

An Area Efficient Mixed Decimation MDF Architecture for Radix 2^2 Parallel FFT

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Abstract - Fast Fourier Transform (FFT) has got many applications in the field of digital signal processing. In this project, an area efficient Mixed decimation Multipath Delay Feedback (M^2DF) methodology have been presented for the radix 2^2 FFT computation. The M^2DF architecture can be employed by using the principle of folding transformation. Thereby the idle period of arithmetic units in Multipath Delay Feedback (MDF) architecture can be mobilized. This is done by the integration of Decimation-In-Time (DIT) operations into the Decimation-In-Frequency (DIF) operated computing units. The M^2DF architecture system design was modified by using Han Carlson adder which is efficient in area and fast in operation. Besides, the M^2DF system is compared with proposed M^2DF system using Han Carlson adder both theoretically and experimentally. From the obtained expressions and statistics, it can be concluded that the proposed system can be used as an area efficient system since it achieves improved efficiency in the consumption of arithmetic resources. The hardware simulation is done in Xilinx Virtex-6 Field-Programmable Gate Array (FPGA), using the programming software ISE 14.2 Vivado Design Suite.

Key Words: Fast Fourier Transform (FFT), Multipath Delay Feedback (MDF), Decimation-In-Time (DIT), Decimation-In-Frequency (DIF), Pipelined architecture.

1. INTRODUCTION

Fast Fourier Transform (FFT) is an efficient algorithm for Discrete Fourier Transform (DFT) computation. Therefore, an efficient implementation of FFT has attracted much consideration and various schemes have been put forward by the hardware designers to achieve reasonable tradeoffs between area and performance. When compared to other hardware structures, pipelined architectures [1]-[6] have a characteristic advantage over other efficient hardware structures in providing high throughputs. Single path Delay Commutator (SDC) structure [1] is one of the most conventional approaches to perform the pipelined FFT computation in the Serial Input Serial Output (SISO) scenario. Single path Delay Feedback (SDF) architecture [1] is proposed in order to decrease the memory banks in SDC pipelines. SDF architecture is composed of many feedback connections in the circuits. These architectures can be used with any algorithm such as radix-2, radix-4, and especially

radix- 2^2 algorithm in order to execute the DFT operation. The radix- 2^2 pipeline is equipped with simpler butterfly units when compared to radix-4 approach while making a better utilization of complex multipliers than the typical radix-2 scheme.

From the perspective of hardware design, radix- 2^2 algorithm acts as an effective alternative to the conventional computation methods. Certainly, the extension of communication service has encouraged a dramatic rise of throughput requirements. The demand for high throughput can be achieved by using another upgraded structures [2]. These structures can be used to calculate the FFT when several samples of the same sequence are received in parallel. So Multipath Delay Commutator (MDC) [3] and Multipath Delay Feedback (MDF) [4] are proposed to improve the throughput rate. MDC and MDF work as the upgrade of SDC and SDF respectively.

In general, multiple interconnected SDF paths are joined together to form MDF structure. Each SDF path is used for managing one of the parallel input streams. This design contributes to efficient utilization of memory resources but it has got only 50 % utilization of adders and provides only less throughput. By contrast, the hardware efficiency of arithmetic units (AUs) can be improved by using the MDC approach. But for either regrouping the samples or folding the streams, additional memories have to be consumed. This will again additionally leads to an increase of computing delay. The feedback structures such as SDF and MDF design afford possible solutions to make a balance between the consumption of hardware resources and the reachable performance [4].

As we move forward the discussion, the hardware resources can be further divided into two categories: arithmetic resources and memory resources. Arithmetic resources are associated with logical or arithmetic operations and memory resources are responsible for caching samples. The MDF scheme [4] has been a great success in a variety of applications due to the outstanding performance in the efficient use of memory resources. Beneath the triumph, the underutilization of arithmetic resources is still an important problem for feedback design and has not been determined satisfactorily.

