Design of Two-Stage Single Speed Gearbox for Transmission System of SAE BAJA Vehicle

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Abstract – This paper focuses on the design of constant speed two stage reduction gearbox for SAE Baja ATV. This season, RMD Sinhgad School of Engineering college Baja team has developed a customised gearbox drivetrain which is efficient, lightweight and durable. The drivetrain consists of a continuously variable transmission device and a single speed two stage reduction gearbox. This gearbox conformed with all SAE BAJA design rules to precisely integrate with all subsystems. The gearbox was modelled on CATIA and analysed on ANSYS workbench and this testing was effective.

Key Words: Off-road vehicle, Single speed Two stage reduction gearbox, Continuously variable transmission device.

1. INTRODUCTION

SAE BAJA is an international competition organised for students pursing engineering course where student teams are provided an opportunity to design and build an ATV. These teams have liberty to design the vehicle as it pleases them although it must conform with the rulebook. However, the teams have to use Briggs and Stratton 10 HP engine in stock condition which is provided [1]. The transmission along with the drive-train is one of the most crucial systems in the vehicle. Its purpose is to transfer the power generated by the engine to the wheels with minimum losses.

An efficient transmission system imparts the vehicle with sufficient torque to run over all the obstacles that can be encountered in a demanding terrain. Simultaneously, the vehicle must be capable of achieving considerable high speed. Thus, the torque and speed requirements must be aptly balanced. If the torque at the wheels is excess of what is needed in the harshest conditions, then there will be torque that will never be used at very low top speeds. Correspondingly, at very high top-speeds there will be low torque and the vehicle may not be able to navigate over terrains. In order to have an efficient transmission system in the vehicle is critical to have a balance between torque and high speed. [3]

1.1 Objective

Our goal is to design a transmission system which provides high torque and moderately high speed. To achieve this two criterium have to be fulfilled which include ability of vehicle to climb 35 degrees gradient and achieve maximum speed of 60 kmph [1].

Fig – 1: CAD model of BAJA ATV

2. TRACTIVE EFFORT

Tractive effort for vehicle to propel over a gradient of 35 degree is calculated by following equations: [2]

1. Drag Resistance \( R_d \) = \( \varrho \times A \times C_d \times \frac{V^2}{2} = 68.08 \) N

\( \varrho \): Density of air = 1.225 kg/m³

\( A \): Cross section area of vehicle = 1 m²

\( C_d \): Coefficient of Drag = 0.4

\( V \): Speed of vehicle = 60 km/h = 16.67 m/s

2. Rolling Resistance \( R_R \) = \( f_r \times m \times g \times R = 106.68 \) N

\( f_r \): Coefficient of rolling resistance = 0.0435

\( m \): Mass of vehicle = 250 kg

3. Static Resistance \( R_s \) = \( \mu \times m \times g \times R = 1594.125 \) N

\( \mu \): Coefficient of friction = 0.65

So, tractive effort \( (F_r) \) is 1768.88 N

Minimum required torque \( (T) \) = \( T \times \tau_{dyn} = 471.76 \) Nm

\( \tau_{dyn} \): Dynamic radius of wheels = 0.2667 m
3. TRANSMISSION RATIO

![Torque Curve](image)

**Fig – 2:** Briggs and Stratton 10 HP torque curve

Tractive effort ($F_T$) = $T_e \cdot T_r \cdot \eta_{all} / r_{dyn}$

- $T_e$: Maximum engine torque = 19.1 Nm
- $T_r$: Transmission ratio
- $\eta_{all}$: Combined efficiency of transmission system = 0.85
- $T_r$ = 30:1

Our transmission system utilises Polaris P-90 continuously variable transmission device (CVT). This CVT is opted because it is quite compact and easily available as compared to other makes despite its expensive price. Also, it provides maximum ratio of 3.83:1 and minimum ratio of 0.76:1 and its adaptability to tuning. [5]

As CVT provides maximum reduction of 3.83:1 so gear ratio of gearbox must be 7.83:1. After considering the space requirements a two-stage constant speed reduction gearbox was decided. Reduction ratio in first stage ($i_1$) is 2.7:1 and second stage reduction ratio ($i_2$) is 2.89:1.

