Isomorphism identification and Structural Similarity & Dissimilarity Among The Kinematic Chains Based On [WSSP] Matrix

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Abstract - *The generation of new well defined kinematic chains is impossible without checking isomorphism among kinematic chains. A new approach based on the weighted squared path [WSSP] technique is introduced in this paper and also determine the structural similarity and dissimilarity in the kinematic chains using the same approach. Two kinematic chains of a family with given number of links and degree of freedom are not completely similar from the structural connectivity point of view. However, some amount of structural similarity may still exist.*

First kinematic chains are represented by the squared shortest path [SSP] matrix and drive weighted Squared shortest path [WSSP] matrix. By comparing the sum of all the elements of [WSSP] matrix, the isomorphism among the kinematic chains can be recognized easily.

Moreover, Weighted Squared shortest path [WSSP] matrix has the more information about the connectivity of the links as well as the type links in the form of mutual interactive effect of the relative weight of the connecting links. The row sum of [WSSP] is determined from [WSSP] matrix hence may be considered more comprehensive to measure structural similarity and dissimilarity among the kinematic chains. In this paper an attempt is made to correlate row sum of [WSSP] matrix which is represented by Rs-i(WSSP) with structural similarity and dissimilarity among kinematic chain

At last, Some examples are carried out to justified the reliability of the method

Key Words: KC, [SSP], [WSSP]

1.INTRODUCTION

One of the keynote areas of structural synthesis of the kinematic chains is to derive all possible mechanisms from a given kinematic chain at the beginning phase of conceptual design of Structural synthesis of the kinematic chain and mechanism. During the enumeration of kinematic chains there are some chance of enumerate duplicate kinematic chain because a lack of reliable method which leads to isomorphism among the kinematic chains and that is the illness of kinematic chains which must have to eliminate before the development of distinct mechanism so that the designer has the numerious option to select the best and desire optimum mechanism depending upon the requirement. As mentioned before In the course of enumeration of kinematic chains, duplication or isomorphism may be possible. So for the recognition of isomorphism, the researchers have proposed many approaches in past decades years. The methods proposed so far are based on distance matrix [1,3], an adjacency matrix [3] to determine the structurally distinct mechanisms of a kinematic chain. The flow matrix method [4], and the row sum of extended adjacency matrix methods [5,6] are used, characteristic polynomial of a matrix [7], Minimum code [8], identification code [9], link path code [10], path matrices [11], a Multivalued Neural Network approach [12], a mixed isomorphism approach [13], Hamming value [14], an artificial neural network approach [15], the theory of finite symmetry groups [16,17], Interactive Weighted Distance Approach [19], the representation set of links by Vijayananda [18], are used to characterize the kinematic chains. Among of these methods either have a lack of uniqueness or very time consuming. Hence, there is a need to develop an efficient and reliable method to detect isomorphism in kinematic chains.

In the recent method, the kinematic chains are represented by the squared shortest path [SSP] matrix which has the information about the type of the links existing in a kinematic chain and their connectivity to each other. The structural invariants are derived from [WSSP] matrix using software Matlab which is the sum of all elements of [WSSP] matrix and called as ∑ [WSSP].

This unique code is treated as a recognition or classification number of the kinematic chain. Therefore [WSSP] code is used to detect isomorphism among the kinematic chains. If ∑ [WSSP] is same for two kinematic chains, they will be treated as isomorphic chains otherwise non isomorphic chains. No counterexample has been found in the detection of isomorphism in 6- link, 8-links, 10-link and 12-links, one of kinematic chains. It is assumed that proposed method will be able to detect isomorphism among the kinematic chains having number of links more than eight.