The objective of this project is to design a new area efficient mixed decimation MDF architecture which can achieve improved efficiency in utilization of arithmetic resources while maintaining the advantages of feedback structures. The theory of folding transformation is employed to derive the proposed scheme namely, the mixed decimation Multi path Delay Feedback (M^2DF) architecture. The integration of Decimation-In-Time (DIT) operations into the Decimation-In-Frequency (DIF) operated building blocks can activate the idle period of arithmetic units in MDF architectures. Thus significant decrease in the consumption of arithmetic resources can be achieved by M^2DF architecture.

2. MIXED DECIMATION MDF SYSTEM

M^2DF architecture is derived from the theory of folding transformation. This will activates the idle period of arithmetic units in MDF architecture. The operations in SDF pipeline are rescheduled to reverse the underutilization of arithmetic modules which is accomplished by integrating the DIT operations into the DIF operated computing units. So there by better efficiency in the consumption of arithmetic resources [4].

2.1 SDF DIF Scheme Using Folding Transformation

The folding transformation offers a methodical procedure to derive many FFT architectures. In folding transformation, several algorithm operations are time multiplexed on a single computing device [5]. The algorithm can be presented using a data flow graph shown in Fig 1, where the nodes represent computations and the directed edges represent data paths. The data flow graph consists of four stages and eight operations are executed within each stage. When $x_i, i = 0, \dots, 15$ arrives in serial, multiple operations in each stage can be time multiplexed to a single computing unit without any collisions and the control circuits are determined systematically by folding transformation. In this way, the FFT Flow graph shown in Fig 1 can be converted into a pipelined form as shown in Fig 2, where operations $A_0, \dots, A_7, B_0, \dots, B_7, C_0, \dots, C_7$ and D_0, \dots, D_7 are performed in the computing unit U_A, U_B, U_C and U_D respectively [4]. The multiple operations included in a computing module are arranged by folding sets. A folding set is an ordered set of operations executed by the same computing unit. Each folding set contains many operations and some of which may be generally called as null operations, Φ .

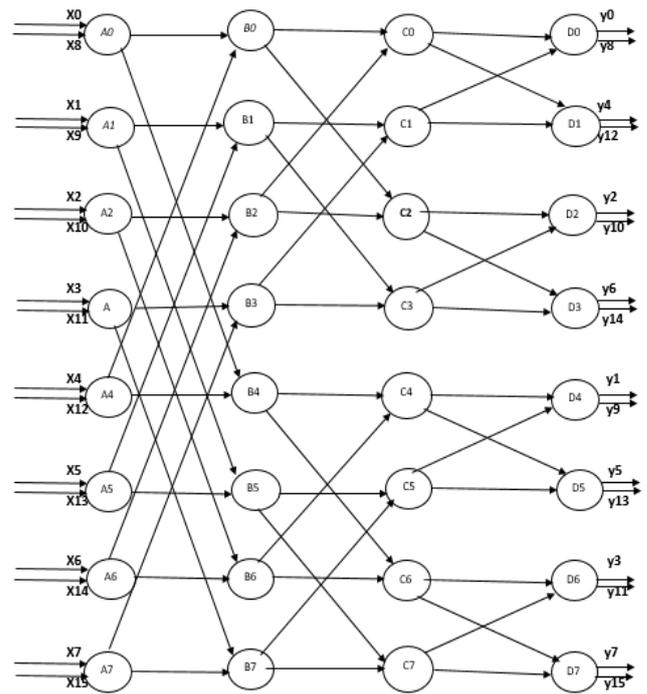


Fig -1: Data flow graph of 16 Point R^2 DIF FFT

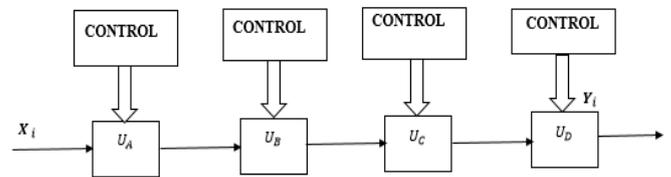


Fig -2: Data flow graph converted into a pipelined version through folding transformation

