

Enhancement of Throughput & Spectrum Sensing of Cognitive Radio Networks

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Abstract - In cognitive radio networks, the secondary network (users) are allowed to utilize the frequency bands of primary network (users) when they are not currently being used. To support this function, the secondary users are required to sense the radio frequency environment, and once the primary user is found to be active, the secondary users have to vacate the channel within certain amount of time. There are two parameters related to channel sensing: probability of detection and probability of false alarm. The higher the detection probability, the better the primary users can be protected. However, from the secondary users' perspective, the lower the false alarm probability, the more chances the channel can be reused, thus the higher the achievable throughput for the secondary users. In this paper, We propose a novel cognitive radio system that exhibits improved throughput & spectrum sensing capabilities compared to the conventional opportunistic spectrum access. We study throughput of proposed system under a single high target detection probability. Finally we provide simulation result, in order to compare throughput of proposed & conventional method.

Key Words: Cognitive radio, opportunistic spectrum access, spectrum sensing, throughput maximization.

1.INTRODUCTION

The core technology that aims to alleviate the spectrum scarcity problem in wireless communications is cognitive radio. Cognitive radio allow access of unlicensed (secondary) users to frequency bands that are allocated to licensed (primary) users, in a way that does not affect the quality of service (QoS) of the licensed networks [1], [2]. With the proliferation of wireless communications technology in the last couple of decades, in many countries, most of the available radio spectrum has been allocated. This results in the spectrum scarcity which poses a serious problem for the future development of the wireless communications industry. On the other hand, careful studies of the usage pattern reveal that most of the allocated spectrum experiences low utilization. Recent measurements by Federal Communications Commission (FCC) show that 70% of the allocated spectrum in US is not utilized. Furthermore, time scale of the spectrum occupancy varies from milliseconds to hours [1]. This motivates the concept of frequency reuse that allows secondary networks to borrow

unused radio spectrum from primary licensed networks (users). The research in cognitive radio has been encouraged by the measurements of the Federal Communications Commission (FCC), which have revealed that there is a significant amount of licensed spectrum which is largely underutilized in vast temporal and geographic dimensions [3]. The FCC recognizing that there is a significant amount of available spectrum that is currently not being used under the current fixed spectrum allocation policy, has recently allowed the access of unlicensed (secondary) users to the broadcast television spectrum at locations where that spectrum is not being used by licensed services [4]. This unused broadcast television spectrum is often termed as "white spaces" and has been the focus of the IEEE 802.22 WRAN standard that aims to provide broadband wireless internet access to rural areas [5].

2. OPPORTUNISTIC SPECTRUM ACCESS

Two main approaches have been proposed for cognitive radio so far, regarding the way that the cognitive radio users can access the licensed spectrum: (i) through opportunistic spectrum access (OSA) [6], [7], according to which the secondary users are allowed to access a frequency band only when it is detected to be idle, and (ii) through spectrum sharing (SS) [8], [9], according to which the secondary users coexist with the primary users under the condition of protecting the latter from harmful interference. In this paper, we are going to focus on the former approach

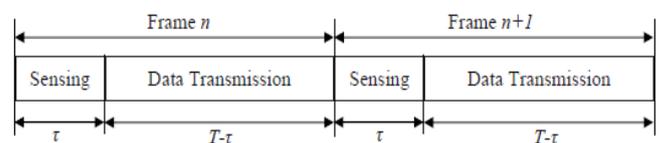


Fig -1 frame structure of the opportunistic spectrum access cognitive radio systems

The frame structure of the opportunistic spectrum access cognitive radio systems studied so far consists of a sensing time slot and a data transmission time slot, as depicted in Fig.1. According to this frame structure, a secondary user ceases transmission at the beginning of each frame and senses for the status of the frequency band (active/idle) for τ units of time, whereas it uses the remaining frame duration $T - \tau$ for data transmission. Therefore, an inherent tradeoff

exists in this frame structure between the duration of spectrum sensing and data transmission, hence the throughput of the cognitive radio system. According to the classical detection theory [10], [11], an increase in the sensing time results in a higher detection probability and lower false alarm probability, which in return leads to improved utilization of the available unused spectrum. However, the increase of the sensing time results in a decrease of the data transmission time, hence the achievable throughput of the cognitive radio system. This sensing-throughput tradeoff was addressed in [12], where the authors studied the problem of finding the optimal sensing time that maximizes the average achievable throughput of an OSA cognitive radio system under a single high target detection probability constraint for the protection of the QoS of the primary users. In this paper, we propose a novel cognitive radio system that overcomes the sensing-throughput tradeoff in opportunistic spectrum access cognitive radio networks by performing spectrum sensing and data transmission at the same time

