PID Control of a Quadrotor UAV

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Abstract - *Quadrotors have a variety of applications in real time e.g. surveillance, inspection, search, rescue and reducing the human force in undesirable conditions. Quadrotor UAV is equipped with four rotors for the purpose of stability but this will make quadrotor more complex to model and control. In this paper, intelligent controller is designed to control attitude of quadrotor UAV. The paper presents a detailed simulation model for a Quadrotor UAV and PID control strategy is designed to implemented for four basic motions; roll, pitch, yaw, and Z Height. The controller presented in this paper is very simple in structure and it is easy to implement. The main objective of this paper is to get the desired output with respect to the desired input. Simulink model and results are shown at the end of the paper*

***Key Words*: Quadrotor, UAV, PD Control, Dynamics, Roll, Pitch, Yaw**

1. INTRODUCTION

A quadrotor or quadcopter can be defined as a “multi-rotor copter with four arms, each of which have a motor and a propeller at their ends” [**1**].

UAVs are classified depending upon type of wings as shown in figure 1.

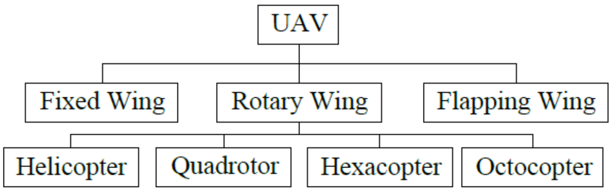

Figure -1: UAV Classification

Quadrotor lies in the category of Rotary wing class of UAVs they are usually used in the applications that required hovering flights such as search and rescue operations, security, journalism, emergency response and in military applications [**2**].

Quadrotors UAV are different from a helicopter for two main reasons

a) The way they are controlled i.e. helicopters are not fully autonomous
b) Second is helicopter can change their blades angle of attack while quadrotor lacks this functionality.

Quadrotors have many advantages over traditional helicopters such as

a) Quadrotors have Small sizes
b) They are safe to use for civilians because of small rotor.
c) Less complex mechanical structure
d) They are very easy to maintain
e) Due to their maneuverability, they are safer in hazardous situations

This paper presents different classical and modern control strategies for the control of quadrotor. Simulations result and comparison of all control techniques are presented at the end of this paper.

2. Dynamic Modeling of Quadrotor

Quadrotor UAV flies with the assistance of four motors as shown in figure below. For the purpose of vertical flight two opposite motors rotates in the similar direction. The combination of opposite motors rotates in the similar direction for stabilization on the x-axis other combination of opposite motors keeps it stabilizes on the y-axis [**3**].

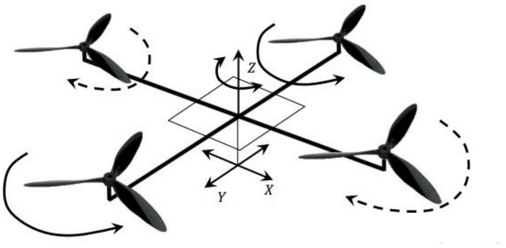

Figure -2: Quadrotor Motion

Quadrotor UAV is a 6 DOF aircraft, so there are 6 variables (x, y, z,$\emptyset$,θ and φ) that are used to express its orientation in space. $\emptyset$, θ, and φ are also known as Euler’s angles. Details of each variable are as follows [**4**]

- x and y: These variables are used to represent the position of Quadrotor in space.
- Z: Defines the altitude of quadrotor
- ϕ: ϕ or Roll angle it represent angle about the x-axis

- θ : θ or Pitch angle it represent angle about the y-axis
- φ : φ or Yaw angle it represent angle about the z-axis

In this Paper, Newton-Euler formalism is used to derive the dynamics of the quadrotor. Following are the assumptions made for the design [5]

- The Structure is rigid and symmetrical
- The propellers are rigid
- Thrust and drag are proportional to square of propellers speed

The model presented in this Paper is considering following equation of motion

$$\begin{aligned}
\ddot{x} &= (\cos\phi\,\sin\theta\,\cos\psi + \sin\phi\,\sin\psi)\frac{1}{m}U_1 \\
\ddot{y} &= (\cos\phi\,\sin\theta\,\cos\psi + \sin\phi\,\sin\psi)\frac{1}{m}U_1 \\
\ddot{z} &= -g + (\cos\phi\,\cos\theta)\frac{1}{m}U_1 \\
\ddot{\phi} &= \dot{\theta}\dot{\psi}\left(\frac{I_y - I_x}{I_x}\right) - \left(\frac{J_r}{I_x}\right)\dot{\theta}\Omega + \frac{l}{I_x}U_2 \\
\ddot{\theta} &= \dot{\theta}\dot{\phi}\left(\frac{I_z - I_x}{I_y}\right) - \left(\frac{J_r}{I_y}\right)\dot{\phi}\Omega + \frac{l}{I_y}U_3 \\
\ddot{\psi} &= \dot{\theta}\dot{\phi}\left(\frac{I_x - I_y}{I_z}\right) - \left(\frac{1}{I_z}\right)U_4
\end{aligned} \tag{1}$$