The folding set corresponding to U_A, U_B, U_C and U_D can be written as :

$$F_A = \{\Phi \ \Phi \ A_0 \ A_1 \ A_2 \ A_3 \ A_4 \ A_5 \ A_6 \ A_7\}$$

$$F_B = \{\Phi \ \Phi \ \Phi \ \Phi \ B_0 \ B_1 \ B_2 \ B_3 \ \Phi \ \Phi \ \Phi \ \Phi \ B_4 \ B_5 \ B_6 \ B_7\}$$

$$F_C = \{\Phi \ \Phi \ C_0 \ C_1 \ \Phi \ \Phi \ C_2 \ C_3 \ \Phi \ \Phi \ C_4 \ C_5 \ \Phi \ \Phi \ C_6 \ C_7\}$$

$$F_D = \{\Phi \ D_0 \ \Phi \ D_1 \ \Phi \ D_2 \ \Phi \ D_3 \ \Phi \ D_4 \ \Phi \ D_5 \ \Phi \ D_6 \ \Phi \ D_7\}$$

When obtaining the folding sets associated with U_A, U_B, U_C and U_D , the supporting circuits used to fulfill the time multiplexing of computing units can be obtained through folding transformation [5]. By introducing the register minimization technique, the implementation scheme can be further optimized, which eventually generates the SDF hardware structure which is shown in Fig 3. So that hardware efficiency can be improved.

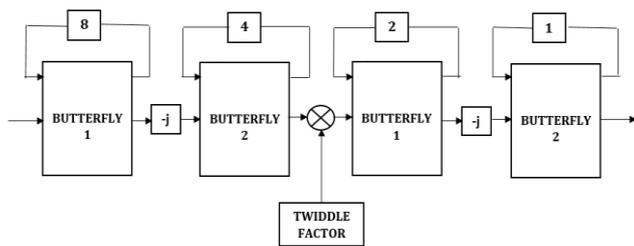


Fig -3: 16 Point R 2² SDF DIF FFT structure

Each stage in radix-2² SDF FFT [6] contains of butterfly I, butterfly II, complex multipliers with twiddle factors. Butterfly I operate on the input data, butterfly II operate on the output data from butterfly I, then multiplied with twiddle factors to get the result of the current stage. Shift registers or Random Access Memory (RAM) are used to store the output data from butterfly elements. Only one complex multiplier is used, since, input data sequence follows a single output path. Butterfly 1 structure [6] is shown in Fig 4. The input for this butterfly comes from the previous component which is the twiddle factor multiplier. The output data from butterfly 1 goes to the next stage which is usually the butterfly II. The control signal C1 has two options. That is either C1=0 or C1=1. When C1=0, multiplexers direct the input data to the feedback registers until they get filled. The other option is C1=1, the multiplexers select the output of the adders and subtractors.

