

The Mathematics of Duckworth – Lewis in Cricket

Prof. K. B. Patel¹, Dr. T. U. Chaudhari², Prof. V. B. Patel³

^{1,2,3} Assistant Professor, College of Agriculture, Waghai, Gujarat, India.

Abstract - Cricket has been a very popular sport around the world. But since most versions of cricket take longer to play than other popular sports, cricket is likely to be affected by unfavorable weather conditions. Duckworth and Lewis developed a method for resetting the target scores for the team batting second in interrupted one-day cricket. Currently, the Duckworth-Lewis method is also used for resetting targets in interrupted matches. The method is based on a simple model involving a two-factor relationship giving the number of runs which can be scored on average in the remainder of an innings as a function of the number of overs is remaining and the number of wickets fallen. It is shown how the relationship enables the target score in an interrupted match to be recalculated to reflect the relative run scoring resources available to the two teams, that is overs and wickets in combination.

Key Words: Cricket, Duckworth-Lewis, Limited overs match, exponential decay.

1. INTRODUCTION

Mathematics is very important in cricket. In fact if one doesn't know quick calculations it is very difficult for him/her to play cricket. Mathematics is very important while chasing. You need to take calculated risk. You have to play according to required run rate. It is also important while batting 1st. You have to take risk according to remaining overs. But most importantly it is required while chasing. You have to calculate in every over while chasing as per no. of balls and runs remaining. There is no cricket without mathematics. The use of mathematics in cricket is at every moment.

Cricket is a team sport thought to be invented as a gentlemen's social game in England, but now played at international level across the world. Despite only being played at the very highest level in 10 countries, the game stretches across 5 continents and is thought to be the second most popular sport in the world, with an estimated 2.5 billion fans. With the introduction of T20 cricket in 2003 the game's popularity reached a new level and since then a number of high profile T20 leagues have been established around the world. As the profile of the game increases more and more money is becoming involved, in terms of players wages, sponsorship and gambling. With so much money now at stake it is more important than ever to have games ending in results and in the event that the game cannot reach its own natural conclusion that a fair method be used to decide upon a winner.

The Duckworth-Lewis Method Cricket is a summer sport and due to safety concerns with the pitch, plus the reluctance of the players to get wet, the game must stop if it rains to allow the pitch to be protected and the players to stay dry. Rain delays are a very common thing across world cricket especially in countries whose summers suffer from the same volume of rain as the British summer. During a test match a rain break just reduces the duration of the game and as there is no restriction on each teams innings' this causes no 3 issues. Test match cricket is also spread across 5 days making it highly unlikely for the whole match to be stopped because of rain. As a limited overs game only lasts a day and the rules dictate that both teams bat for a set number of overs, the same treatment of rain delays cannot occur. Duckworth and Lewis point out that the reason limited overs cricket was invented was because too many first class games were ending in draws and players and fans alike craved a shorter format of the game that produced a result in just one day. Therefore considering a rain affected limited overs game as a draw is contrary to the reason this format of the game exist, so a fair method of deciding the result of a rain affect limited overs game needed to be devised. Many methods were trialed but finally the Duckworth and Lewis method, devised by Frank Duckworth and Anthony Lewis in 1998, was settled upon. Duckworth and Lewis (1998) devised a two factor relationship between the number of overs a team had remaining and the number of wickets they had lost in order to quantify the total resources a team had remaining. This was then divided by the resources each team had at the start of the game to obtain a resource percentage, which Duckworth and Lewis tabulated allowing remaining resource to be calculated for all combinations of overs remaining and resources lost. No matter where a stoppage in play occurred, using this resources table and simple calculations outlined by Duckworth and Lewis its possible to calculate the percentage of their full resources any team has received. And therefore reset a teams target such that both team have to score proportionally the same number of runs in the resources available to them.

2. Duckworth – Lewis method

The Duckworth-Lewis (D/L) method is a mathematical formulation designed to calculate the target score for the team batting second in a limited overs cricket match interrupted by weather or other circumstances. It is generally accepted to be the most accurate method of setting a target score. The D/L method was devised by two English statisticians, Frank Duckworth and Tony Lewis[1]. After their retirements Professor Steven Stern became the custodian of the method. In November 2014, it was renamed

the Duckworth–Lewis–Stern method (or D/L/S method). When overs are lost, setting an adjusted target for the team batting second is not as simple as reducing the run target proportionally to the loss in overs, because a team with ten wickets in hand and 25 overs to bat can play more aggressively than if they had ten wickets and a full 50 overs. The D/L method is an attempt to set a statistically fair target for the second team's innings, which is the same difficulty as the original target. The basic principle is that each team in a limited overs match has two resources available with which to score runs (overs to play and wickets remaining), and the target is adjusted proportionally to the change in the combination of these two resources.

