Analysis of Logarithmic Zero

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Abstract - *This paper represent a way to evaluate value of log (0) under assumption that log (0) has some value and it can be evaluated. Under such assumption we have evaluated a value of logarithmic zero in complex number form. We further assume that every number or every complex number can be represented in complex number form, so we carried further analysis on log (0), which further reduced it to arithmetic constant running from 0 to infinite.*

Key Words: Log of Complex Number, Hyperbolic function.

1. INTRODUCTION

Logarithm is a mathematical tool to deal with large value number, but it has some limitation. Limitation is, it fails at value zero and cannot be evaluated. But while solving Euler's equation, I came across a equation which provide value of log(0) in form of constant running from zero to infinity.

1.1 Basic definitions

The purpose of this section is to recall some results required in this paper. From basics of trigonometry we know that, Trigonometric tangent function of complex number are expressed as

$$\tan(x + iy) = \sin(x + iy) / \cos(x + iy)$$

we multiply the numerator and denominator by complex conjugate of denominator i.e. by cos(x - iy)

$$\tan(x + iy) = \sin(x + iy) \cos(x - iy) / \cos(x + iy) \cos(x - iy)$$

After some simplification, we get

$$\tan(x + iy) = \{ \sin(2x) + i \sinh(2y) \} / \{ \cos(2x) \cosh(2y) \} \quad ....(1)$$

Hyperbolic function equation as

$$\cosh x = (e^{x} + e^{-x})/2$$

$$\sinh x = (e^{x} - e^{-x})/2$$

Now assume a complex number as a+ib , So logarithm of complex number will be

$$\ln(a+ib) = (1/2) \ln(a^2 + b^2) + i \tan^{-1} (b/a)$$

put a =0 and b=1 in above equation we get

$$\ln (i) = (1/2) \ln (1) + i \tan^{-1} (1/0)$$

$$\ln (i) = i\pi/2$$

which can be generalized further as

$$\ln(i) = i (2m + (1/2))\pi$$

where m = 0,1,2,3,4,5,.........

$$i \ln(i) = - (2m + (1/2))\pi \quad .....(2)$$

where m = 0,1,2,3,4,5,.........

2. Logarithmic zero

It is well known from euler's formula that, for any real number x,

$$e^{ix} = \cos(x) + i \sin(x)$$

Similarly we can define

$$e^{i \ln x} = \cos(\ln x) + i \sin(\ln x)$$
$$x^{i} = \cos(\ln x) + i \sin(\ln x)$$

Put x=0 in above equation we get,

$$0^{i} = \cos(\ln 0) + i \sin(\ln 0)$$

Here we assume that ln0 behave like an value also 0 power anything is 0
therefore

$$0^{i} = 0 = \cos(\ln 0) + i \sin(\ln 0)$$

now divide both side by cos(ln 0) ,we get

$$1 + i \tan(\ln 0) = 0 \quad ....(3)$$

so eqn(3) is true only when

$$\tan(\ln 0) = i$$

indirectly when

$$\ln 0 = \tan^{-1} (i) \quad ....(4)$$

which represent value of natural logarithm of zero since it is in complex form so it cannot be represented in the real number graph of ln(x) and x where x is real number. Let us solve above equation further under the assumption that numbers can be represented in a form of complex number

$$\ln 0 = \tan^{-1}(i) = x + iy \qquad ....(5)$$

Let us solve for x and y as shown below,

$$i = \tan(x + iy)$$

Above equation can be expanded using eq^n(1) , As

$$i = [\sin(2x) + i\sinh(2y)] / [\cos(2x) + \cosh(2y)]$$

since it is a complex form we can compare real and imaginary separately to get value of x and y. solving real part of above equation which equals 0,

$$\sin(2x) = 0$$

Since sine of some angle is 0 for angle equals nπ, where n = 0,1,2,3....
so

$$2x = n\pi$$
$$x = n\pi/2 \qquad .....(6)$$
$$\text{where } n = 0,1,2,3,4....$$

solving imaginary part

$$\cos(2x) + \cosh(2y) = \sinh(2y)$$
$$\cos(2x) = \sinh(2y) - \cosh(2y)$$

$$\cos(2x) =$$
$$[(e^{2y} - e^{-2y})/2] - [(e^{2y} + e^{-2y})/2]$$

$$\cos(2x) = -e^{-2y}$$

Since we have value of x, substitute it in other equation to get value of y, as shown below

$$\cos(n\pi) = -e^{-2y}$$

Since we have $\cos(n\pi) = (-1)^n$

$$y = -[(n+1)\ln(-1)]/2$$
$$iy = -[(n+1)\, i \ln(-1)]/2$$
$$iy = -[(n+1)\, i \ln(i)]$$

Using eqn(2) ,we can expand above equation further as ,

$$iy = [(n+1)\ (2m + (1/2))\pi] \qquad .....(7)$$
$$\text{where } m = 0,1,2,3,4,5,.....$$

Now substitute the value of x and iy from eqn (6) and (7) in eqn (5) , to obtain equation of ln0

$$\ln 0 = \tan\text{-}1\,(i)$$
$$= x + iy$$
$$= n\pi/2 + [(n+1)\ (2m + (1/2))\pi]$$

further reduced to

$$\ln 0 = \tan\text{-}1\,(i)$$
$$= x + iy$$
$$= \pi\{n + 1/2 + 2m(n+1)\} \qquad ....(8)$$

$$\text{where } n = 0,1,2,3,4,5,......$$
$$\text{and } m = 0,1,2,3,4,5,.......$$

Since we are not aware of combination of m and n which will provide single and exact value of log(0).

3. CONCLUSIONS

This paper shows that log(0) can be evaluated under certain assumption. Since we do not know about combination of random number m and n due to which exact value cannot be evaluated.

REFERENCES

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