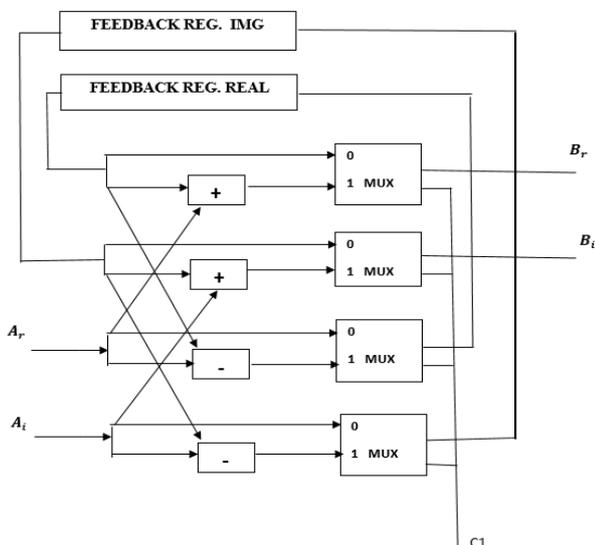


Fig -4: Butterfly 1 structure

Butterfly 2 structure is shown in Fig 3. The butterfly 2 structure [6] is same as that of butterfly 1. But in additional, there is a swap-mux provided for -j multiplication. The multiplication by -j contains swapping between real part and imaginary part and sign inversion. Swap-MUX is efficiently used for swapping real and imaginary part and the

sign inversion is handled by switching between the adding and the subtracting operations by mean of swap-MUX. The control signals C1 and C2 will be one when there is a need for multiplication by -j .

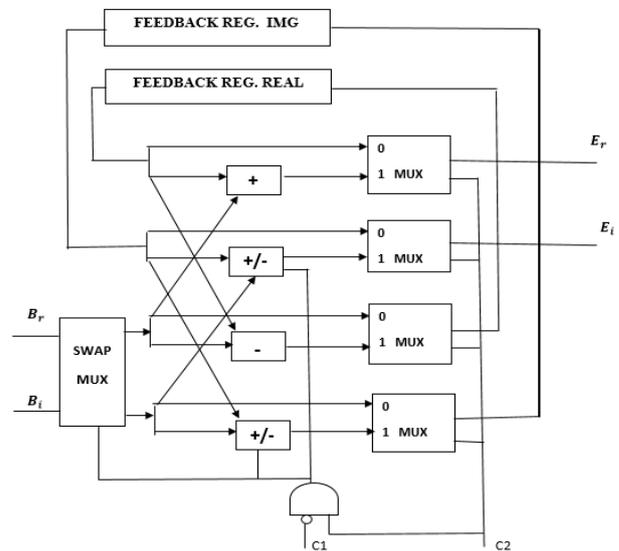


Fig -5: Butterfly 2 structure

2.2 SDF DIT Scheme Using Folding Transformation

The DIT algorithm can also be represented using a data flow graph shown in Fig 6. Similar processes as in DIF can be applied to analyze DIT algorithm. As shown in the Fig 6, the flow graph contain four stages and eight operations are performed within each stage. When $x_i, i = 0, \dots, 15$ arrives in serial, multiple operations in each stage can be time multiplexed to a single computing unit without any collisions and the control circuits are determined systematically by folding transformation [5]. In this way, the FFT Flow graph shown in Fig 6 can be converted into a pipelined form through folding transformation as shown in Fig 7, where operations A0, ..., A7, B0, ..., B7, C0, ..., C7 and D0,.....D7 are executed in the computing unit U_A, U_B, U_C and U_D respectively. The folding set corresponding to U_A, U_B, U_C and U_D can be written as :