In equation (1) *m [kg]* represent the mass of quadrotor helicopter whereas I_{xx} *[Nms²]*, I_{yy}-*[Nms²]*, I_{zz} *[Nms²]* describes the factors of inertia matrix expressed in body system, *J [Nms⁻¹]* is the angular momentum and *Ω [rads⁻¹]* is the speed of propeller. U_1, U_2, U_3, U_4 are the inputs or translation vector factors. Basic motions and the speed of the propeller can be depicted by following equation 2 [6]

$$\begin{aligned}
U_1 &= b\left(\Omega_1{}^2 + \Omega_2{}^2 + \Omega_3{}^2 + \Omega_4{}^2\right) \\
U_2 &= lb\left(-\Omega_2{}^2 + \Omega_4{}^2\right) \\
U_3 &= lb\left(-\Omega_1{}^2 + \Omega_3{}^2\right) \\
U_4 &= d\left(-\Omega_1{}^2 + \Omega_2{}^2 - \Omega_3{}^2 + \Omega_4{}^2\right) \\
\Omega &= \left(-\Omega_1{}^2 + \Omega_2{}^2 - \Omega_3{}^2 + \Omega_4{}^2\right)
\end{aligned} \tag{2}$$

In equation (2) *l[m]*, *b[Ns²]* and *d[Nms²]* describe the distance between propeller center and quadrotors center, lift and drag respectively. Ω_1 *[rads⁻¹]*, Ω_2*[rads⁻¹]*, Ω_3*[rads⁻¹]* and Ω_4*[rads⁻¹]* are front, right, back and left propeller's velocity.

3. PID Control

A PID controller has three terms a Proportional, Integral and Derivative. PID is used in automatic control field since the dawn of last century. Nowadays most of the electrical devices are using PID control either in the form of stand-alone or in functional blocks for example in PLCs and DCSs. PID control can be implemented in following ways [7]

- Feedback Control
- On-Off Control

The simulation in this thesis is using the parameters as mentioned in ***Bouabdallah*** PHD thesis as shown in table below [5].

Table -1: Quadrotor Parameters

I_{XX}	0.0075
I_{YY}	0.0075
I_{ZZ}	0.0130
J_r	$6.50\text{x}10^{-5}$
B	$3.13\text{x}10^{-5}$
D	$7.50\text{x}10^{-5}$
L	0.23
M	0.65

Simulink is used to develop the controller. For the control of quadrotor four PD type controllers are used in order to achieve the desired output. In this paper we are only going to discuss the Roll Controller and rest remains similar.
For the purpose to control the roll angle of the quadrotor control input can be defined as [**8**]

$$U_2=K_p.(\phi_d-\phi)+K_d.(\dot{\phi}_d-\dot{\phi}) \tag{3}$$

Where

U_2: Control Input K_p: Proportional Gain
K_d: Derivative Gain ϕ_d: Roll Desired
ϕ: Actual Roll

The equation above can be implemented in Simulink as shown in figure below

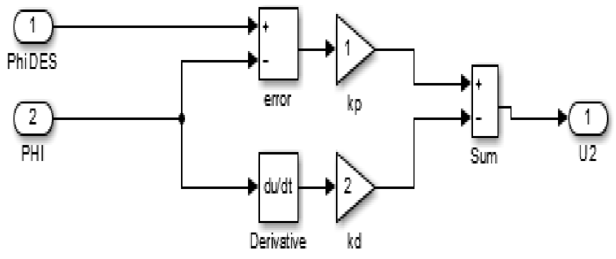

Figure -3: PHI PD Controller

To achieve the desired output proportional gain was adjusted as 1 and derivative as 2.

4. Simulation and Results

General Scheme of the whole system in Matlab is shown in the picture below

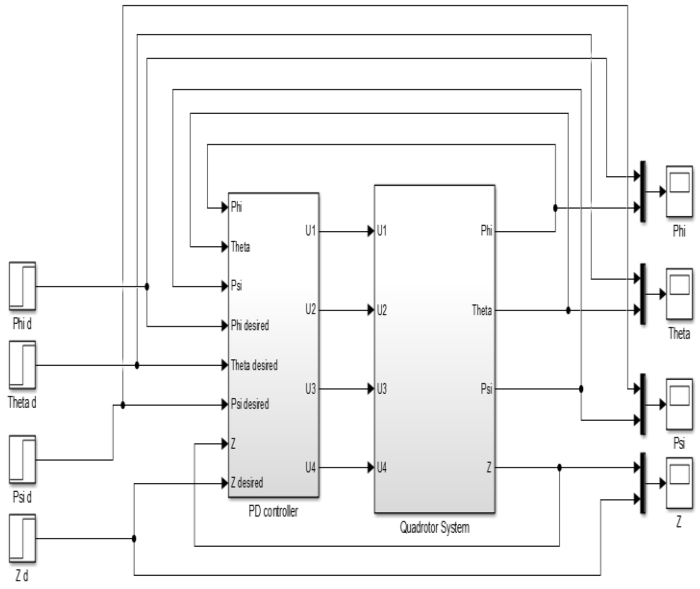

Figure -4: Block Diagram

Here in this case the desired input is unit step and desired output is also unit step. Simulation results with respect to the desired input is shown in the figures below

