Mathematical Modelling and Simulation of Compensator for Dynamical System

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Abstract - *As the technology is growing the new applications are coming out which need both speed as well as position control for better and efficient performance. Brushless DC motor find a various application in control system due to its advantages over conventional DC motor. But there are different problems in work operation due to fast dynamics and instability. Therefore, in dynamical system, controller plays an important role to achieve stability and to get desired results. To meet this requirement, the response of brushless dc drive system need to be improved. The performance of the typical brushless drive system found to be sluggish. So, it is necessary to improve the performance of such system to expected level as this system find an application in most of the guided missiles with movable control surfaces or fins. To overcome the shortcomings of conventional method, this system gives compensator design used to improve steady state response and transient response of the system to accommodate required application.*

Key Words: **Mathematical Modelling, Control system, PID Controller, BLDC Motor, Compensator, etc**

1. Introduction

In electromechanical system, command signals are translated to physical movement by an electric motor with the help of a precision gear train. The electromechanical actuator assembly mainly consists of the Brushless Direct Current (BLDC) Motor, Gear trains, Feedback sensors, electronics controller, aeronautical or control surfaces. All these are packed together and mounted at the tail end of the missile from the inner side. This forms the missile's Electromechanical Fin Actuator System. Common problems faced in the actuator modeling are that they may be saturated due to their physical constraints such as voltage, current, or slew rate limit. The conventional servo motors do not ensure the required accuracy and dynamic performance. The performance of the typical brushless drive system found to be sluggish. So, it is necessary to improve the performance of such system to expected level. To overcome the shortcomings of conventional method, this system gives methodology of controller design to improve stability and performance of a guided missile system. So depending upon the requirements of the system we have chosen lag compensator as controller to improve response of the system.

To ensures the stability and performance of a guided missile system we will have to develop an algorithm for lag compensator

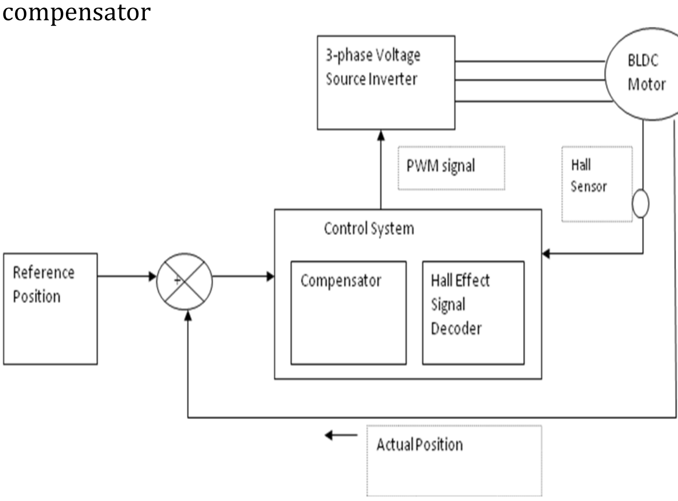

Fig.1 Basic block diagram of BLDC Drive Servo System

The BLDC motor setup consist of motor block, position estimation block provided with hall sensors, control system block provided with compensator, gate pulse generator block with commutation logic and finally inverter block to supply the voltage to the motor. The block diagram of position control of BLDC motor in fig.1, represents a closed loop position control of BLDC motor system. The position feedback is sensed by a position sensor is fed back and compared with the reference position command signal. The error signal is given to the control system block which consist of compensator and its output forms the reference current. The compared output is used to generate the PWM signal which is fed to the gating pulse generation. The six gating signals are provided to the inverter which generates phase voltages to run the BLDC motor. Each dynamic state of motor is modelled using MATLAB/SIMULINK. Control system block containing compensator can be designed to improve response of the compared output [1].

1. Role of Compensator for Dynamical System

Compensator used to improve response of the control system to accommodate required application. It is used to improve transient as well as steady state response by improving rise time, settling time, overshoot, steady state error, phase margin, gain margin etc. A compensator, or controller, placed forward path of control system will modify the shape of the loci if it contains additional poles and zeros [7]. It is used to alter phase characteristics of given system. Also help in improving time and frequency response of the system.