$$F_A = \{\Phi A0 \Phi A1 \Phi A2 \Phi A3 \Phi A4 \Phi A5 \Phi A6 \Phi A7\}$$

$$F_B = \{\Phi \Phi B0 B1 \Phi \Phi B2 B3 \Phi \Phi B4 B5 \Phi \Phi B6 B7\}$$

$$F_C = \{\Phi \Phi \Phi \Phi C0 C1 C2 C3 \Phi \Phi \Phi \Phi C4 C5 C6 C7\}$$

$$F_D = \{\Phi \Phi \Phi \Phi \Phi \Phi \Phi \Phi D0 D1 D2 D3 D4 D5 D6 D7\}$$

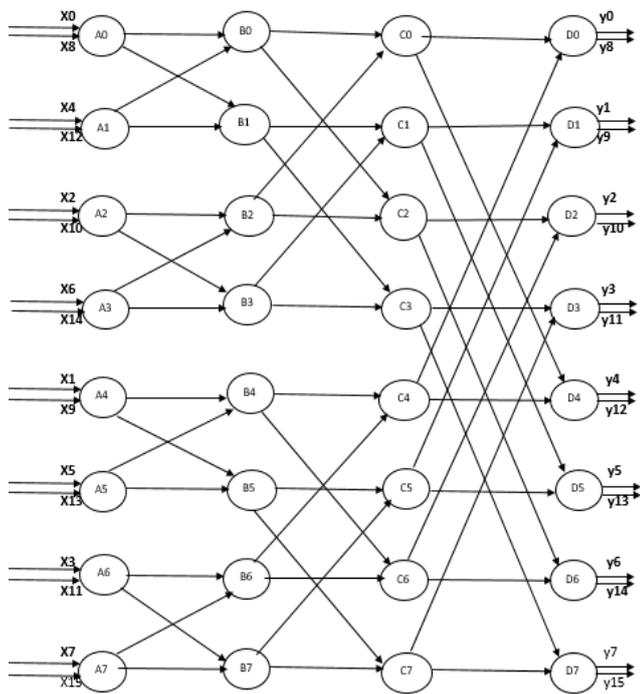


Fig -6: Data flow graph of 16 Point R^2^2 DIT FFT

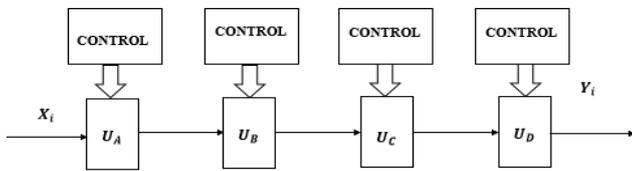


Fig -7: Data flow graph converted into a pipelined version through folding transformation

When obtaining the folding sets associated with U_A , U_B , U_C and U_D , the supporting circuits used to fulfill the time multiplexing of computing units can be obtained through folding transformation [5]. By using register minimization method implementation system can be further optimized which eventually generates the SDF hardware structure which is shown in Fig 8. Thereby hardware efficiency can be enhanced. The DIT SDF hardware structure is exactly the reverse of DIF hardware scheme.

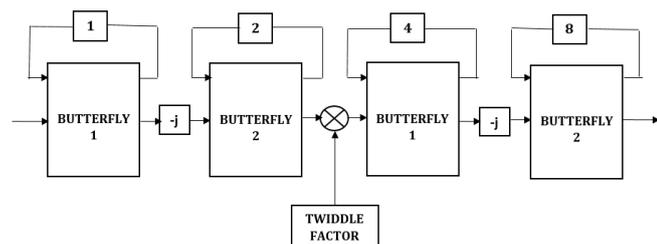


Fig -8: 16 Point R^2^2 SDF DIT FFT structure

The folding sets conveys the fact that, whether the designers adopt the DIF approach or DIT scheme to construct the SDF circuit, the existence of null operations in folding sets will always reduce the efficiency of arithmetic components. This will lead to an approximate 50% utilization of complex adders merely. To address this issue, we rearrange the operations in SDF pipelines to activate the idle intervals of computing units [4]. From the SDF DIF and SDF DIT hardware structures we can clearly understand that both DIF and DIT approach uses same hardware resources. But only difference is that DIT is the reverse of DIF scheme. So by integrating the DIT operations into DIF operated computing units, the standby time of arithmetic modules in feedback architectures can be eliminated. The integration of DIF and DIT pipelined processor to fully utilize the computing units is shown in Fig 9.