Furthermore, Two kinematic chains of a family with given number of links and degree of freedom are not completely similar from the structural connectivity point of view. However, some amount of structural similarity may still exist in the form of either same number of binary, ternary –n- nary links and/or same number of E-, Z,-D -and V-chains in the two kinematic chains. But on the bases of arrangements of E-, Z,-D, -and V- chains in the kinematic chains the structural similarity and dissimilarity comes into the picture. The Weighted squared shortest path distance matrix [WSSP] as discussed in section, has the more information of the connectivity of the links as well as the type links in the form of mutual interactive effect of the relative weight of the connecting links as compare to the usual (0,1) adjacency matrix. The row sum of [WSSP] are determined from [WSSP] matrix hence may be considered more comprehensive to measure structural similarity and dissimilarity among the kinematic chains. In this dissertation an attempt is made to correlate row sum of [WSSP] matrix which is resented by Rs-i(WSSP) with structural similarity and dissimilarity among kinematic chain

2.0 Basic Concepts

2.1 Definition of isomorphism

An isomorphism is a homomorphism that is one-to-one and onto for more precise definition an isomorphism is homomorphism and bijection

Homomorphism is a kind of function that sustain the group structure in each group, it is a tool for comparing two group for similarities. Some time two groups are more than similar they are identical in this case the groups are no longer called homomorphism, instead of called isomorphism

Homorphism function does not have to be 1-1(aka injection), it is not have to be injection
Suppose there are two group X and Y. It is possible many elements of X are mapped to the same elements in Y similarly,

Function does not have to be onto, it is not have to be surjection but for group X and Y to be identical we need the homorphism function to be one to one(1-1) and onto, function need to be injection and surjection

These way we compare each element in X with unique element in Yand vice versa

2.2 Isomorphic Graphs

Two graphs X and Y are referred to be isomorphic if , their number of elements (vertices and edges) are equal and their edge connectivity is preserved.

There must present a function 'f' from vertices of X to vertices of Y
[f: V(X) ⇒ V(Y)], such that
case (1): f is a one-to-one and onto (bijection)
case (2): f preserves adjacency of vertices,
Suppose if the edge {U, V} ∈ X, then the edge {f(U), f(V)} ∈ Y, then X ≡ Y(Isomorphic graph)

2.3 Isomorphism among the kinematic chains

Isomorphic kinematic chains have neighboring kind of relationship between the links and the presence of one to one correspondence between them. If the chains falsly recognized as isomorphics, result in lesser distinct kinematic chains. The chance of dublication may occur because of adaptation of non relable method for development of new kinematic chains

2.4 Degree of the link d (li):

The degree of a link actually represents the type of the link, such as binary, ternary, quaternary links etc. Let the degree of i^{th} link in a kinematic chain be designated d(li) and d(li) = 2, for binary link, d(li) = 3, for ternary link, d(li) = 4, for quaternary link and d(li) = n, for n-nary link.

2.5 Squared shortest path matrix [SSP]:

The path between two links i and j is an alternating sequence of links and joints starting from link i and terminating at link j. The sum of the joints in the path is called path length or path distance, the shortest of all the paths is called shortest path distance. The path distance does not consider the degree of links in the path, i.e. the shortest path length will be counted as two if on the shortest path of two links there is either a binary, ternary, quaternary or any polygonal link. In the present work a new matrix is proposed. The proposed squared shortest path distance is the least of the summation of the squared values of degrees of links between i and j. The [SSP] is represented as a square symmetric matrix of size n x n, where n is the number of links in a KC.

$$[SSP] = \{d_{ij}\}_{nxn}$$

$$\{d_{ij}\}_{nxn} = \begin{cases} 1, \text{ if } i^{th} \text{ and } j^{th} \text{ are directly connected} \\ \text{Summation of squared values of degrees of links between } i^{th} \text{ and } j^{th} \text{ links on shortest path for } i \neq j \\ (dli)^2, \text{ i.e Square of the degree of } i^{th} \text{ link if } i = j \end{cases}$$

Where

dij= Summation of squared values of degrees of links between i and j on the shortest path for i ≠ j, 1 if i and j are directly connected and 0 for i = j.

2.6 Relative weight of degree of links:

The squared shortest path distance [SSP] matrix does not provide any information about the exact sequence of types of links (i.e binary, ternary, quaternary etc) existing in a KC in a given path.