So, maximum torque available at wheels is 573 Nm at 2800 rpm.

4. DESIGN CALCULATIONS

4.1. Gearbox Design Calculation

Material selected for gears is 20MnCr5 because its high tensile and yield strength along with appreciable hardness.

Ultimate Tensile Strength ($S_u$): 1000 - 1300 N/mm²

Yield Strength, ($S_y$): 700 – 900 N/mm²

Hardness: 750 BHN

First Stage reduction: [4]

- Number of teeth on pinion ($Z_p$) = 18
- Number of teeth on gear ($Z_g$) = 49

Speed of Pinion ($N_p$) = 875 rpm

Gear Ratio ($i_1$) = 2.7

$$\sigma_{bp} = \frac{S_u}{3} = \frac{1100}{3} = 366.67 \text{ N/mm}^2$$

$$\sigma_{bg} = \frac{S_u}{3} = \frac{1100}{3} = 366.67 \text{ N/mm}^2$$

Lewis Form Factor ($Y$) = 0.484-(2.87/$Z$)

- $Y_p$=0.484-(2.87/$Z_p$) = 0.484-(2.87/18) = 0.324
- $Y_g$=0.484-(2.87/$Z_g$) = 0.484-(2.87/49) = 0.425

As Pinion is weaker than gear it is necessary to design pinion.

Diameter of pinion ($d_p$) = $m \times Z_p$

= 18m

Face width ($b$) = 10m

Ratio Factor ($Q$) = $2 \times Z_g / (Z_g + Z_p)$

= $2 \times 49 / (49+18)$

= 1.463

Load Stress Factor ($K$) = 0.16 × (BHN/100)²

= 0.16 × (650/100)²

= 6.76 N/m²

Beam Strength: $F_b = \sigma_{bp} \times b \times m \times Y_p$

= $366.67 \times 10 \times 18 \times 0.324$

= 11187.97 m² N

Wear Strength ($F_w$) = $d_p \times b \times Q \times K$

= 18m × 10m × 1.463 × 6.76

= 1779.79 m² N
As, \( F_b < F_w \), Gear Pair is weak in bending hence it is designed for safety against bending.

Torque at Pinion (\( T_1 \)) = 61.12 Nm

Pitch line velocity at first stage pinion (\( V_1 \))

\[
V_1 = \pi \times d_p \times N_p / 60 \times 1000
\]

= \( \pi \times 18 \times m \times 875 / 60 \times 1000 \)

= 0.8246 m/sec

Tangential Force at tooth (\( F_t \)) = \( 2 \times T_1 / d_{p1} \)

= \( 2 \times 61.12 / 18 \times m \)

= 6791.11/m

Velocity Factor (\( K_v \)) = \( 6 / (6+V_1) \)

= \( 6 / (6+0.8246) \)

= 0.8246

Service Factor (\( K_m \)) = 1.3

\[
F_{eff} = \{K_m \times F_1 \} / K_v
\]

= \( \{1.3 \times (6791.11/m)\} / (6/6+0.8246) \)

= 1471.407 \times (6+0.8246) /m

Calculating Module

\[
F_b = FOS \times F_{eff}
\]

1187.97 \times m^3 = 1.5 \times 1471.407 \times (6+0.8246m)

m = 2.47

m = 2.5 mm

Checking gear pair for wear strength

\[
F_w = FOS \times F_{eff}
\]

11123.687 = FOS \times 4744.699

FOS = 2.34

Load Stress Factor (\( K \)) = 0.16 \times (BHN/100)^2

Diameter of pinion (\( d_p \)) = \( m \times Z_p \)

= 20 m

Face width (\( b \)) = 10m

Ratio Factor (\( Q \)) = \( 2 \times Z_g / (Z_g + Z_p) \)

