full range of flow conditions, from laminar to turbulent.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{5.02\log_{10}\left(R_e / 4.518\log_{10}\left(R_e / 7\right)\right)}{R_e\left(1 + R_e^{\,0.52} / 29(D/k)^{0.7}\right)} + \frac{1}{3.7(D/k)}\right]$$

Where,

f = friction factor

k = sand roughness coefficient

D = Diameter of pipe

Re = Reynold's number

6. CHEN'S FRICTION EQUATION

Chen's friction equation is given by,

$$\frac{1}{\sqrt{f}} = -4\log\left(\frac{k/D}{3.7065} - \frac{5.0452}{R_e}\log\left[\frac{(k/D)^{1.1096}}{2.8257} + \left(\frac{7.149}{R_e}\right)^{0.8961}\right]\right)$$

Where,

f = friction factor

k = sand roughness coefficient

D = Diameter of pipe

Re = Reynold's number

7.ALGORITHM OUTLINE

The comparison of friction values, pressure heads and discharges are done with the use of different friction equations. This is an outline of the algorithm to calculate the pressure head Hi j and the discharge Qi j for the pipe flow hydraulic transient.

[Here 'i' refers to the section no. of length along the pipe, 'j' refers to the reference no. of the time step]

1. Enter known parameters such as L, D, H0, Q0, a, g, tmax, Vo, n, m.

(Here,

L = Length of the pipe

D = Diameter of the pipe

H0 = Pressure head at inlet

Q0 = Initial discharge

a = Velocity of pressure wave

g = Acceleration due to gravity

tmax = Time taken for complete valve closure

V0 = Initial velocity of the water in the pipe

n = No. of sections along the length axis of the pipe

m = No. of section along the time axis)

2. Calculate constant values such as A, Av, Δx, Δt, xi for i = 1,2,…, n+1, and tj for j =   1,2,…,m+1.

(Here,

A = Cross sectional area of the pipe

Av = Cross sectional area of the valve

Δx = Length of each section of the pipe

Δt = Value of each individual time step

3. Create matrices Hij = H(xi,tj), and Qij = Q(xi,tj), and initialize them to zero.

4. Load the initial conditions at time=0, i.e. Qi1 = Qo , and Hi1 =H0

5. Loop on time steps, i.e., for j = 1, 2… m

6. Calculate boundary values Q1 j+1 and H 1 j+1

7. If tj<tmax (valve in process of closing),

8. Use VO(t) = VOo(1-t/tmax) to calculate valve opening.

9. Use CD(t) = CD0(1-t/tmax) to calculate coefficient of discharge at various time steps.

10. Calculate H n+1 j+1

11. If tj ≥ tmax (valve already closed),

12. Make Qn+1 j+1 =0, and Hn+1 j+1 =CP2.

13. Calculate Qi j+1 and H i j+1 at the inner points, i.e., i = 2, 3, …, n. Use left division or inverse matrices to solve the matrix equation.

8. IMPLEMENTATION OF NUMERICAL MODEL ON DATA of Saikia M.D. and Sarma A.K. (2006)[19]

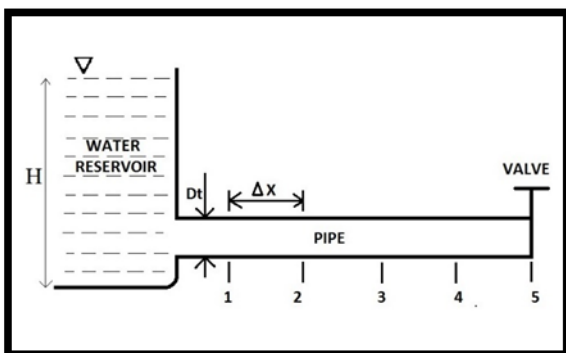Figure 1: Schematic representation of water hammer situation without surge tank.