Therefore, this information is included in the weighted squared shortest path distance matrix [WSSP] in the form of relative importance of the degree of ith link and jth link and vice versa. The relative weight of the degree of the links (wij) is defined as the ratio between the degree of ith link and jth link and given as

wij= d (li) / d (lj)
wji= d (lj) / d (li)

$$w_{ij} = d(l_i)/d(l_j)$$

and

$$w_{ji} = d(l_j)/d(l_i)$$

2.7 Mutual interactive effects of relative weights:

Mutual interactive effect of relative weights is defined as

$$W_{ij} = \frac{w_{ij} + w_{ji}}{2}$$

The weigh matrix [W] derived from the relative weights of degree of links is given as

$$[W] = \{W_{ij}\}_{nxn}$$

The weight matrix [W] derived from the relative weights of degree of links is given as
{Wij}nxn

2.8 Weighted squared shortest path: [WSSP] matrix:

Weighted squared shortest path distance matrix [WSSP] takes the combined influence of squared shortest path distance between the links and their relative weights. Each row of [WSSP] represents a link of the KC. The weighted squared shortest path distance matrix is a square matrix of size n x n and defined as:

WSSPD = [SSP] * [W]

$$[WSSP] = \{g_{ij}\}_{nxn}$$

2.9 Structural Invariant Of A Kinematic Chain:

The proposed [WSSP] matrix contains all necessary information of the type of the link and their arrangements. Therefore the sum of all the elements of the [WSSP] matrix is considered as an invariant of a kinematic chain which may be used to detect isomorphism

$$\sum [WSSP] = \left\{ \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} g_{ij} \right\}$$

Where i, j = 1, 2 3 ……………………n

3. Architect Of The Proposed Method:

3.1 Procedure of identifying isomorphism

Step1: Development of [SSP] from given KC.
Step2: Development of [WSSP] from [SSP] and [W] matrix.
Step3: The sum of all the elements of the [WSSP] matrix is considered as an invariant of a kinematic chain which may be used to detect isomorphism

4. Representation Of row sum Of [WSSP] Matrix With Polar Diagram

For n- link kinematic chains, n row sum of [WSSP] matrix obtained. These values can be represented on the polar diagram. The steps involved in the construction of polar diagram are discussed as under.

- Choose a point 'O' as origin and draw an equal radial line/rays at 3600/n angles from the Origin 'O' and mark them as 1, 2, 3-----n.
- Mark the point RS-1 on the radial line 0-1. The distance 0 RS-1 actually represents the row sum of [WSSP] matrix of kinematic chain link first.
- Similarly mark other point RS-1, RS-2, RS-3, -------- RS-n on lines 0-2, 0-3, 0-4,-----0-n taking lengths 0 RS-2, 0 RS-3,0 RS-4 ,----0 RS-n equal to

2nd,3rd,4th,-----nth

- Represents the row sum of[WSSP] matrix of kinematic chain respectively.

- Joint the point RS-1, RS-2, RS-3, ------ RS-n in order by straight lines. Polygon RS-1, RS-2, RS-3,RS-4,RS-5 ------ RS-n is obtained. The polygon is called as the polar diagram of row sum of [WSSP] of the kinematic chain as shown in figure 2(a)

- Following step 1 and 4, polar diagram of the structural Eigen spectrum of second Kinematic chain under comparison is drawn marking the point as RS-1', RS-2',RS-3', RS-4', RS-5', ------ RS-n' as shown in fig 2(b)

- Now super impose the polar diagram of to kinematic chains having the same origin 'O' fig 2(c)

5. Structural Similarity And Dissimilarity Based On Polar Diagrams

Row sum of [WSSP] matrix represents the structural pattern of a kinematic chain uniquely. As the polar diagram is the graphical representation of the row sum of [WSSP] matrix, hence intern represents the structural pattern/parameter of the kinematic chain without ambiguity. Therefore, the polar diagram can be used to mepattern/parameter of the kinematic chain without ambiguity. Therefore, the polar diagram can be used to measure the structural similarity and dissimilarity very useful for optimum selection of kinematic chains at the conceptual stage of mechanism-design problem.

