Cordial labelings in the context of triplication

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Abstract - In this paper, we introduce the extended triplicate graph of a ladder and investigate the existence of cordial labeling, total cordial labeling, product cordial labeling, total product cordial labeling and prime cordial labeling for the extended triplicate graph of a ladder graph by presenting algorithms.

Key Words: Ladder graph, Triplicate graph, Graph labelings.

1. INTRODUCTION

Graph theory has various applications in the field of computer programming and networking, marketing and communications, business administration and so on. Some major research topics in graph theory are Graph coloring, Spanning trees, Planar graphs, Networks and Graph labeling. Graph labeling has been observed and identified for its usage towards communication networks. That is, the concept of graph labeling can be applied to network security, network addressing, channel assignment process and social networks [3].

In 1967, Rosa introduced the concept of graph labeling [4]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices (edges) then the labeling is called a vertex (an edge) labeling.

In 1987, Cahit introduced the notion of cordial labeling [2]. A graph G is said to admit a cordial labeling if there exists a function \( f : V \rightarrow \{0, 1\} \) such that the induced function \( f^* : E \rightarrow \{0, 1\} \) defined as \( f^*(v,v_i) = |f(v) - f(v_i)| \) or \( (f(v_i) + f(v)) \mod 2 \) satisfies the property that the number of vertices labeled '1' differ by atmost one and the number of vertices labeled '0' and the number of edges labeled '1' differ by atmost one. A graph that admits product cordial labeling is called product cordial graph.

A graph is called total product cordial graph if there exists a function \( f : V \rightarrow \{0, 1\} \) such that the induced function \( f^* : E \rightarrow \{0, 1\} \) defined as \( f^*(v,v_i) = (f(v) \times f(v_i)) | v_i \in E \) satisfies the property that the number of vertices labeled '0' and the number of edges labeled '1' differ by atmost one and number of edges labeled '0' and the number of edges labeled '1' differ by atmost one.

In 2011, Bala and Thirusangu introduced the concept of the extended triplicate graph of a path \( P_n (ETG(P_n)) \) and proved many results on this newly defined concept [1]. Let \( V = \{ v_1, v_2, ..., v_n \} \) and \( E = \{ e_1, e_2, ..., e_n \} \) be the vertex and Edge set of a path \( P_n \). For every \( v_i \in V \), construct an ordered triple \( \{ v_1, v_i, v_i' \} \) where \( 1 \leq i \leq n+1 \) and for every edge \( v_i \in E \), construct four edges \( v_i v_i', v_i v_i'' \) and \( v_i v_i''' \) where \( j = i+1 \), then the graph with this vertex set and edge set is called a Triplicate Graph of a path \( P_n \). It is denoted by \( TG(P_n) \). Clearly the Triplicate graph \( TG(P_n) \) is disconnected. Let \( V_1 = \{ v_1, v_2, ..., v_{3n+1} \} \) and \( E_1 = \{ e_1, e_2, ..., e_{3n} \} \) be the vertex and edge set of \( TG(P_n) \). If n is odd, include a new edge \( v_{n+1} v_1 \) and if n is even, include a new edge \( v_n v_1 \) in the edge set of \( TG(P_n) \). This graph is called the Extended Triplicate of the path \( P_n \) and it is denoted by \( ETG(P_n) \).

In 2014, Thirusangu et.al proved some results on Duplicate Graph of Ladder Graph [7].

A ladder graph \( L_n \) is a planar undirected graph with 2n vertices and 3n - 2 edges. It is obtained as the cartesian product of two path graphs, one of which has only one edge: \( L_{n+1} = P_n \times P_2 \), where \( n \) is the number of rungs in the ladder.

Motivated by the study, the present work is aimed to provide label for the extended triplicate graph of a ladder graph and prove the existence of cordial labeling, total cordial labeling, product cordial labeling.
and total product cordial labeling for the extended triplicate graph of a ladder graph.
Throughout this work, graph \( G = (V, E) \), we mean a simple, finite, connected and undirected graph with \( p \) vertices and \( q \) edges.

K.Thirusangu and E.Bala (2011) introduced the concept of triplicate graph and proved many results on this newly defined concepts.

Motivated by the study, the present work is aimed to provide label for the extended triplicate graph of a ladder graph and prove the existence of cordial labeling, total cordial labeling, product cordial labeling and total product cordial labeling for the extended triplicate graph of a ladder graph.

Throughout this work, graph \( G = (V, E) \), we mean a simple, finite, connected and undirected graph with \( p \) vertices and \( q \) edges.

2. STRUCTURE OF THE EXTENDED TRIPlicate GRAPH OF LADDER

In this section we discuss about the structure of the extended triplicate graph of ladder by presenting algorithm.