3. PROPOSED COGNITIVE RADIO SYSTEM

We consider the cognitive radio system that is presented in Fig. 2. Let g and h denote the instantaneous channel power gains from the secondary transmitter (SU-Tx) to the secondary receiver (SU-Rx) and the primary receiver (PU-Rx), respectively. The channel power gains g and h are assumed to be ergodic, stationary and known at the secondary users similar to [8], [9], [13], [14], [15], [17], whereas the noise is assumed to be circularly symmetric complex Gaussian (CSCG) with zero mean and variance $\sigma^2 n$, namely $(0, \sigma^2 n)$. It should be noted here that knowledge of the precise channel power gain h is very difficult to be obtained in practice and therefore our results serve as upper bounds on the achievable throughput of the cognitive radio system.

The proposed cognitive radio system operates as follows. In the beginning, an initial spectrum sensing is performed, in order to determine the status (active/idle) of the frequency band. When the frequency band is detected to be idle, the secondary transmitter accesses it for the duration of a frame by transmitting information to the secondary receiver. The latter decodes the signal from the secondary transmitter, strips it away from the received signal, and uses the remaining signal for spectrum sensing, in order to determine the action of the cognitive radio system in the next frame. At the end of the frame, if the presence of primary users is detected, namely if the primary users started transmission after the initial spectrum sensing was performed, data transmission will be ceased, in order to protect the primary users from harmful interference. In the opposite case, the secondary users will access the frequency band again in the next frame. Finally, the process is repeated.

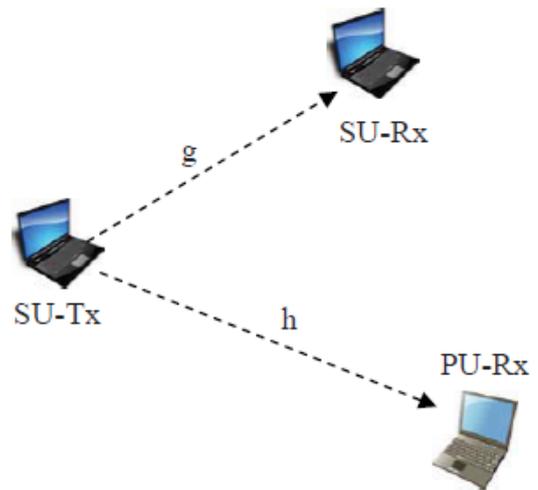


Fig -2 System model

3.1 Receiver structure

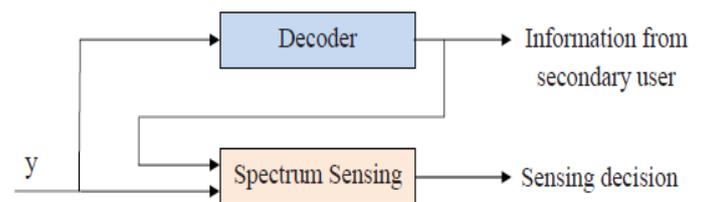


Fig -3 Receiver structure of proposed system

The receiver structure of the proposed cognitive radio system is presented in Fig. 3. The received signal at the secondary receiver is given by

$$y = \theta xp + xs + n \tag{1}$$

Where θ denotes the actual status of the frequency band ($\theta = 1$ if the frequency band is active and $\theta = 0$ if it is idle), xp and xs represent the received (faded) signal from the primary users and the secondary transmitter, respectively, and finally n denotes the additive noise. The received signal y is initially passed through the decoder, as depicted in Fig. 3, where the signal from the secondary transmitter is obtained. In the following, the signal from the secondary transmitter is cancelled out from the aggregate received signal y ; and the remaining signal

$$\tilde{y} = \theta xp + n \tag{2}$$

is used to perform spectrum sensing. This is the same signal that the secondary receiver would receive if the secondary transmitter had ceased data transmission, which is the conventional way that was proposed to perform spectrum sensing. Here, instead of using a limited amount of time τ , the whole duration of the frame T can be used for

spectrum sensing. This way, we are able to perform spectrum sensing and data transmission at the same time, thus maximizing the duration of both.