For this purpose, polar diagram for diagram for two chains to be compared, based on the row sum of [WSSP] matrices are drown and then to polar diagram are super imposed on the same origin 'O' as discussed in previous section

5.1 Analysis of Superimposed Polar Diagram

The following observations are obtained from the superimposed polar diagram shown in figure 2(c):
Polygon RS-1, RS-2, RS-3, RS-4, ------RS-n represents the polar diagram of the row sum of the [WSSP] matrix of given kinematic chains.

Polygon RS -1', RS-2', RS-3', RS-4',------RS'-n represents the polar diagram of the Eigen spectrum for the second kinematic chain.

The area in super imposed diagram, which is common to both polygons, is the measure of the structural similarity of two kinematic chains.

The hatched area within the polygon line of super imposed diagram is responsible for the structural dissimilarity of two kinematic chains. The hatched area is a function of difference of absolute row sum of [WSSP] matrix and may be written as.

Hatched area =f {(RS-i- RS-i'), ϴ}

A critical observation reveals that the distances (RS-1- RS-1') (RS-2- RS-2') ----(RS-n- RS-n') are directly proportional to the hatched area shown in fig.8.1 (c). Hence, we can say that the sum of difference in corresponding absolute row sum of [WSSP] values is the measure of structural dissimilarity of two kinematic chains.

5.2 Coefficient of structural dissimilarity (Cds)

It may be define as the ratio between sum of the difference of the corresponding absolute row sum of [WSSP] matrix to the maximum absolute of row sum of the [WSSP] matrix for two kinematic chains, designated as Cds and given as under:

$$Cds(i) = 1/B\left\{\sum_{i=1}^{i=n} |RS-i-RS-i'|\right\}$$

Where

$$B = \max.of\left\{\sum_{i=1}^{i=n} |RS-i| \; and \sum_{i=1}^{i=n} |RS-i'|\right\}$$

$$\left\{\sum_{i=1}^{i=n} |RS-i-RS-i'|\right\}$$

= sum of the difference of the corresponding absolute row sum of the [WSSP] matrix
Cds = coefficient of structural dissimilarity.

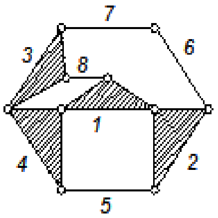
5.3 Coeffiecient Of Structural Similarity (Cs)

The coefficient of structural similarity (Cs) is define by the equation below

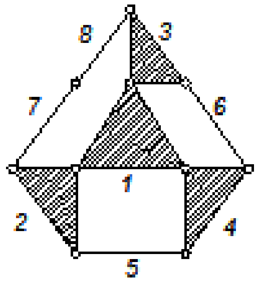
Cs = (1-Cds)

6. ILLUSTRATIVE EXAMPLE

To determine the coefficient of structural similarity and dissimilarity, let us consider the example of 8-links, 10-joints, 1-F kinematic chains as shown in fig 1 and 2. the row sum of [WSSP] matrix of these kinematic chains are determined using software of Matlab



(a)



(b)

Fig. 1 (a) and (b) 8-links, 10-joints, 1-F kinematic chains

Step-1

Degree Vector

The degree vector for the kinematic chain shown in figure 1(a) and (b) are written as,
d1=[3 3 3 3 2 2 2 2 2 2]
d2=[3 3 3 3 2 2 2 2 2 2]

Step-2

Squared shortest path distance matrix [SSP] of Fig. 1(a) and (b)

0	4	4	4	1	1	1	13
4	0	4	1	13	13	1	1
4	4	0	1	1	13	13	1
4	1	1	0	9	1	9	9
1	13	1	9	0	9	9	9
1	13	13	1	9	0	9	18
1	1	13	9	9	9	0	9
13	1	1	9	9	18	9	0

0	1	1	1	9	9	9	9
1	0	8	4	1	13	1	4
1	8	0	4	13	1	4	1
1	4	4	0	1	1	13	13
9	1	13	1	0	9	9	13
9	13	1	1	9	0	13	9
9	1	4	13	9	13	0	1
9	4	1	13	13	9	1	0

Step-3

Sum of all the elements of [WSSP]matrix of fig 1(a)

2933.3

Sum of all the elements of [WSSP]matrix of fig 1(b)

2916.7

Our method reports that both the KC shown in Fig.1 (a) and Fig.1 (b) are non-isomorphic as the values of sum of all the elements of [WSSP] of Fig. 1 (a) and (b) are different for both the KC.