2.1 Calculation summary

The essence of the D/L method is 'resources'. Each team is taken to have two 'resources' to use to make as many runs as possible: the number of overs they have to receive; and the number of wickets they have in hand. At any point in any innings, a team's ability to score more runs depends on the combination of these two resources. Looking at historical scores, there is a very close correspondence between the availability of these resources and a team's final score, a correspondence which D/L exploits [2].

The D/L method converts all possible combinations of overs (or, more accurately, balls) and wickets left into a combined resources remaining percentage figure and these are all stored in a published table or computer. The target score for the team batting second ("Team 2") can be adjusted up or down from the total the team batting first ("Team 1") achieved using these resource percentages, to reflect the loss of resources to one or both teams when a match is shortened one or more times.

In the version of D/L most commonly in use in international and first-class matches, the target for Team 2 is adjusted simply in proportion to the two teams' resources, i.e.

$$\text{Team 2's par score} = \text{Team 1's par score} \times \frac{\text{Team 1's resources}}{\text{Team 2's resources}} \quad (1)$$

If, as usually occurs, this 'par score' is a non-integer number of runs, then Team 2's target to win is this number rounded up to the next integer, and the score to tie (also called the par score), is this number rounded down to the preceding integer. If Team 2 reaches or passes the target score, then they have won the match. If the match ends when Team 2 has exactly met (but not passed) the par score then the match is a tie. If Team 2 fail to reach the par score then they have lost.

2.2 Mathematical theory

The original D/L model started by assuming that the number of runs that can still be scored (called R), for a given number of overs remaining (called O) and wickets lost (called W), takes the following exponential decay relationship[3].

$$R(O, W) = R_0(W)(1 - e^{-b(w)O}) \quad (2)$$

Where the constant R_0 is the asymptotic average total score in limited overs and b is the exponential decay constant. Both vary with w (only). The values of these two parameters for each w from 0 to 9 were estimated from scores from 'hundreds of one-day internationals' and 'extensive research and experimentation', though were not disclosed due to 'commercial confidentiality'. [3]

Finding the value of R for a particular combination of O and W and dividing this by the score achievable at the start of the innings, i.e. finding

$$P(O, W) = \frac{R(O, W)}{R(O = 50, W = 0)} \quad (3)$$

gives the proportion of the combined run scoring resources of the innings remaining when O overs are left and W wickets are down[3]. These proportions can be plotted in a graph.

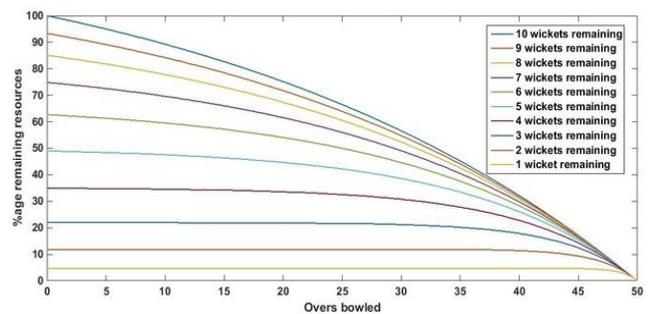


Chart -1: Name of the chart

When it was introduced, it was necessary that D/L could be implemented with a single table of resource percentages, as it could not be guaranteed that computers would be present. Therefore this single formula was used giving average resources. This method relies on the assumption that average performance is proportional to the mean, irrespective of the actual score. This was good enough in 95 per cent of matches, but in the 5 per cent of matches with very high scores, the simple approach started to break down[4]. To overcome the problem, an upgraded formula was proposed with an additional parameter whose value depends on the Team 1 innings [5]. This became the Professional Edition.

3. CONCLUSIONS

Actually, there is no right or wrong answer to the point in which the model should be fair to both teams and this is simply a value judgment made by the model builders and the people who regulate cricket. Originally Duckworth and Lewis set their model up so that it would be fair to both sides

at the start of the season. The D/L resource table is considered to do a pretty good job of doing this and is thus considered to be fair to every team at the start of the season. Although this method is little bit complex for fans who are not person of mathematical field. But overall this method is quite satisfactory by ignoring some odd cases.

REFERENCES

- [1] "A Decade of Duckworth–Lewis". BBC. 1 January 2007. Retrieved 21 March 2009.
- [2] Data Analysis Australia's detailed mathematical analysis of the Duckworth–Lewis Method daa.com.au.
- [3] Duckworth, F.C.; Lewis, A. J. (1998). "A fair method for resetting the target in interrupted one-day cricket matches". *Journal of the Operational Research Society*. 49(3):220–227. doi:10.1057/palgrave.jors.2600524.
- [4] Duckworth, F.C. (2008). "The Duckworth/Lewis method: an exercise in Maths, Stats, OR and communications" (PDF). *MSOR Connections*. 8 (3): 11–14.
- [5] Duckworth, F.C.; Lewis, A. J. (2004). "A successful Operational Research intervention in one-day cricket". *Journal of the Operational Research Society*. 55 (7):749–759. doi:10.1057/palgrave.jors.2601717.