3.2 Frame structure

The frame structure of the proposed cognitive radio system is presented in Fig. 4 and consists of a single slot during which both spectrum sensing and data transmission are performed at the same time, using the receiver structure presented in the previous subsection. The advantage of the proposed frame structure is that the spectrum sensing and data transmission time are simultaneously maximized, whereas, more specifically, they are equal to the frame duration T . The significance of this result is twofold. Firstly, the increased sensing time:

i) Enables the detection of very weak signals from the primary users, the detection of which under the frame structure of Fig. 1 would significantly reduce the data transmission time, hence the throughput of the cognitive radio network,

ii) Leads to an improved detection probability, thus better protection of the primary users from harmful interference,

iii) Results to a decreased false alarm probability, which enables a better use of the available unused spectrum,

iv) Facilitates the use of more complex spectrum sensing techniques that exhibit increased sensing capabilities, but require higher sensing time (such as Cyclostationary detection [18] or several covariance-based spectrum sensing techniques [19], [20]), which prohibits their application for quick periodical spectrum sensing under the frame structure presented in Fig. 1,

v) The calculation of the optimal sensing time is no longer an issue, since it is maximized and equal to the frame duration T

vi) Continuous spectrum sensing can be achieved under the proposed cognitive radio system, which ensures better protection of the quality of service (QoS) of the primary networks.

The second important aspect is that the sensing time slot τ of the frame structure of Fig. 1 is now used for data transmission, which leads to an increase in the throughput of the cognitive radio network on the one hand, and facilitates the continuity of data transmission on the other

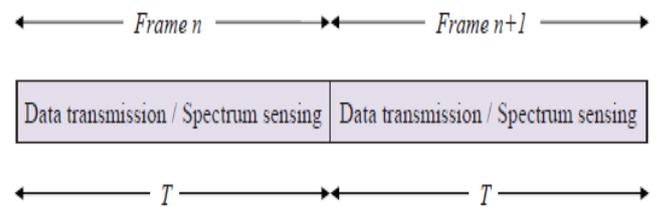


Fig -3 Frame structure of proposed system

4. AVERAGE ACHIEVABLE THROUGHPUT OF THE PROPOSED COGNITIVE RADIO SYSTEM UNDER A HIGH TARGET DETECTION PROBABILITY CONSTRAINT

In this section, we study the average achievable throughput of the proposed cognitive radio system and compare it with the respective achievable throughput of the cognitive radio system that operates based on the conventional frame structure depicted in Fig. 1. We consider, similar to the work in [12], a single high target detection probability constraint for the protection of the primary users from harmful interference. Considering the fact that the priority of a cognitive radio system is and should be the protection of the quality of service (QoS) of the primary network, a high target detection probability is required, in order to ensure that no harmful interference is caused to the licensed users by the secondary network. For instance, the target probability of detection in the IEEE 802.22 WRAN standard [5] is chosen to be 90% for a signal-to-noise ratio (SNR) as low as -20 dB for the primary user's signal at the secondary detector. We denote this target detection probability in the following by P_d . More specifically, we consider as in [12] the energy detection scheme [21] as a spectrum sensing technique, in order to determine the status (active/idle) of the frequency band. The detection and false alarm probability under the energy detection scheme are given by

$$\mathcal{P}_d = Q \left(\left(\frac{\epsilon}{\sigma_n^2} - \gamma - 1 \right) \sqrt{\frac{\tau f_s}{2\gamma + 1}} \right), \quad (3)$$

$$\begin{aligned} \mathcal{P}_{fa} &= Q \left(\left(\frac{\epsilon}{\sigma_n^2} - 1 \right) \sqrt{\tau f_s} \right) = \\ &= Q \left(\sqrt{2\gamma + 1} Q^{-1}(\mathcal{P}_d) + \sqrt{\tau f_s} \gamma \right), \quad (4) \end{aligned}$$

Respectively [12], where ϵ denotes the decision threshold of the energy detector, γ the received signal-to-noise ratio (SNR) from the primary user at the secondary detector, τ denotes the sensing time and finally f_s represents the sampling frequency. For a given target detection probability $P_d = P_d$, the decision threshold ϵ is given by

$$\epsilon = \sigma_n^2 \left(\sqrt{\frac{2\gamma + 1}{\tau f_s}} Q^{-1}(\bar{\mathcal{P}}_d) + \gamma + 1 \right). \quad (5)$$