Structural Similarity And Dissimilarity Based On Polar Diagrams

Row sum of [WSSP] of kinematic chain a shown in Fig. 1(a)

RS-i = [316.6667, 233.3333, 233.3333, 316.6667, 500.0000, 416.6667, 416.6667, 500.0000]

Row sum of [WSSP] of kinematic chain a shown in Fig. 1(b)
RS-I'= [325.0000, 266.6667, 266.6667, 308.3333, 458.3333, 458.3333, 416.6667, 416.6667]

The polar diagram for RS-I and Rs-I' are shown in fig 2(a) and(b) respectively. The superimposed polar diagram for the chains is represented in fig 2(c)

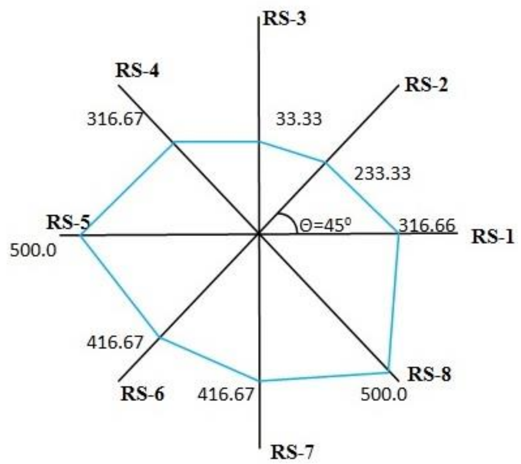


Fig. 2.(a) Polar plot of the row sum of [WSSP] matrix for kinematic chain as shown in Fig. 1(a).

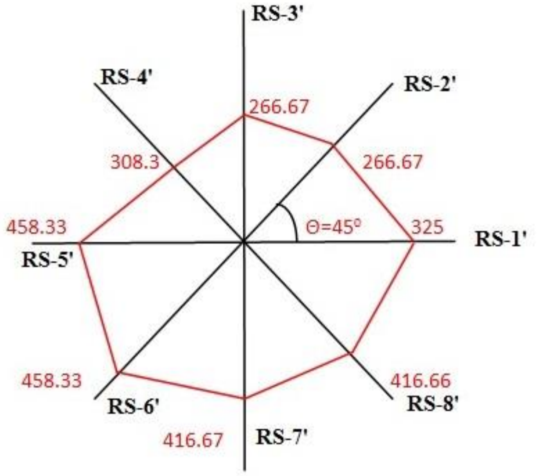


Fig. 2.(b) Polar plot of the row sum of [WSSP] matrix for kinematic chain as shown in Fig. 1(b)

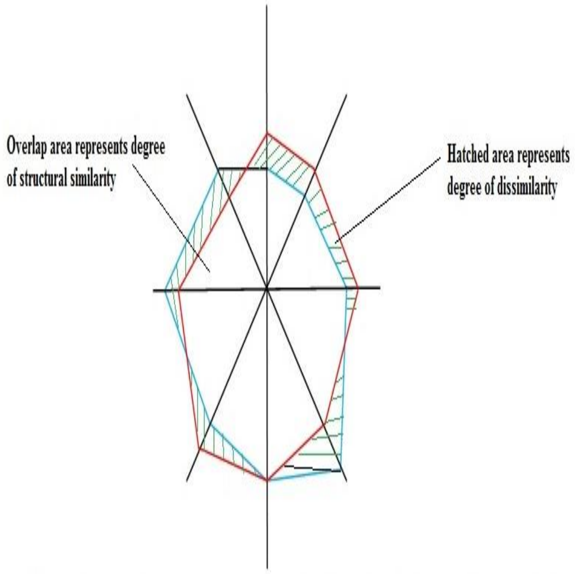


Fig. 2.(c) Super imposed Polar plot of the row sum of [WSSP] matrix for kinematic chain 1(a) and 1(b) as shown in Fig. 1