2. Types of Compensator

2.1. Lag Compensator:

In this, phase of output voltage lags the phase of input voltage. It is low pass filter. It improves steady state response. Lag compensator reduces the gain in high frequency range to improve phase margin. Transfer function for Lag compensator is given by,

$$C(s)=K\frac{(s+a)}{(s+b)}, \text{ Where a>b}$$

Where 'a' is zero, 'b' is pole and 'K' is compensator gain

2. 2. Lead Compensator:

In this, phase of output voltage leads the phase of input voltage. It is high pass filter. It improves transient response. Lag compensator improves phase margin but unnecessarily increase in gain of system at high frequency due to which performance get sluggish. It helps to improve phase margin, speed response, bandwidth. Transfer function for Lead compensator is given by,

$$C(s)=K\frac{(s+a)}{(s+b)}, \text{ Where b>a}$$

Where 'a' is zero, 'b' is pole and 'K' is compensator gain

2.3. Lag lead compensator:

In this, both phase lag and phase lead occur in the output but in different frequency region. It helps to improve speed response of the system. It helps to reduced percentage overshoot Transfer function for Lag-lead compensator is given by,

$$c = K\frac{(s+a1)(s+a2)}{(s+b1)(s+b2)}, \text{ where a1>b1 and b2>a2}$$

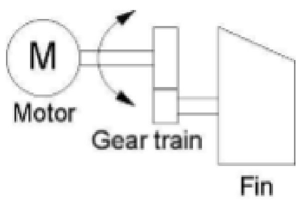

Fig.3 Motor and Fin coupled system

The drive motor used is a brushless DC motor. The schematic of the electric circuit of DC motor is as shown in Fig.4

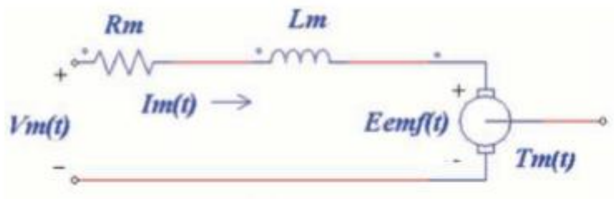

Fig.4 Equivalent circuit of the BLDC motor for a single phase

Following are the various system parameters used for Mathematical modeling:

Vm: Input voltage of the motor (V),

Im: Input armature current (A),

Rm: Motor terminal resistance (Ohm),

Lm: Motor terminal inductance (H),

Eemf: Back emf (V),

θm: Motor shaft position (rad),

Km: Motor back emf constant (V.s/rad), Jm: Motor inertia (kg.m2),

Tm: Motor torque (N.m),

Tl: Load torque (N.m),

Kg: Total gear ratio,

θl: Position of load angle (rad),

Bv: Viscous damping co-efficient (Nm/rad/s),

Kt: Motor torque constant (N.m/A),

ω: angular velocity of motor shaft (rad/s).

From the Fig.4 ,Apply Kirchhoff's law to circuit,

$$Vm = Rm.Im + Lm.\frac{dIm}{dt} + Emf \text{ ....(1)}$$

$$Emf = Km.\omega = Km.\frac{d\theta}{dt}$$

Refer motor parameters from FAULHABER 3564K024B datasheet.

Applying Newton's Second law of motion at motor shaft is given by,

$Jm.\ddot{\theta}m + Bv.\dot{\theta}m = Tm - Tl = Teq$ ....(2)

Where Teq is equivalent torque of motor such that $Teq = Kt.Im$ and $\theta m = Kg.\theta l$

The above equation gives,

$Jm.Kg.\ddot{\theta}l + Bv.Kg.\dot{\theta}l = Kt.Im$ ....(3)

The value of Im from equation (1) can be obtained as

$Im = \frac{Vm}{Rm} - \frac{Lm}{Rm}\frac{dIm}{dt} - \frac{Kb.Kg\dot{\theta}l}{Rm}$ ....(4)

Substituting above value of Im in equation (3) we get,

$$Jm.Kg.\ddot{\theta}l + Bv.Kg.\dot{\theta}l = Kt(\frac{Vm}{Rm} - \frac{Lm.dIm}{dt} - \frac{Kb.Kg.\dot{\theta}l}{Rm}$$

Solving above equation gives,

Hence, Open loop transfer function of 3-phase BLDC motor is given as,

$$G(s) = \frac{\frac{1}{Kb.Kg}}{\tau m.\tau e.s^2 + \tau m.s + 1}$$

Substituting the values of motor parameters from table ,G(s) becomes

$G(s) = \frac{0.0197}{(11*10^{-3}*150*10^{-6})s^2 + (11*10^{-3})s+1}$ ....(11)

$\therefore G(s) = \frac{0.0197}{(1.65*10^{-6})s^2+(11*10^{-3})s+1}$ ....(12)

This equation (12) gives open loop transfer function of BLDC motor

3. Selection of Compensator for Proposed System:

The compensation scheme is designed as per the requirements. In order to increase it phase margin, lead compensator is introduced in the feed forward path. But due to inserting a lead compensator it is seen that phase margin of system is increasing but this increase unnecessary gain of the system at high frequency. Increasing gain will decrease the stability margin, will tend to lower overshoot but system will be unacceptably sluggish [10]. However, lag compensator permits higher gain at low frequencies which improves steady state error and reduces gain in the higher critical range of frequencies, so as to improve the phase margin. Hence, we have chosen lag compensator as controller for proposed system.