= 2 \times 58 / (58+20)

= 1.487

Diameter of Gear of first stage (\( d_{g1} \)) = \( m \times Z_g \)

= 2.5 \times 49

= 122.5 mm

Wear Strength (\( F_w \)) = \( d_p \times b \times Q \times K \)

= 20 \times 10 \times 1.487 \times 6.76

Number of teeth on pinion (\( Z_p \)) = 20

Number of teeth on gear (\( Z_g \)) = 5

Speed of Pinion (\( N_p \)) = 324.07 rpm

Gear Ratio (\( i_2 \)) = 2.89

\[
\sigma_{bp} = S_{ult} / 3 = 1100 / 3
\]

= 366.66 N/mm²

\[
\sigma_{bg} = S_{ult} / 3 = 1100 / 3
\]

= 366.66 N/mm²

Lewis Form Factor (\( Y \)) = 0.484-(2.87/Z)

\[
Y_p = 0.484-(2.87/Z_p)
\]

= 0.484-(2.87/20)

= 0.3405

\[
Y_g = 0.484-(2.87/Z_g)
\]

= 0.484-(2.87/58)

= 0.434

\[
\sigma_{bp} \times Y_p < \sigma_{bg} \times Y_g
\]

As Pinion is weaker than gear it is necessary to design pinion.

Diameter of pinion (\( d_p \)) = \( m \times Z_p \)

= \( 20 \times m \)

\[
\sigma_{bp} \times Y_p < \sigma_{bg} \times Y_g
\]

}\[
\sigma_{bp} \times Y_p < \sigma_{bg} \times Y_g
\]
As, \( F_b < F_w \), Gear Pair is weak in bending hence it is designed for safety against bending.

Torque at Pinion \( (T_2) = 165.024 \) Nm

Pitch line velocity at second stage pinion \( (V_2) \)
\[
V_2 = \pi \times d_p \times N_p / 60 \times 1000
\]
\[
= 0.3393 \text{ m/s}
\]

Tangential Force at tooth \( (F_t) = 2 \times T_2/d_p \)
\[
= 2 \times 165.024/20 \text{m}
\]
\[
= 16502.4/ \text{m}
\]

Velocity Factor \( (K_v) = 6/(6+V_2) \)
\[
= 6/6.03393m
\]

Service Factor \( (K_m) = 1.3 \)

\[
F_{eff} = \{K_m \times F_t \} / K_v
\]
\[
= (1.3 \times (16502.4/m)) / (6/6+0.3393m)
\]
\[
= 3575.52 \times (6+0.3393m) / m
\]

Calculating Module

\[
F_b = FOS \times F_{eff}
\]
\[
= 1248.477 \times m^2 = 1.5 \times 1471.407 \times (6+0.8246m)
\]
\[
m = 2.70
\]
\[
m = 3 \text{ mm}
\]

Checking gear pair for wear strength

\[
F_w = FOS \times F_{eff}
\]
\[
= 18093.816 = FOS \times 8364.21
\]
\[
FOS = 2.16
\]

Diameter of pinion in second stage \( (d_{p2}) = m \times Z_p \)
\[
= 3 \times 20
\]
\[
d_{p2} = 60 \text{ mm}
\]

Diameter of Gear in second stage \( (d_{g2}) = m \times Z_g \)
\[
= 3 \times 58
\]
\[
d_{g2} = 174 \text{ mm}
\]

**4.2. Shaft Design Calculation**

Material of shaft is the same as gears (20MnCr5). This material is chosen because it has high tensile strength as well as yield strength along with appreciable fatigue strength and rigidity. [4]

The shaft design is based on ASME code.