Coefficient of structural dissimilarity (Cds)

The coefficient of structural dissimilarity between two kinematic chains is determined using 8.1, 8.2 and 8.3 as under

Sum of absolute row sum values derived from [WSSP] matrix is

$$RS-i=\left\{\sum_{i=1}^{i=n}|RS-i|\right\}$$

|316.6667|+| 233.3333|+ |233.3333|+| 316.6667|+ |500.0000|+ | 416.6667|+| 416.6667|+| 500.0000|

= 2933.3

$$RS-i'=\left\{\sum_{i=1}^{i=n}|RS-i|\right\}$$

|325.0000|+|266.6667|+ | 266.6667|+| 308.3333|+ | 458.3333|+| 458.3333|+ | 416.6667|+ | 416.6667|

= 2916.7

$$B=\max.of\left\{\sum_{i=1}^{i=n}|RS-i|\; and \sum_{i=1}^{i=n}|RS-i'|\right\}$$

= 2933.3

$$Cds(i)=1/B\left\{\sum_{i=1}^{i=n}|RS-i-RS-i'|\right\}$$

= 1/2933.3{|316.6667-325.0000|+|233.3333-266.6667|+|233.3333-266.6667|+ |316.6667-308.3333|+|500.0000- 458.3333|+|416.6667-458.3333|+ |416.6667 - 416.6667|+|500.0000-416.6667|

= 250.0001/2933.3

= 0.08522

= 8.55%

Coefficient of structural similarity between two kinematic chains shown in fig is determined using equation as

Cs = 1-Cds

= (1- 0.08522)

= 0.91477

= 91.477%

Hence we can say that8.55 % dissimilarity and 91.477% similarity exit between two kinematic chains shown in fig 2. (a) and (b). in this way we can compare two kinematic chains on the basis of coefficient of structural similarity and dissimilarity for optimum selection chains at conceptual stage of mechanism- design problem.

7. RESULT

The proposed method is presented for the identification code for the given simple jointed kinematic chain. The methodology is applied on 8-links, 10-joints, 1-F simple jointed kinematic chains

For storing and retrieving the structural information in the computer, the KC's are synthesized with the help of Weighted squared shortest path distance matrix: [WSSP]. Applying the Weighted squared shortest path distance matrix: [WSSP] in the C-programming and running the program in the MATLAB software to obtain desired results.

The only one structural invariant derived from Weighted squared shortest path distance matrix: [WSSP] by using the MATLAB software. This invariant is same for structurally equivalent chains and different for distinct chains. These invariants are used as the identification number of simple jointed kinematic chains and to detect isomorphism in the multiple jointed kinematic chains. If these invariants are the same the two simple jointed kinematic chains are isomorphic otherwise not.

8. Conclusions

The kinematic structural synthesis is the systematic development of kinematic chains and mechanisms derived from the kinematic chains. The kinematic chains can be represented by their Skelton which is an abstract representation of kinematic chains. During the course of development of kinematic chains duplication is possible. To avoid this duplication, an isomorphic test is required. For this purpose, numbers of methods are proposed in recent years. But those methods have either the lack of uniqueness or sometimes fail in the detection of isomorphism among the kinematic chains. Therefore scope of further research is needed to detect isomorphism. In the proposed method, the kinematic chains are represented by the weighted squared shortest path distance [WSSP] matrix which has the information of the type of the links existing in a kinematic chains and their connectivity to each other. The structural invariants is derived from [WSSP] matrix using software Matlab which is the sum of all elements of [WSSP] matrix and called as ∑[WSSP]. This unique invariants is treated as an identification or characterization number of the kinematic chain. Therefore [WSSP] invariants are used to detect isomorphism among the kinematic chains. If ∑ [WSSP] is same for two kinematic chains, they will be treated as isomorphic chains otherwise non isomorphic chains. No counter example has been found in detection of isomorphism in 6- link & 8- link, one dof kinematic chains. It is expected that proposed method will be able to detect isomorphism among the kinematic chains having number of links more than eight

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