4.Algorithm of Lag Compensator:

- Identify system Requirement
- Find Value of gain K to set the required Phase margin
- Determine the gain crossover frequency(wg)
- Set zero to be wg/10
- Set Pole to be wg/50
- Substituting obtained values of gain K,zero and pole from step 2,4 and 5 respectively in following transfer function of lag compensator

$$c = K\frac{(s+wg/10)}{(s+wg/50)}$$

3. Simulation Results

3.1. Simulink model of proposed system:

According to given requirement, proposed system is designed using MATLAB/Simulink shown in fig.5

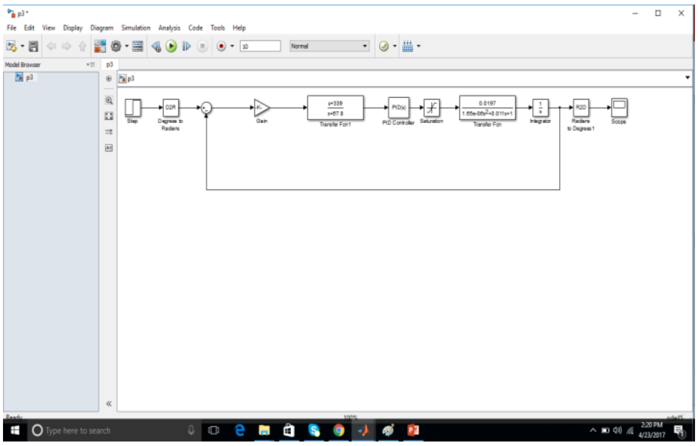

Fig.5 Simulink model for proposed system

3.2. Output Step response with PID controller:

The proposed system with PID as controller gives following output shown in fig.6

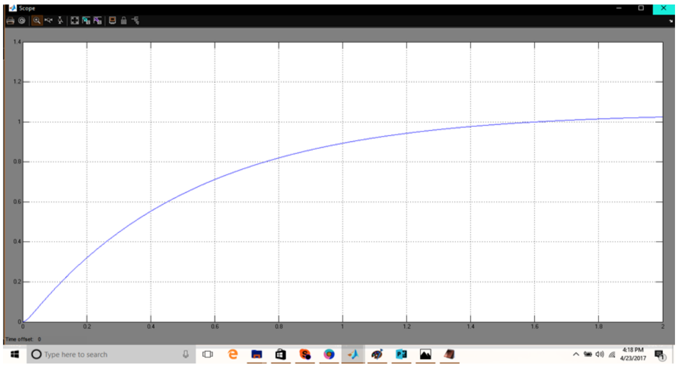

Fig.6 Output step response with PID controller

3.3 Output step response with Lag compensator:

The proposed system with Lag compensator as controller gives following output shown in fig.7

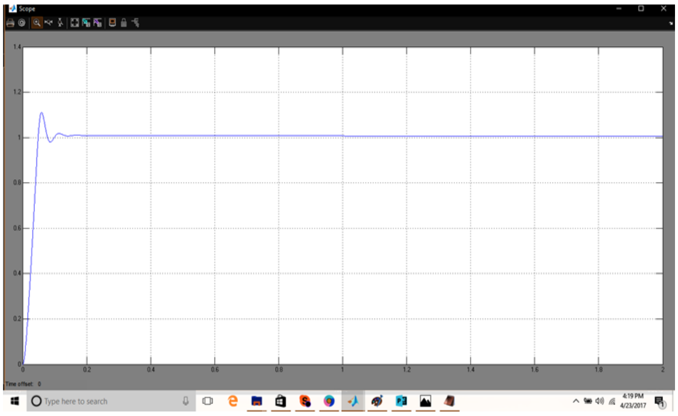

Fig.7 Output step response with lag compensator

4. Comparison of results obtained between system with PID controller and system with Lag compensator:

	Rise Time(sec)	Settling Time(sec)	%Overshoot
With PID controller	0.81	1.2	1.1
With Lag Compensator	0.045	0.11	0.85

Table.2 Comparative study

CONCLUSION

For one degree step command input, output response of system with lag compensator is better than that of output response of system with PID controller. From results, it analysed that system with lag compensator have less rise time, less settling time and less percentage overshoot. This conclude that designed lag compensator improves response of the given system.

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