**Algorithm 2.1:**

**Input** ladder graph \( L_n \)

**procedure** triplicate of graph \( L_n \)

for \( i = 1 \) to \( n \) do

\[ V \leftarrow \{ v_i \cup v_i' \cup v_i'' \cup u_i \cup u_i' \cup u_i'' \} \]

end for

for \( i = 1 \) to \( n-1 \) do

\[ E_1 \leftarrow (v_i v_i+1) \cup (v_i' v_i'+1) \cup (u_i u_i+1) \cup (u_i'' u_i''+1) \]

end for

for \( i = 2 \) to \( n \) do

\[ E_2 \leftarrow (v_i v_i-1) \cup (u_i u_i-1) \cup (u_i'' u_i''-1) \cup (v_i' v_i''-1) \]

end for

for \( i=1 \) to \( n \) do

\[ E_3 \leftarrow (v_i' u_i') \cup (u_i v_i') \cup (v_i'' u_i''') \cup (v_i' v_i''') \]

end for

\[ E \leftarrow E_1 \cup E_2 \cup E_3 \]

**end procedure**

**output** : Triplicate graph of ladder \( L_n \)

From the above algorithm 2.1, the triplicate graph of a ladder denoted by \( TG(L_n) \) is a disconnected graph with 6\( n \) vertices and 12\( n \) - 8 edges. To make it as a connected graph, for convenience, we include an edge \( v_i'' u_i' ' \) to the edge set \( E \) as defined in the above algorithm. Thus the graph so obtained is called an extended triplicate graph of ladder \( L_n \) and is denoted by \( ETG(L_n) \). By the construction, it is clear that, the graph \( ETG(L_n) \) has 6\( n \) vertices and 12\( n \) - 7 edges.

**Illustration 2.1:**

The structure of extended triplicate graph of ladder \( ETG(L_n) \) is given in figure 1.

3 CORDIAL AND TOTAL CORDIAL LABELING

In this section, we present an algorithm and prove the existence of cordial and total cordial labeling of the extended triplicate graph of ladder \( (ETG(L_n)) \).

**Algorithm 3.1**

**procedure** (cordial labeling for \( ETG(L_n) \))

for \( i = 1 \) to \( n \) do

\[ V \leftarrow \{ v_i \cup v_i' \cup v_i'' \cup u_i \cup u_i' \cup u_i'' \} \]

end for

for \( i = 1 \) to \( n \) do

\[ u_i'' \leftarrow u_i'' \; \text{if} \; i \equiv 1 \; \text{(mod 2)} \]

end for

end procedure

**output** labeled vertices of \( ETG(L_n) \)

**Theorem 3.1**

The extended triplicate graph of a ladder graph admits cordial labeling.

**Proof:**

We know that, the extended triplicate graph of a ladder has 6\( n \) vertices and 12\( n \) - 7 edges. Consider the...
arbitrary vertex \( v_i \in V \). To label the vertices, using algorithm 3.1, define a map \( f : V \rightarrow \{0,1\} \). Clearly the number of vertices labeled with '0' is \( 3n \) and '1' is \( 3n \).

Thus the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one.

In order to get the labels for the edges, define the induced map \( f^* : E \rightarrow \{0,1\} \) such that for any \( v_i, v_j \in E \), \( f^*(v_i, v_j) = (f(v_i) + f(v_j)) \mod 2 \). Thus,

(i) For \( 1 \leq i \leq n - 1 \), the edges receives the following labels:

\[
\begin{align*}
    v_i v_{i+1}' &= v_i' v_{i+1}'' = u_i u_{i+1}' = u_i' u_{i+1}'' = \begin{cases} 0, & i \equiv 1 \mod 2 \\ 1, & \text{otherwise} \end{cases} \\
\end{align*}
\]

(ii) For \( 2 \leq i \leq n \), the edges receives the labels as follows:

\[
\begin{align*}
    v_i v_{i-1}' &= v_i' v_{i-1}'' = u_i u_{i-1}' = u_i' u_{i-1}'' = \begin{cases} 0, & i \equiv 1 \mod 2 \\ 1, & \text{otherwise} \end{cases} \\
\end{align*}
\]

(iii) For \( 1 \leq i \leq n \), the edges receives the labels as follows:

\[
\begin{align*}
    v_i u_i' &= v_i' u_i'' = \begin{cases} 0, & i \equiv 1 \mod 2 \\ 1, & \text{otherwise} \end{cases} \\
\end{align*}
\]

(iv) \( v_n'' u_n' = 1 \)

Clearly the number of edges labeled with '0' is \( 6n - 4 \) and '1' is \( 6n - 3 \).

Thus, the number of edges labeled with '0' and '1' differ by at most one.

Hence ETG(L\(_n\)) admits cordial labeling.

**Theorem 3.2**

Extended triplicate graph of ladder admits total cordial labeling.

**Proof:**

By Theorem 3.1, using the map \( f \) on \( V \) and there by the induced map \( f^* \) on \( E \), the total number of vertices and edges labeled together with '0' and '1' is \( 9n - 4 \) and \( 9n - 3 \) respectively.

