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FORMATION OF TRIPLES CONSIST SOME SPECIAL NUMBERS WITH INTERESTING PROPERTY

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Abstract- In this communication, we discover the triple (a,b,c) involving some figurate numbers such that the sum of any two of them is a perfect square. Also, we find some interesting relations among the triples.

Key words: Diophantine m -tuples, polygonal numbers.

INTRODUCTION: Let n be an integer. A set of positive integers $\{a_1,a_2,a_3,......a_m\}$ is said to have the the property $D(n)$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ such a sets is called a Diophantine m-tuple. such sets were studied by Diophantus [1].The set of numbers $\{1,2,7\}$ is Diophantine triple with property D(2).For an extensive review of various articles one may refer [2-12]. In this communication, we find the triple (a,b,c) involving centered pentagonal numbers, decagonal numbers, Gnomonic numbers, Kynea numbers and Jacobsthal-lucas numbers such that the sum of any two of them is a perfect square. Also, a few interesting relations among the triples are presented.

Notations:

Let $CP_n = \frac{5n^2+5n+2}{2}$ be a centered pentagonal number of rank n.

$t_{10,n} = 4n^2 - 3n$ be a decagonal number of rank n.

$Gno_n = (2n-1)$ be a Gnomonic number of rank n.

$K_n = 2^{2n} + 2^{n+1} - 1$ be a Kynea number of rank n.

$j_n = 2^n + (-1)^n$ be a Jacobshthal-lucas number of rank n.

Method of analysis:

The procedure for finding the triple (a,b,c) involving some interesting numbers such that the sum of any two of them is a perfect square is given in the following two sections.

Section- A

Let $a(n) = 2CP_{2n+1}$

$b(n) = Dec_{2n+2} + Gno_{2n+2}$

which are equivalent to the following two equations

$$a(n) = 20n^2 + 30n + 12, b(n) = 16n^2 + 30n + 13$$

Now, we assume that

$$a(n) + b(n) = \alpha^2$$

Let $c(n)$ be any non-zero integer such that

$$b(n) + c(n) = \beta^2 \quad (1)$$

$$a(n) + c(n) = \gamma^2 \quad (2)$$

Subtracting (2) from (1), we get

$$\beta^2 - \gamma^2 = b(n) - a(n) \quad (3)$$

Put $\beta = A+1, \gamma = A$ in (3), we get

$$\gamma = A = -2n^2 \quad (4)$$

Substituting (4) in (2), the values of c are represented by

$$c(n) = 4n^4 - 20n^2 - 30n - 12 \quad (5)$$

Hence,

$\{20n^2 + 30n + 12, 16n^2 + 30n + 13, 4n^4 - 20n^2 - 30n - 12\}$ is a triple in which the sum of any two of them is a perfect square.

Table-1: Some numerical examples are illustrated below:

n	$a(n)$	$b(n)$	$c(n)$	$a(n)+b(n)$	$a(n)+c(n)$	$b(n)+c(n)$
1	62	59	-58	11^2	2^2	1^2
2	152	137	-88	17^2	8^2	7^2

3	282	247	42	23^2	18^2	17^2
4	452	389	572	29^2	32^2	31^2
5	662	563	1838	35^2	50^2	49^2

A few interesting relations among the numbers are given below:

1. $a(n)+b(n)=36\Pr o_n+12Gno_{n-1}+13$
2. $(1/4)[a(n)+c(n)]$ is a bi-quadratic integer
3. $(1/4)[b(n)-c(n)-36\Pr o_n-12Gno_n-37]$ is a bi-quadratic integer
4. $(1/4)[a(n)-c(n)-40\Pr o_n-10Gno_n-34]$ is a bi-quadratic integer

SECTION-B

Let $a(n)=K_{2n}$

$b(n)=8j_{2n}+j_1+j_4$

which are equivalent to the following two equations

$$a(n)=2^{4n}+2.2^{2n}-1, b(n)=8.2^{2n}+26$$

Now, we assume that

$$a(n)+b(n)=\alpha^2$$

Let $c(n)$ be any non-zero integer such that

$$b(n)+c(n)=\beta^2 \quad (6)$$

$$a(n)+c(n)=\gamma^2 \quad (7)$$

Subtracting (7) from (6), we get

$$\beta^2-\gamma^2=b(n)-a(n) \quad (8)$$

The choices $\beta=A+1, \gamma=A$ lead (8) to

$$\gamma=A=3.2^{2n}-2^{4n-1}+13 \quad (9)$$

Substituting (9) in (7), the values of c are represented by

$$c(n)=8.2^{4n}+2^{8n-2}-6.2^{6n-1}+76.2^{2n}-26.2^{4n-1}+170$$

Hence,

$$\{2^{4n}+2.2^{2n}-1, 8.2^{2n}+26, 8.2^{4n}+2^{8n-2}-6.2^{6n-1}+76.2^{2n}-26.2^{4n-1}+170\}$$

is a triple in which the sum of any two of them is a perfect square.

Table-2: Some numerical examples are illustrated below:

n	$a(n)$	$b(n)$	$c(n)$	$a(n)+b(n)$	$a(n)+c(n)$	$b(n)+c(n)$
1	23	58	266	9^2	17^2	18^2
2	287	154	4202	21^2	67^2	66^2
3	4223	538	3392426	69^2	1843^2	1842^2
4	66047	2074	572	261^2	31987^2	31986^2
5	1050623	8218	2.716515166×10^{11}	1029^2	521203^2	521202^2

CONCLUSION:

In this communication, we discover the triple involving various special numbers in such a way that the sum of any two of them is a perfect square. In this manner, one may seek out other triples and quadruples satisfying some other properties.

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