The numerical model is implemented to the given data of Sakia M.D. and Sarma A.K. (2006)[19]. The pipe is divided into 4 sections of equal length, which means there are 5 locations for the calculations. The lab data are given as follows:-

Length of the pipe, L = 12,000 ft

Discharge, Q = 20 ft3/sec

Initial Pressure Head at the different locations:

Location 1 (Reservoir end) = 600 ft

Location 2 = 587.5 ft

Location 3 = 565 ft

Location 4 = 547.5 ft

Location 5 (Valve end) = 530 ft

Diameter of pipe, Dt = 2 ft

Area of valve opening = 3.1416 ft2

Surface roughness coefficient, k = 0.007093 ft

Kinematic Viscosity, vis = 0.000001 ft2/sec

Coefficient of discharge, Cd = 0.90

Velocity of pressure wave, V = 3000 ft/sec

Now we have applied all these input conditions to our developed numerical model and the results are plotted as given below:-
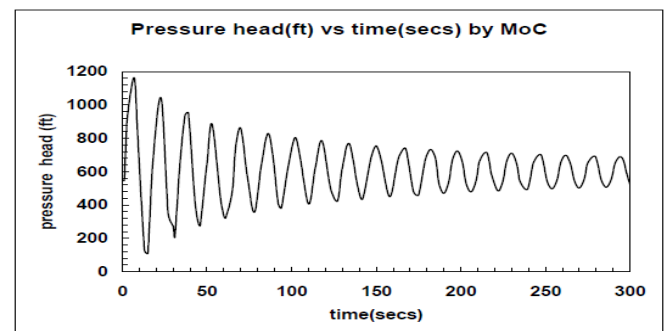

Figure 2: Pressure Head v/s time at pipe position, x=5 (from Numerical Model of Saikia M.D. and Sarma A.K. (2006)[19])
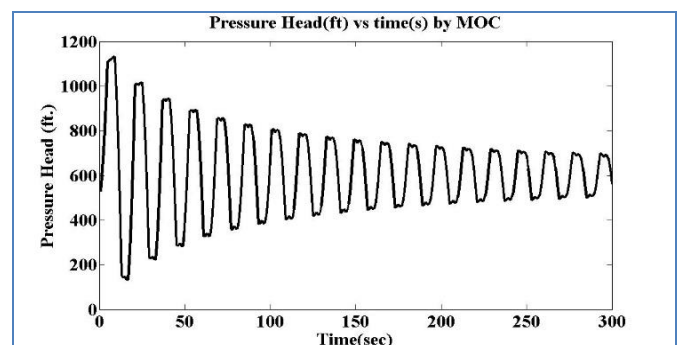

Figure 3: Pressure head vs. time at pipe position, x=5 (Developed Numerical Model using Barr's friction equation using data of Saikia M.D. and Sarma A.K. (2006)[19])
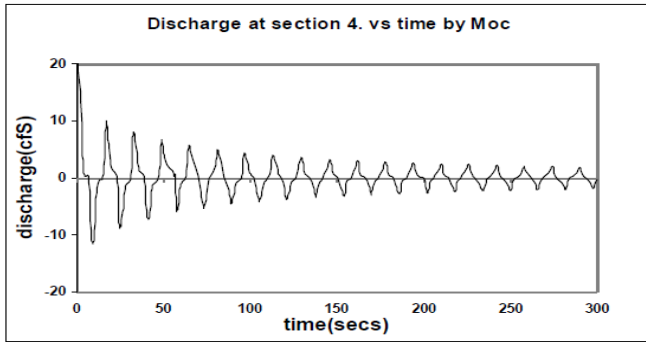
Figure 4: Discharge v/s time at pipe position, x=4 (from Numerical Model of Saikia M.D. and Sarma A.K. (2006)[19])
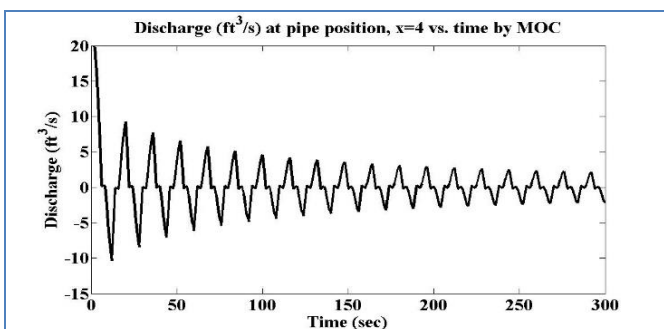

Figure 5: Discharge v/s time at pipe position =4 (Developed Numerical Model using Barr's friction equation using data of Saikia M.D. and Sarma A.K. (2006)[19])

9. FURTHER VERIFICATION OF THE DEVELOPED NUMERICAL MODEL

Now applying the input conditions of Jinping LI, Peng WU and Jiandong YANG (2010)[21] to the developed numerical model as used in the existing water hammer situation for gradual valve closure, the following observations are obtained.