Thus for all \( n \), the number of zeroes on the vertices and edges taken together differ by atmost 1 with the number of one's on vertices and edges taken together.

Hence the extended triplicate graph of ladder admits total cordial labeling.

**Illustration 3.1**

ETG(L\(_4\)) with its cordial labeling is given below in figure 2.

![Fig-2: ETG(L\(_4\)) and its cordial labeling](image)

**4 PRODUCT CORDIAL AND TOTAL PRODUCT CORDIAL LABELING**

In this section we present an algorithm and prove the existence of product cordial and total product cordial labelings for the extended triplicate graph of ladder (ETG(L\(_n\))).

**Algorithm 4.1**

**procedure** (product cordial labeling for ETG(L\(_n\)))

for \( i = 1 \) to \( n \) do

\[
V \leftarrow \{ v_i, \ v_i', \ v_i'', \ u_i, \ u_i', \ u_i'' \}
\]

end for

for \( i = 1 \) to \( n \) do

\[
\begin{align*}
    u_i' &\leftarrow u_i \leftarrow v_i' \leftarrow \begin{cases} 1, & i \equiv 1 \mod 2 \\ 0, & \text{otherwise} \end{cases} \\
    v_i' &\leftarrow v_i \leftarrow u_i' \leftarrow \begin{cases} 0, & i \equiv 1 \mod 2 \\ 1, & \text{otherwise} \end{cases}
\end{align*}
\]

end for

**output** labeled vertices of ETG(L\(_n\))

**Theorem 4.1**

The extended triplicate graph of a ladder admits product cordial labeling.

**Proof:**

The extended triplicate graph of twig has \( 6n \) vertices and \( 12n - 7 \) edges. Using Algorithm 4.1, define the function \( f : V \rightarrow \{0,1\} \) to label the vertices. Thus the number of vertices labeled with '0' is \( 3n \) and '1' is \( 3n \).
To obtain the edge labels, define the induced function \( f^* : E \to \{0,1\} \) such that for any \( v_i v_j \in E \), 
\[
 f^*(v_i v_j) = (f(v_i) \times f(v_j)) \pmod{2}.
\]

(i) For \( 1 \leq i \leq n - 1 \), the edges receive the following labels:
\[
 v_i v_{i+1} = u_i u_{i+1} = \begin{cases} 0, & i \equiv 1 \pmod{2} \\ 1, & \text{otherwise} \end{cases}
\]
\[
 u_i u'_{i+1} = v_i v''_{i+1} = \begin{cases} 1, & i \equiv 1 \pmod{2} \\ 0, & \text{otherwise} \end{cases}
\]

(ii) For \( 2 \leq i \leq n \), the edges receive the labels as follows:
\[
 v_i v_{i-1} = u'_i u''_{i-1} = \begin{cases} 0, & i \equiv 1 \pmod{2} \\ 1, & \text{otherwise} \end{cases}
\]
\[
 v'_i v''_{i-1} = u_i u''_{i-1} = \begin{cases} 1, & i \equiv 1 \pmod{2} \\ 0, & \text{otherwise} \end{cases}
\]

(iii) For \( 1 \leq i \leq n \), the edges receive the labels as follows:
\[
 v_i u_i = u'_i v'_i = \begin{cases} 0, & i \equiv 1 \pmod{2} \\ 1, & \text{otherwise} \end{cases}
\]
\[
 v'_i u''_i = u_i v'_i = \begin{cases} 1, & i \equiv 1 \pmod{2} \\ 0, & \text{otherwise} \end{cases}
\]

(iv) \( u''_n u''_n = 0 \)

Clearly the number of edges labeled with '0' is \( 6n - 3 \) and '1' is \( 6n - 4 \).

Thus, for all \( n \), the number of edges labeled with '0' and '1' differ by at most one.

Hence \( ETG(L_n) \) admits product cordial labeling.

**Illustration 4.1:**

\( ETG(L_4) \) with its product cordial labeling is given below in figure 3.

![ETG(L_4) with its product cordial labeling](image)

**Theorem 4.2**

The extended triplicate graph of ladder admits total product cordial labeling.

**Proof**

By theorem 4.1, using the map \( f \) on \( V \) and there by the induced map \( f^* \) on \( E \), we have the total number of vertices and edges labeled together with '0' and '1' is \( 9n - 3 \) and \( 9n - 4 \) respectively.

Thus for all \( n \), the number of zeroes on the vertices and edges taken together differ by at most 1 with the number of one's on vertices and edges taken together.

Hence the extended triplicate graph of ladder admits total product cordial labeling.

**5 CONCLUSION**

In this paper, we have introduced and proved the existence of cordial labeling, total cordial labeling, product cordial labeling and total product cordial labeling for the extended triplicate graph of ladder by presenting algorithms.

**REFERENCES**