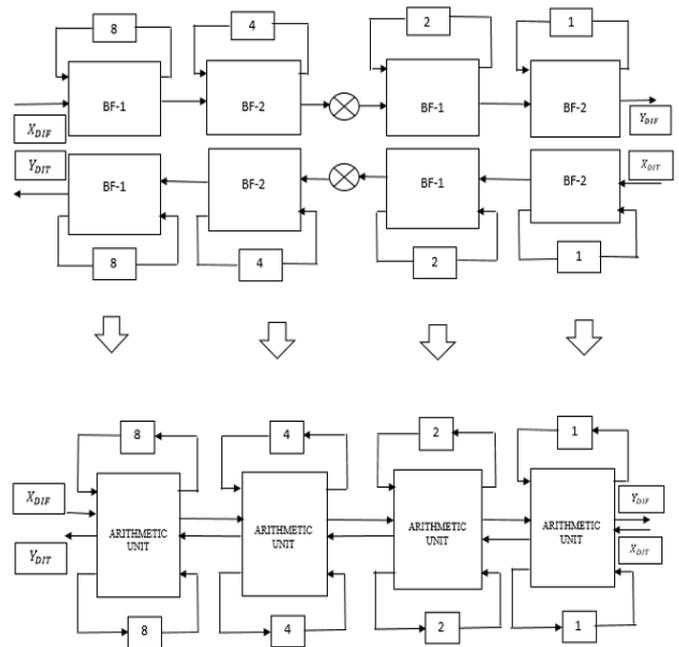


Fig -9: Integration of DIF and DIT pipelined processor

The SDF hardware structure can be upgraded by using another hardware structure called Type 1 [4] which is shown in Fig 11. The multiplexors and selectors in the circuit can be divided into two categories, white for category I and gray for category II. When arithmetic modules are in service, the components within each category share the identical logic signal. The control scheme, which is synchronized with the DIF computing stream is summarized as follows: For the first M samples of the stream, the components belonging to category I are controlled by logic 1, while others in category II are controlled by logic 0, where M is the length of shift register sets in the arithmetic module. During the next M samples, the control signals should be inverted.

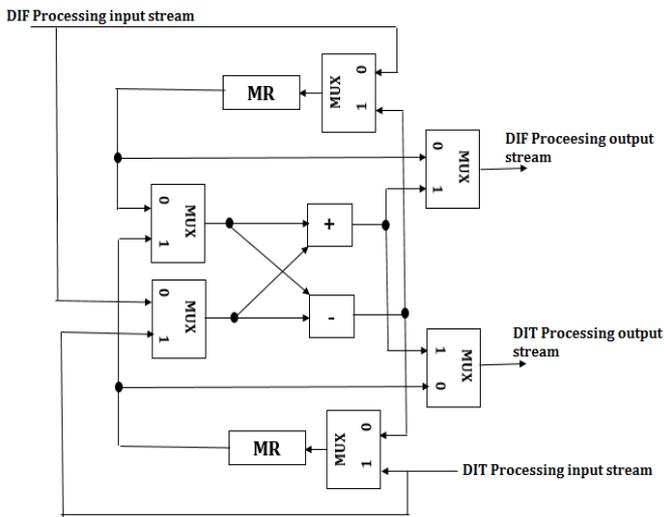


Fig -10: Type 1 structure

Final system M^2DF design can be constructed by using this Type 1 structure. Fig 12 shows the system design of two parallel M^2DF design for 32 point FFT. During 32 point DFT processing, X_0 and X_1 execute DIF and DIT scheme respectively. X_0 represent first 16 input samples and X_1 represent next 16 input samples. In addition, as input samples required to be rearranged as bit reversal before participating in DIT computation. The DIT output samples are multiplied by necessary twiddle factors for 32 point FFT computation. Finally parallel streams are connected to simple radix 2 butterfly unit to get 32 output samples for FFT [4].