Maximum shear stress \( (\Gamma_{\text{max}}) \)
\[
\Gamma_{\text{max}} = 0.18 \times S_{ut}
\]
\[
= 0.18 \times 1100
\]
\[
= 198 \text{ MPa}
\]

Shafts have keyways hence these values have to be reduced by 25%

Modified Maximum shear stress \( (\Gamma_{\text{max}}) \)
\[
\Gamma_{\text{max}} = 0.75 \times 198
\]
\[
= 148.5 \text{ Nmm}
\]

1) Input Design

In vertical plane,

\( R_{AV} \) and \( R_{BV} \) are reactions at end A and B in vertical plane respectively

\[
R_{AV} + R_{BV} = 2718.2
\]
\[
(R_{AV} \times 0) + (2718.45 \times 40.5) = (R_{BV} \times 126)
\]
\[
R_{BV} = 873.787 \text{ N}
\]
\[
R_{AV} = 1844.663 \text{ N}
\]

Maximum bending moment in vertical plane \( (M_V) \)
\[
M_V = 1844.63 \times 40.5
\]
\[
= 74708.85 \text{ Nmm}
\]

In horizontal plane,
Fig – 3: Forces on input shaft in horizontal plane

$R_{AH}$ and $R_{BH}$ are reactions at end A and B in horizontal plane respectively

$R_{AH} + R_{BH} = 988.70$

$(R_{AH} \times 0) + 988.7 \times 40.5 = (R_{BH} \times 126)$

$R_{BH} = 317.796 \text{ N}$

$R_{AH} = 670.90 \text{ N}$

Maximum bending moment in horizontal plane ($M_H$)

$M_H = 670.90 \times 40.5$

$= 27171.594 \text{ Nmm}$

Equivalent bending moment ($M_e$)

$M_e = \sqrt{(74708.85)^2 + (27171.594)^2} = 79496.589 \text{ Nmm}$

Equivalent torque ($T_e$)

$T_e = T_1 = 61110 \text{ Nmm}$

Diameter of input shaft ($d_1$)

Combined shock and fatigue factor for bending ($K_b$) = 1.5

Combined shock and fatigue factor for torsion ($K_t$) = 1.5

$\Gamma_{max} = \pi \times d_1^2 \times ((K_b M_e)^2 + (K_t T_e)^2)^{1/2}/16$

$d_1 = 17.27 \text{ mm}$

We have selected $d_1 = 20 \text{ mm}$.

2) Intermediate Design

In vertical plane,

$R_{AV} + R_{BV}$ are reactions at end A and B respectively

$R_{AV} + R_{BV} = 8227.17$

$(2722.76 \times 39.5) + (5504.41 \times 84.5) = (R_{BV} \times 124)$

$R_{BV} = 4618.31 \text{ N}$

$R_{AV} = 3608.85 \text{ N}$

Maximum bending moment in vertical plane at gear ($M_{GV}$)

$M_{GV} = 1844.63 \times 39.5$

$= 142549.59 \text{ Nmm}$

Maximum bending moment in vertical plane at pinion ($M_{PV}$)

$M_{PV} = 1844.63 \times 42$

$= 93969.02 \text{ Nmm}$

In horizontal plane,

$R_{AH} + R_{BH} = 2984$

$(984 \times 39.5) + (2000 \times 82) = (R_{BH} \times 124)$

$R_{BH} = 1636.03 \text{ N}$

$R_{AH} = 1347.96 \text{ N}$

$M_{BH} = 1347.96 \times 39.5$

$= 53244.42 \text{ Nmm}$

$M_{BH} = 1636.03 \times 42$
Equivalent bending moment at gear ($M_{Ge}$)

$$M_{Ge} = \{(142549.59)^2 + (53244.42)^2\}^{1/2} = 152168.838 \text{ Nmm}$$

Equivalent bending moment at pinion ($M_{Pe}$)

$$M_{Pe} = \{(193969.02)^2 + (68713.26)^2\}^{1/2} = 205780.26 \text{ Nmm}$$

Equivalent Torque ($T_e$)

$$T_e = T_2 = 165000 \text{ Nmm}$$

Diameter of intermediate shaft ($d_2$)

Combined shock and fatigue factor for bending ($K_b$) = 1.5

Combined shock and fatigue factor for torsion ($K_t$) = 1.5

$$\Gamma_{max} = \pi \times d_2^3 \times \{(K_bM_e)^2 + (K_tT_e)^2\}^{1/2} / 16$$

$$d_2 = 22.14 \text{ mm}$$

We have selected $d_2 = 20 \text{ mm}$. 

