FORMATION OF TRIPLES CONSIST SOME SPECIAL NUMBERS WITH INTERESTING PROPERTY

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Abstract- In this communication, we discover the triple \((a, b, c)\) involving some figurate numbers such that the sum of any two of them is a perfect square. Also, we find some interesting relations among the triples.

Key words: Diophantine m-tuples, polygonal numbers.

INTRODUCTION: Let \(n\) be an integer. A set of positive integers \(\{a_1, a_2, a_3, \ldots, a_m\}\) is said to have the property \(D(n)\) if \(a_i + a_j = n\) is a perfect square for all \(1 \leq i < j \leq m\) such a set is called a Diophantine m-tuple. Such sets were studied by Diophantus [1]. The set of numbers \(\{1, 2, 7\}\) is Diophantine triple with property \(D(2)\). For an extensive review of various articles one may refer [2-12]. In this communication, we find the triple \((a, b, c)\) involving centered pentagonal numbers, decagonal numbers, Gnomonic numbers, Kynea numbers and Jacobsthal-lucas numbers such that the sum of any two of them is a perfect square. Also, a few interesting relations among the triples are presented.

Notations:

Let \(CP_n = \frac{5n^2 + 5n + 2}{2}\) be a centered pentagonal number of rank \(n\) .

\(t_{10,n} = 4n^2 - 3n\) be a decagonal number of rank \(n\) .

\(Gno_n = (2n - 1)\) be a Gnomonic number of rank \(n\) .

\(K_n = 2^{2n} + 2^{n+1} - 1\) be a Kynea number of rank \(n\) .

\(j_n = 2^n + (-1)^n\) be a Jacobsthal-lucas number of rank \(n\) .

Method of analysis:

The procedure for finding the triple \((a, b, c)\) involving some interesting numbers such that the sum of any two of them is a perfect square is given in the following two sections.

Section- A

Let \(a(n) = 2CP_{2n+1}\)

\(b(n) = Dec_{2n+2} + Gno_{2n+2}\)

which are equivalent to the following two equations

\(a(n) = 20n^2 + 30n + 12, b(n) = 16n^2 + 30n + 13\)

Now, we assume that

\(a(n) + b(n) = \alpha^2\)

Let \(c(n)\) be any non-zero integer such that

\(b(n) + c(n) = \beta^2\) \hspace{1cm} (1)

\(a(n) + c(n) = \gamma^2\) \hspace{1cm} (2)

Subtracting (2) from (1), we get

\(\beta^2 - \gamma^2 = b(n) - a(n)\) \hspace{1cm} (3)

Put \(\beta = A + 1, \gamma = A\) in (3), we get

\(\gamma = A = -2n^2\) \hspace{1cm} (4)

Substituting (4) in (2), the values of \(c\) are represented by

\(c(n) = 4n^4 - 20n^2 - 30n - 12\) \hspace{1cm} (5)

Hence, \(\{20n^2 + 30n + 12, 16n^2 + 30n + 13, 4n^4 - 20n^2 - 30n - 12\}\) is a triple in which the sum of any two of them is a perfect square.

Table-1: Some numerical examples are illustrated below:

<table>
<thead>
<tr>
<th>n</th>
<th>(a(n))</th>
<th>(b(n))</th>
<th>(c(n))</th>
<th>(a(n) + b(n))</th>
<th>(a(n) + c(n))</th>
<th>(b(n) + c(n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62</td>
<td>59</td>
<td>58</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>152</td>
<td>137</td>
<td>88</td>
<td>338</td>
<td>338</td>
<td>338</td>
</tr>
</tbody>
</table>

A few interesting relations among the numbers are given below:
1. \( a(n) + b(n) = 36 \rho_0 n + 12Gn_{n-1} + 13 \)
2. \((1/4)[a(n) + c(n)]\) is a bi-quadratic integer
3. \((1/4)[b(n) - c(n) - 36 \rho_0 n - 12Gn_{n-1} - 37]\) is a bi-quadratic integer
4. \((1/4)[a(n) - c(n) - 40 \rho_0 n - 10Gn_{n-1} - 34]\) is a bi-quadratic integer

**SECTION-B**

Let \( a(n) = K_{2n} \)
\( b(n) = 8j_{2n} + j_1 + j_4 \)
which are equivalent to the following two equations
\( a(n) = 2^{4n} + 2.2^{2n} - 1, b(n) = 8.2^{2n} + 26 \)

Now, we assume that
\( a(n) + b(n) = \alpha^2 \)

Let \( c(n) \) be any non-zero integer such that
\( b(n) + c(n) = \beta^2 \)
\( a(n) + c(n) = \gamma^2 \)

Subtracting (7) from (6), we get
\( \beta^2 - \gamma^2 = b(n) - a(n) \)

The choices \( \beta = A + 1, \gamma = A \) lead (8) to
\( \gamma = A = 3.2^{2n} - 2^{4n-1} + 13 \)

Substituting (9) in (7), the values of \( c \) are represented by
\( c(n) = 8.2^{4n} + 2^{8n-2} - 6.2^{6n-1} + 76.2^{2n} - 26.2^{4n-1} + 170 \)

Hence, \( \{2^{4n} + 2.2^{2n} - 18.2^{2n} + 26.8^{2n} + 2^{8n-2} - 6.2^{6n-1} + 76.2^{2n} - 26.2^{4n-1} + 170\} \)
is a triple in which the sum of any two of them is a perfect square.

**Table-2: Some numerical examples are illustrated below:**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( a(n) )</th>
<th>( b(n) )</th>
<th>( c(n) )</th>
<th>( a(n) + b(n) )</th>
<th>( a(n) + c(n) )</th>
<th>( b(n) + c(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23</td>
<td>58</td>
<td>266</td>
<td>9^2</td>
<td>17^2</td>
<td>18^2</td>
</tr>
<tr>
<td>2</td>
<td>287</td>
<td>154</td>
<td>4202</td>
<td>21^2</td>
<td>67^2</td>
<td>66^2</td>
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<td>3</td>
<td>4223</td>
<td>538</td>
<td>3392426</td>
<td>69^2</td>
<td>1843^2</td>
<td>1842^2</td>
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<td>4</td>
<td>66047</td>
<td>2074</td>
<td>572</td>
<td>261^2</td>
<td>31987^2</td>
<td>31986^2</td>
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<tr>
<td>5</td>
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<td>8218</td>
<td>271651566-10^1</td>
<td>1029^2</td>
<td>521203^2</td>
<td>521202^2</td>
</tr>
</tbody>
</table>

**CONCLUSION:**

In this communication, we discover the triple involving various special numbers in such a way that the sum of any two of them is a perfect square. In this manner, one may seek out other triples and quadruples satisfying some other properties.

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