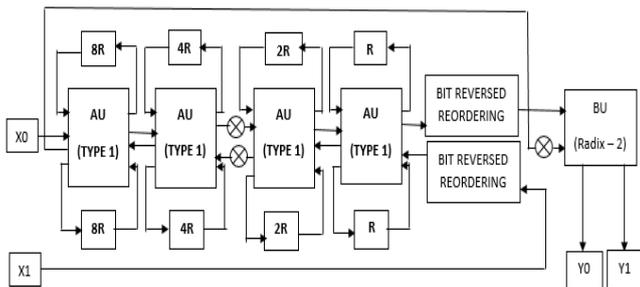


Fig -11: System design of two parallel M^2DF architecture for 32 Point FFT

3. PROPOSED M^2DF SYSTEM

In any of FFT architectures, Butterfly unit is the important block to calculate data addition and subtraction. An adder is the main building block of butterfly element. The key requirement of adder is that the adder should be fast in operation and efficient in terms of chip area. So the

performance of butterfly unit can be further enhanced by using high speed and an area efficient adder [7].

3.1 M^2DF Design by Using Han Carlson Adder

The simplest method of doing binary addition is to connect the carry-out from the previous bit to the carry-in of the next bit. Each bit takes carry-in as one of the inputs and gives sum and carry-out as output bit and therefore named as ripple carry adder. This type of adders is made by cascading 1 bit full adders. But ripple carry adder has high propagation delay. So M^2DF system designed by using ripple carry adder have only small performance [7].

The M^2DF system design is modified by using high speed and an area efficient Han Carlson adder. Han Carlson adder is a parallel prefix tree [8]. Parallel prefix adders are high performance adders. Han Carlson adder is shown in Fig 13 and it uses only even bits for performing carry-merge operations. Generate and propagate signals of odd bits are transferred down the prefix tree. The true carry bits are produced by the recombination of generate and propagate signals with even bit carry signals. So by using high speed and area efficient Han Carlson adder, system design can give better performance.

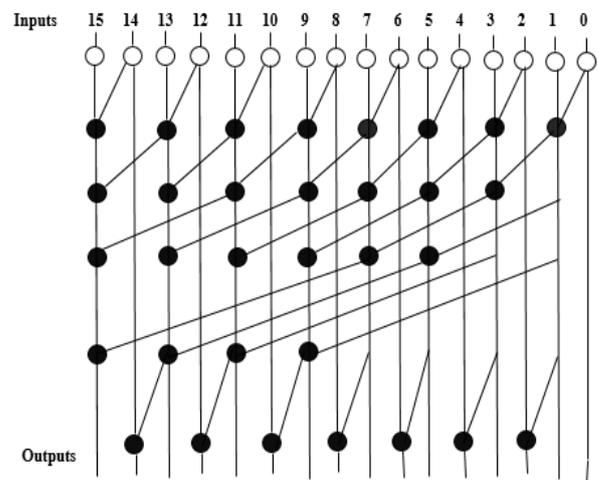


Fig -12: 16 Bit Han Carlson adder

4. RESULTS

Simulation results of M^2DF system are given below. Coding was done using Verilog HDL (Hardware Description Language). The hardware simulation is done in Xilinx Virtex-6 Field-Programmable Gate Array (FPGA), XC6VLX240T-3FF784 using the programming software ISE 14.2.

4.1 M²DF System

Fig 14 shows the simulation result of two parallel 32 point R² M²DF FFT. a1 and a2 represent two 16 point parallel DIF and DIT input samples respectively. t_r and t_img are the 32 point output samples for given input samples and clock signal,clk.