The pipe from the reservoir is divided into 5 sections with 6 nodal points. The nodal points are designated as x=1, x=2, x=3, x=4, x=5, x=6 from the reservoir to the valve end respectively.

The input parameters used in the research paper by LI Jinping are listed below.

Length of the pipe from reservoir to the valve end = 600 m

Discharge = 14 m3/s

Pressure Head at the reservoir H = 150 m

Pressure Head at the valve end (i.e. at pipe position x=6) = 140 m

Diameter of pipe = 1 m

Viscosity of water at 15 degree celsius = 1.1386 mm2/s

Coefficient of discharge = 0.90

Velocity of pressure wave = 1200 m/s

The developed numerical program is run for 10 seconds and the results are plotted as shown in below.

Figure 6: Pressure head v/s time and Discharge v/s time plot at pipe position, x=2 by developed numerical model using data from Jinping Et Al (2010) [21]

The Pressure head v/s time and Discharge v/s time graph from Jinping et al (2010)[21] is plotted below for comparison.

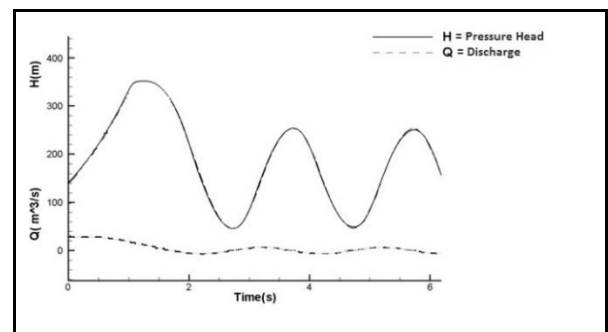Figure 7: Pressure vs. time and Discharge vs. time plot at pipe position, x=2 obtained from Jinping Et Al (2010) [21].

Comparing Fig.6 and Fig.7, it is observed that the present numerical model for water hammer simulation in pipe is approximately accurate with the existing results by JI Jinping et al (2010)[21]. Hence the developed model can be used for predicting the pressure and discharge variation in water pipeline system for water hammer situation.

On comparing the graphs, we observe that the numerical model we obtained is quite accurate and thus we can use it to compare different friction model equations. Therefore in this study we have considered the comparison between friction model by Barr's friction equation and Chen's friction equation in case of turbulent flow through pipe. The results are plotted below:-

10. COMPARISON BETWEEN TWO FRICTION MODEL: BARR'S FRICTION EQUATION AND CHEN'S FRICTION EQUATION

The variations in Pressure Head, Discharge and Friction values for unsteady flow in a conduit, using Barr's and Chen's equation are as follows:-

for this study the input parameters are taken from the water hammer problem as discussed on the article No as illustrated in the above analysis.

A) The respective output graphs using Barr's friction equation by the developed numerical model are as shown below:-
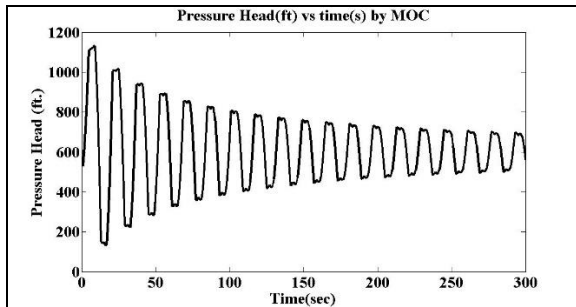

Figure 8: Variation in Pressure head, H vs. time (at pipe position x=5) using Barr's friction equation by Developed Numerical Model using data from Saikia M.D. and Sarma A.K. (2006)[19]
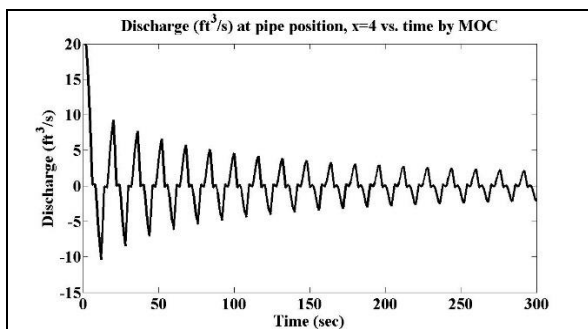