We can now focus on the average achievable throughput of the cognitive radio system. The instantaneous transmission rate of the cognitive radio system when the frequency band is actually idle (\mathcal{H}_0) is given by

$$r_0 = \log_2 \left(1 + \frac{gP}{\sigma_n^2} \right). \quad (6)$$

However, considering the fact that perfect spectrum sensing may not be achievable in practice due to the nature of wireless communications that includes phenomena such as shadowing and fading, we consider the more realistic scenario of imperfect spectrum sensing, where the actual status of the primary users might be falsely detected. Therefore, in this paper, we also consider the case that the frequency band is falsely detected to be idle, when in fact it is active (\mathcal{H}_1). Following the approach in [15], [22], the instantaneous transmission rate in this case is given by

$$r_1 = \log_2 \left(1 + \frac{gP}{\sigma_n^2 + \sigma_p^2} \right), \quad (7)$$

Where σ_p^2 denotes the received power from the primary users

The average achievable throughput of the cognitive radio system that operates based on the conventional frame structure of Fig. 1 is given by

$$\bar{R}(\tau) = \bar{R}_0(\tau) + \bar{R}_1(\tau), \quad (8)$$

where $\bar{R}_0(\tau)$ and $\bar{R}_1(\tau)$ are given by

$$\bar{R}_0(\tau) = \frac{T - \tau}{T} \mathcal{P}(H_0) (1 - \mathcal{P}_{fa}(\tau)) r_0, \quad (9)$$

$$\bar{R}_1(\tau) = \frac{T - \tau}{T} \mathcal{P}(H_1) (1 - \mathcal{P}_d(\tau)) r_1, \quad (10)$$

Respectively. In the equations above, T represents the frame duration, $\mathcal{P}(\mathcal{H}_0)$ the probability that the frequency band is idle, and $\mathcal{P}(\mathcal{H}_1)$ the probability that the frequency band is active.

Under the proposed cognitive radio system, spectrum sensing is performed simultaneously with data transmission, whereas the sensing time and data transmission time are equal to the frame duration T , as seen in Fig. 4. Therefore,

the average achievable throughput of the proposed cognitive radio system is given by

$$\bar{C} = \bar{C}_0 + \bar{C}_1, \quad (11)$$

Where \bar{C}_0 and \bar{C}_1 denote the average achievable throughput when the frequency band is actually idle and active (but falsely detected to be idle), respectively, and are given by respectively.

$$\bar{C}_0 = \mathcal{P}(H_0) (1 - \mathcal{P}_{fa}(T)) r_0, \quad (12)$$

$$\bar{C}_1 = \mathcal{P}(H_1) (1 - \mathcal{P}_d(T)) r_1, \quad (13)$$

For a target probability of detection P_d , we can now show that the proposed cognitive radio system exhibits higher average achievable throughput compared to the cognitive radio system that operates based on the conventional frame structure shown in Fig. 1. Following the FCC requirements in [4], the secondary users should detect a worst-case SNR from the primary users, regardless if the spectrum sensing is performed at the receiver or the transmitter. This worst-case SNR is denoted here by $\bar{\gamma}$. From the classical detection theory [10], [11], it is known that for a target probability of detection P_d , the higher the sensing time, the lower the probability of false alarm P_{fa} . Therefore, for a target probability of detection $P_d = P_d$ and sensing time $0 < \tau \leq T$, it results from the equation (4) that

$$\begin{aligned} \mathcal{P}_{fa}(\tau) &= Q \left(\sqrt{2\bar{\gamma} + 1} Q^{-1}(\bar{\mathcal{P}}_d) + \sqrt{\tau f_s \bar{\gamma}} \right) \\ &\geq Q \left(\sqrt{2\bar{\gamma} + 1} Q^{-1}(\bar{\mathcal{P}}_d) + \sqrt{T f_s \bar{\gamma}} \right) = \mathcal{P}_{fa}(T), \end{aligned} \quad (14)$$