3) Outside Design

In vertical plane,

[Fig 6: Forces on output shaft in vertical plane]

Resultant force at end A of input shaft ($R_{RA}$)

$$R_{RA} = \{(R_{AV})^2 + R_{AH}^2\}^{1/2}$$

$$R_{RA} = 5517.69 \text{ N}$$

$$R_{AV} = 2758.88 \text{ N}$$

$$R_{AH} = 2758.88 \text{ N}$$

$$M_V = 2758.88 \times 65.5 = 180704.347 \text{ Nmm}$$

In horizontal plane,

[Fig 7: Forces on output shaft in horizontal plane]

We have selected $d_3 = 35 \text{ mm}$. 

4.3. Bearing Selection

As only spur gear pairs are used there is only radial and tangential load on the shafts. So, deep groove ball bearings will suffice our need. [4]

Life of bearing in million hours ($L_{10h}$) for our application = 10000 hours.

1) Bearing for Input Shaft

Required inner diameter of bearing = 20 mm

Resultant force at end A of input shaft ($R_{RA}$)

$$R_{RA} = (R_{AV}^2 + R_{AH}^2)^{1/2}$$
Resultant force at end B of input shaft ($R_{RB}$)

$$R_{RB} = \sqrt{R_{BV}^2 + R_{BH}^2}$$

$$= \sqrt{(873.78^2 + 317.79^2)}$$

$$= 929.78 \text{ N}$$

Maximum Load on one bearing = 1962.78 N

Life of bearing in million revolutions ($L_{10}$)

$$L_{10} = \frac{60 \times n_1 \times L_{10h}}{10^6}$$

$$= 60 \times 992.17 \times 10000 / 10^6$$

$$= 220.48 \text{ million revolutions}$$

Basic Dynamic Load Rating ($C$) = $P \times (L_{10})^{1/3}$

$$C = 4899.52 \times (220.48)^{1/3}$$

For Bearing 6405, $C = 35800 \text{ N}$

Angular speed of input shaft ($n_1$) = 992.17 rpm when the engine is running at 3800 rpm.

L10 = 60 x n1 x L10h / 10^6

= 60 x 992.17 x 10000 / 10^6

= 220.48 million revolutions

Basic Dynamic Load Rating ($C$) = $P \times (L_{10})^{1/3}$

$$C = \frac{P \times (L_{10})^{1/3}}{n_1}$$

For Bearing 6405, $C = 35800 \text{ N}$

3) Bearings for Output Shaft

Resultant force at end A of output shaft ($R_{RA}$)

$$R_{RA} = \sqrt{R_{AV}^2 + R_{AH}^2}$$

$$= \sqrt{(2758.88^2 + 997.67^2)}$$

$$= 2933.66 \text{ N}$$

Maximum Load on one bearing = 2933.66 N

Life of bearing in million revolutions ($L_{10}$)

$$L_{10} = \frac{60 \times n_3 \times L_{10h}}{10^6}$$

$$= 60 \times 992.17 \times 10000 / 10^6$$

$$= 75.50 \text{ million revolutions}$$

Basic Dynamic Load Rating ($C$) = $P \times (L_{10})^{1/3}$

$$C = \frac{P \times (L_{10})^{1/3}}{n_3}$$

For Bearing 6007, $C = 15900 \text{ N}$

Angular speed of output shaft ($n_3$) = 125.85 rpm when the engine is running at 3800 rpm.