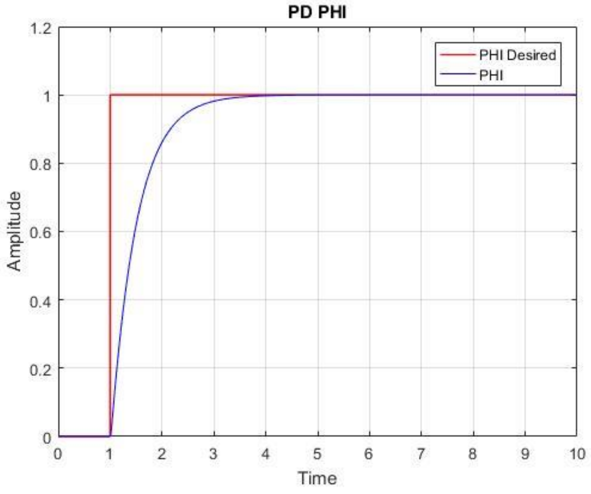

Figure -5: PHI Response

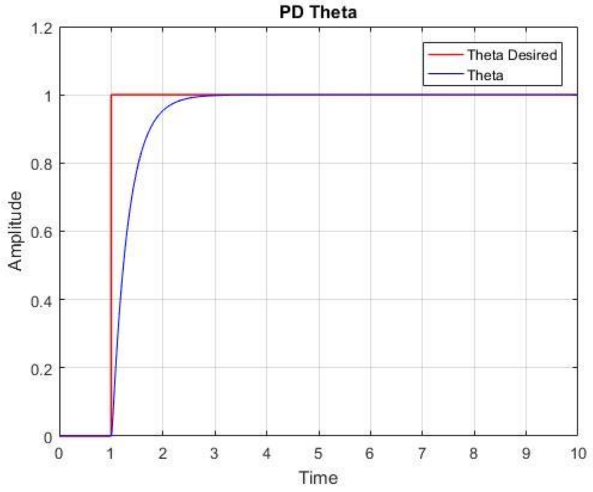

Figure -6: Theta Response

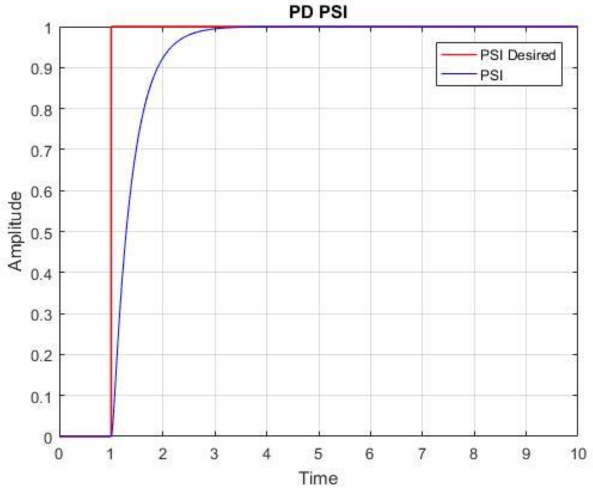

Figure -7: PSI Response

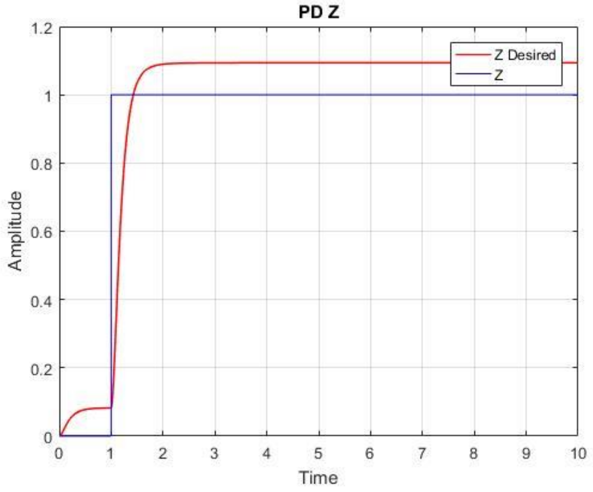

Figure -8: Z/Altitude Response

5. CONCLUSIONS

In this paper a nonlinear mathematical model of quadrotor is presented and implementation of the presented model is done through Matlab/Simulink. The presented model also considered rotor dynamics and aerodynamic effects which in most of the literatures are not considered during modeling. By looking at the responses shown in figures above we can observe that the behavior of Roll, Pitch, Yaw and Z are almost the same as the provided input. Further investigation can be made by using the same controller design and implement it on hardware.

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BIOGRAPHIES

Muhammad Awais Sattar is a graduate of Electrical Engineering from Comsats Institute of technology Lahore (CIIT) and currently pursuing his Master's degree in Electrical Engineering, specializing in Control System, at Rochester Institute of Technology (RIT), Dubai Campus.

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