Considering the fact that the complementary cumulative distribution function of the standard Gaussian $\mathcal{Q}(x)$ is a decreasing function of x . As a result, for a sensing time $0 < \tau \leq T$, it results from the equations (8)-(14) that

$$\begin{aligned} \bar{R}(\tau) &= \bar{R}_0(\tau) + \bar{R}_1(\tau) = \frac{T - \tau}{T} \mathcal{P}(H_0) (1 - \mathcal{P}_{fa}(\tau)) r_0 + \\ &\quad + \frac{T - \tau}{T} \mathcal{P}(H_1) (1 - \bar{\mathcal{P}}_d) r_1 \\ &< \mathcal{P}(H_0) (1 - \mathcal{P}_{fa}(\tau)) r_0 + \mathcal{P}(H_1) (1 - \bar{\mathcal{P}}_d) r_1 \\ &\leq \mathcal{P}(H_0) (1 - \mathcal{P}_{fa}(T)) r_0 + \mathcal{P}(H_1) (1 - \bar{\mathcal{P}}_d) r_1 \\ &= \bar{C}_0 + \bar{C}_1 = \bar{C}, \end{aligned} \quad (15)$$

i.e. that the average achievable throughput of the proposed cognitive radio system for a target detection probability $P_d = P_d$ is higher compared to the respective of the cognitive

radio system that employs the frame structure depicted in Fig. 1, namely it results that

$$\bar{C} > \bar{R}(\tau) \tag{16}$$

For a sensing time $0 < \tau \leq T$

5. SIMULATION RESULTS

In this section, we present the simulation results for the proposed opportunistic spectrum access cognitive radio system using the energy detection scheme as a spectrum sensing technique. The frame duration is set to $T=100$ ms, the probability that the frequency band is idle is considered to be $P(\mathcal{H}0) = 0.6$, whereas the sampling frequency f_s is assumed to be 6 MHz. The channels g and h are assumed to follow the Rayleigh fading model and more specifically, they are the squared norms of independent CSCG random variables that are distributed as $\mathcal{CN}(0, 1)$ and $\mathcal{CN}(0, 10)$, respectively.

In chart 1, the average achievable throughput versus the sensing time τ is presented for the proposed cognitive radio system (solid line) and the cognitive radio system that employs the conventional frame structure of Fig. 1 (dashed line), for the case of a single high target detection probability constraint. The received signal-to-noise ratio (SNR) from the secondary transmitter at the secondary receiver is considered to be $SNR_s = 20$ dB as in [12], the target probability of detection is set to $P_d = 99.99\%$, in order to effectively protect the primary users from harmful interference, whereas different values of the target detection signal-to-noise ratio from the primary user (denoted by SNR_p) are presented. One can clearly see that the average achievable throughput of the proposed cognitive radio system (solid line) is significantly higher compared to the respective achievable throughput of the cognitive radio system that employs the conventional frame structure of Fig. 1 (dashed line). This throughput improvement can be explained by the fact that the whole duration of the frame T is used for data transmission, as opposed to the conventional frame structure of Fig. 1, where only a part of the frame is used for data transmission (i.e. $T - \tau$). Moreover, the improved sensing capabilities of the proposed cognitive radio system also contribute to the throughput improvement of the cognitive radio system by enabling a more efficient usage of the available unused spectrum. More specifically, it can be seen from Fig. 5 and the equation (4) that for the same target probability of detection P_d , the probability of false alarm P_{fa} for the optimal sensing time under the conventional frame structure is higher compared to the respective false alarm probability of the proposed cognitive radio system. The latter remark can be explained by the fact that the whole duration of the frame T is used for spectrum sensing in the proposed system, as opposed to merely a part of the frame under the conventional frame structure of Fig. 1.

In chart 2, the average achievable throughput is presented versus the target probability of detection P_d , for a target detection signal-to-noise ratio from the primary user equal to $SNR_p = -22$ dB. It can be clearly seen from chart 2 that the average achievable throughput under the proposed cognitive radio system is significantly higher compared to the respective achievable throughput of the system that employs the frame structure presented in Fig. 1, whereas the decrease in the average achievable throughput as the target probability of detection P_d receives higher values is small, especially compared to the respective of the secondary users that employ the conventional frame structure of Fig. 1. This means that the proposed cognitive radio system can provide better protection for the primary users on the one hand, while achieving an increased throughput for its users on the other, even for very high values of target detection probability and very weak signals from the primary users. This can be further seen from chart 3, where the average achievable throughput is presented versus the target detection signal-to-noise ratio from the primary users (SNR_p), for a target probability of detection equal to $P_d = 99.99\%$.