Life of bearing in million revolutions ($L_{10}$)

$$L_{10} = \frac{60 \times n_3 \times L_{10h}}{10^6}$$

$$= 60 \times 992.17 \times 10000 / 10^6$$

$$= 75.50 \text{ million revolutions}$$

Basic Dynamic Load Rating ($C$) = $P \times (L_{10})^{1/3}$

$$C = \frac{P \times (L_{10})^{1/3}}{n_3}$$

For Bearing 6007, $C = 15900 \text{ N}$

Required inner diameter of bearing = 35 mm

Resultant force at end B of output shaft ($R_{RB}$)

$$R_{RB} = \sqrt{R_{BV}^2 + R_{BH}^2}$$

$$= \sqrt{(2758.88^2 + 997.67^2)}$$

$$= 2933.66 \text{ N}$$

Maximum Load on one bearing = 2933.66 N

Angular speed of output shaft ($n_3$) = 125.85 rpm when the engine is running at 3800 rpm.

Life of bearing in million revolutions ($L_{10}$)

$$L_{10} = \frac{60 \times n_3 \times L_{10h}}{10^6}$$

$$= 60 \times 992.17 \times 10000 / 10^6$$

$$= 75.50 \text{ million revolutions}$$

Basic Dynamic Load Rating ($C$) = $P \times (L_{10})^{1/3}$

$$C = \frac{P \times (L_{10})^{1/3}}{n_3}$$

For Bearing 6007, $C = 15900 \text{ N}$

Required inner diameter of bearing = 25 mm

Resultant force at end A of intermediate shaft ($R_{RA}$)

$$R_{RA} = \sqrt{R_{AV}^2 + R_{AH}^2}$$

$$= \sqrt{(3608.85^2 + 1347.96^2)}$$

$$= 3852.37 \text{ N}$$

Resultant force at end B of intermediate shaft ($R_{RB}$)

$$R_{RB} = \sqrt{R_{BV}^2 + R_{BH}^2}$$

$$= \sqrt{(4618.31^2 + 1636.03^2)}$$

$$= 4899.52 \text{ N}$$

Maximum Load on one bearing = 4899.52 N

Angular speed of intermediate shaft ($n_2$) = 367.47 rpm when the engine is running at 3800 rpm.
Bearing 6007 was selected for output shaft

Fig - 8: CAD model of Gearbox

5. STRESS ANALYSIS

The Gear Train was thoroughly tested in ANSYS. (Forces in vertical plane are summation of tangential forces and weights of gears and the forces in horizontal plane are radial forces on gears)

Table - 1: Forces and Torques on shafts

<table>
<thead>
<tr>
<th>Shafts</th>
<th>Torque (Nm)</th>
<th>Forces in vertical plane (N)</th>
<th>Forces in horizontal plane (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>T₁ = 61.12</td>
<td>Fₚ₁ = 2718.20</td>
<td>Fᵣ₃ = 988.70</td>
</tr>
<tr>
<td>Lay</td>
<td>T₂ = 165.02</td>
<td>Fₚ₂ = 5504.41</td>
<td>Fᵣ₃ = 2000.00</td>
</tr>
<tr>
<td>Output</td>
<td>T₃ = 476.84</td>
<td>Fₚ₂ = 5517.69</td>
<td>Fᵣ₃ = 1995.20</td>
</tr>
</tbody>
</table>

Fig - 9: Static structural analysis showing equivalent stress

Fig - 10: Static structural analysis showing total deformation

6. CONCLUSION

The objective was to design an automatic transmission system for an SAE BAJA vehicle. As a Continuously Variable Transmission device was installed, it was logical decision to couple it to a two-stage single speed gearbox. Although the CVT device provides a good range of speeds it does not provide the required reduction ratio hence a gearbox is needed. Thus, the output from the CVT must be further deepened to achieve the required speed. The entire system was designed, built and then tested on the vehicle for over 300 km. The system showed no fatigue and achieved all the performance parameters for which it was designed. However, the system can be further improved if a light weight and compact CVT device is used.

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REFERENCES