Figure 9: Discharge at pipe position. x=4 vs. time using Barr's friction equation by Developed Numerical Model using data from Saikia M.D. and Sarma A.K. (2006)[19]
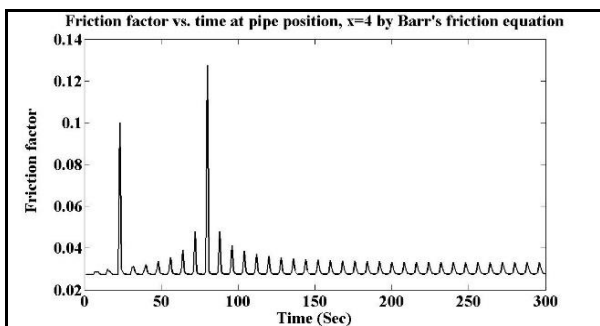

Figure 10: Friction factor at pipe position, x= 4 vs. time using Barr's friction equation by Developed Numerical Model using data from Saikia M.D. and Sarma A.K. (2006)[19]

B) The respective output graphs using Chen's friction equation by the developed numerical model are as shown below:-
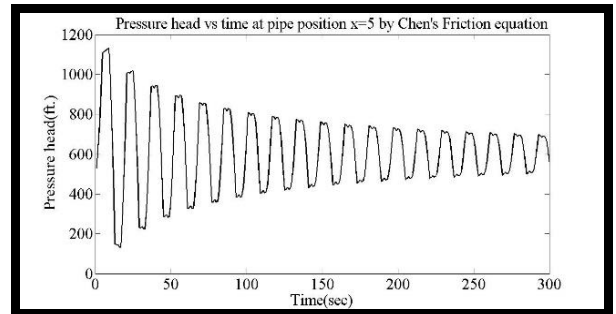

Figure 11: Pressure head vs time plot at pipe position x=5 using Chen's Friction equation by Developed Numerical Model using data from Saikia M.D. and Sarma A.K. (2006)[19]
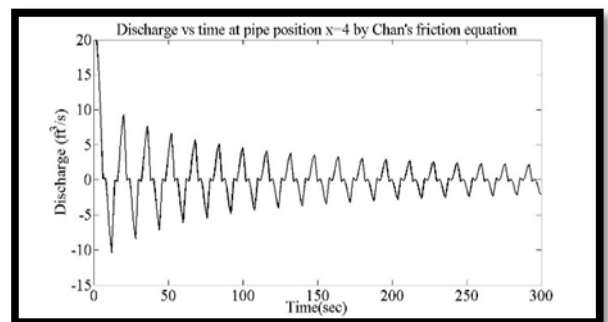

Figure 12: Discharge vs time plot at pipe position, x=4 using Chen's Friction equation by Developed Numerical Model using data from Saikia M.D. and Sarma A.K. (2006)[19]
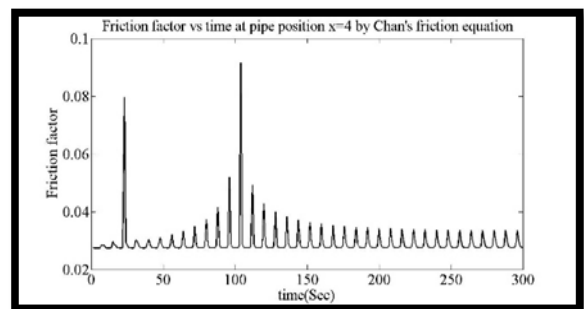

Figure 13: Friction factor vs time plot at pipe position, x= 4 using Chen's Friction equation by Developed Numerical Model using data from Saikia M.D. and Sarma A.K. (2006)[19]

11. CONCLUSION

The observations made from the above comparison are:-

- The maximum pressure head calculated considering Barr's friction equation is equal to the maximum pressure head calculated considering Chen's friction equation, therefore we can conclude that both the friction model yields similar value for maximum pressure head inside the pipe due to water hammer.

- By comparing the discharge curves, which are calculated from Barr's friction equation and Chen's friction equation, we can conclude that both the friction model yields same discharge value.

- The maximum value of the friction factor calculated by using Barr's equation is 0.1274 and the maximum value of the friction factor by using Chen's equation is 0.0915

- Friction factor value (in both the cases of friction calculation) at the initial time (i.e., at steady state) is 0.0274

- The variation of friction factor with respect to time is nearly similar for both the cases (i.e., Barr's friction equation and Chen's friction equation).

- Two friction factor picks are also observed in both the cases of friction calculation, these picks may arise due to unsteady behaviour of water flow inside the pipe.

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