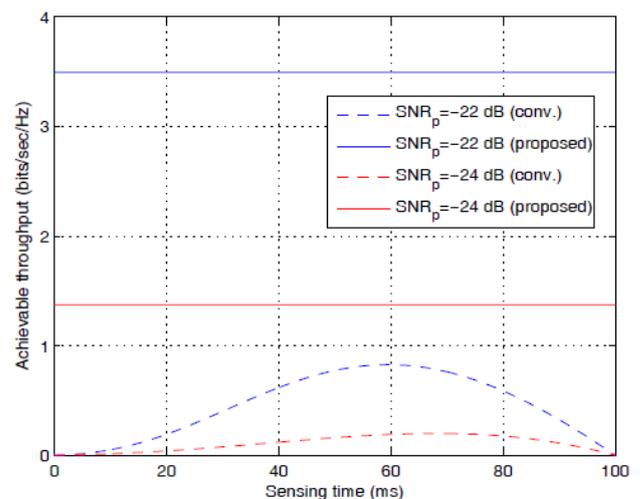


Chart -1 Average achievable throughput of the proposed and conventional opportunistic spectrum access cognitive radio system versus the sensing time τ , for various values of the target detection SNR from the primary user (SNR_p) and for a target detection probability $P_d = 99.99\%$.

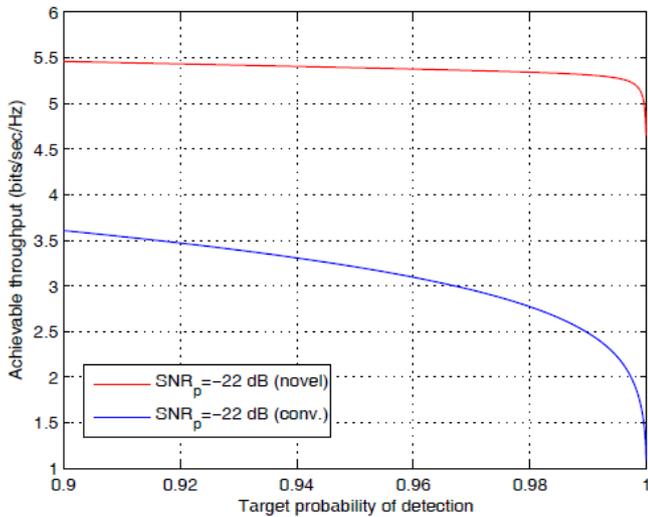


Chart -2 Average achievable throughput of the proposed and conventional opportunistic spectrum access cognitive radio system versus the target probability of detection P_d for various values of the target detection SNR from the primary user (SNR_p)

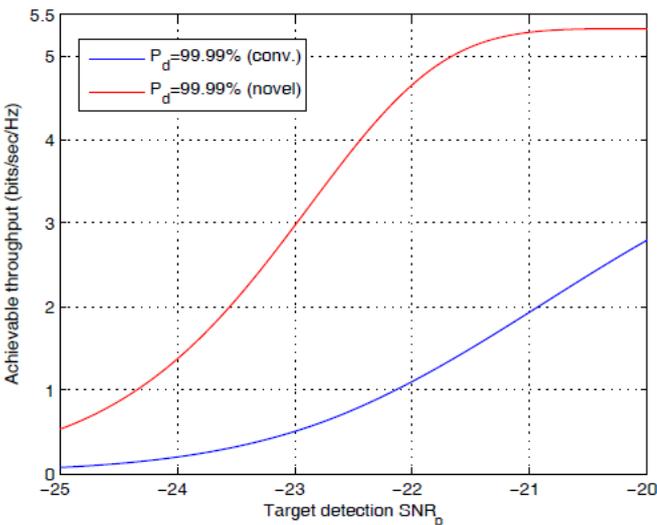


Chart -3: Average achievable throughput of the proposed and conventional opportunistic spectrum access cognitive radio system versus the target detection SNR from the primary user (SNR_p) for a target detection probability $P_d = 99.99\%$.

6. CONCLUSIONS

We proposed a novel cognitive radio system that significantly improves the achievable throughput of opportunistic spectrum access cognitive radio systems by performing data transmission and spectrum sensing at the same time. More specifically, we studied the average achievable throughput of the proposed cognitive radio

system under a single high target detection probability constraint and showed that it can achieve significantly improved throughput compared to the respective conventional cognitive radio systems. Furthermore, we provided simulation results, from which it is clear that throughput & spectrum sensing is improved in proposed cognitive radio system than that of conventional cognitive radio system.

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