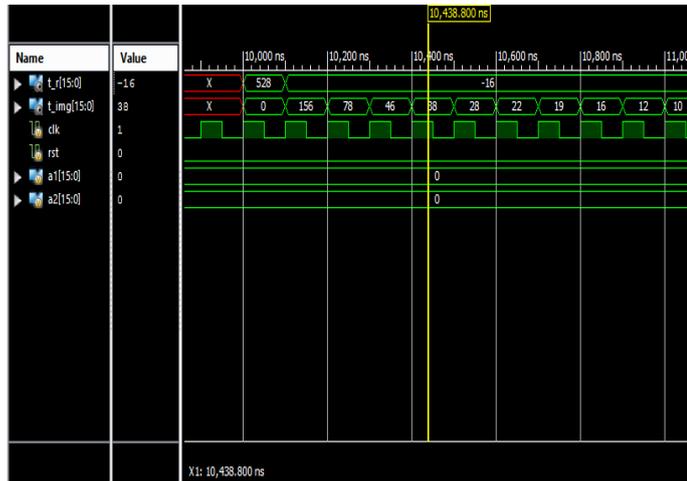


Fig -13: Simulation result of two parallel 32 point R² M²DF FFT

4.2 Proposed M²DF System

M²DF architecture is modified by using high speed and an area efficient Han Carlson adder. So the Proposed M²DF System is found to be area efficient. Fig 15 shows the simulation result of proposed two parallel 32 point R² M²DF FFT. The simulation result obtained is same for both system design and proposed scheme.

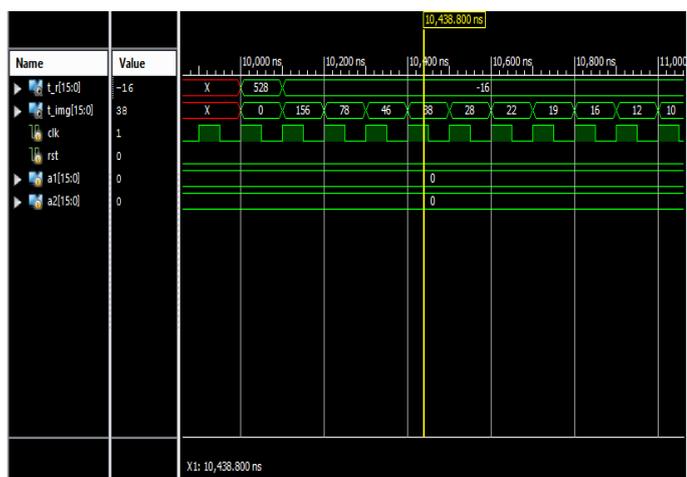


Fig -14: Simulation result of two parallel 32 point R² M²DF FFT

4.3 Performance Analysis

The performance analysis was done on both system design and proposed M²DF system design. The occupation of slices is evaluated for comparing hardware efficiency. Table 1 shows the design summary of system design and proposed M²DF system for 32 point FFT.

Table -1: Device utilization summary of system design and proposed M²DF system

Structures	Slice LUTs	Slice registers	LUT FF pairs	Total
M ² DF Architecture	1561	7722	1082	10365
Proposed M ² DF Architecture	1527	7717	1078	10322

From the device utilization summary, we can understand that when M²DF system was modified by using area efficient Han Carlson adder, there is decrease in the utilization of slice registers, slice LUTs and no of fully used LUT FF pairs than the M²DF system designed with ripple carry adder. There by we obtained a new area efficient hardware for M²DF architecture. Chart 1 shows the graphical representation of above obtained test results in table 1.

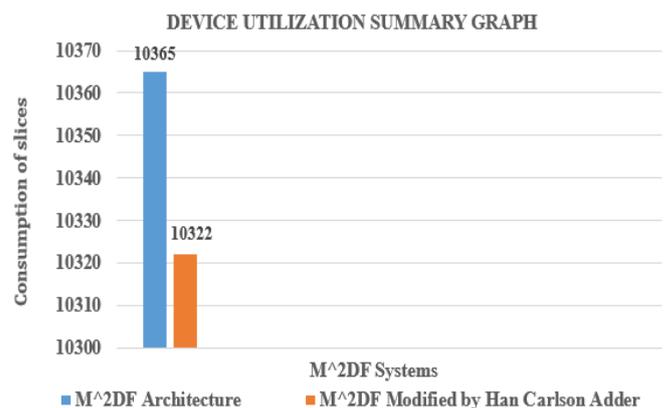


Chart -1: Device utilization summary of system design and proposed M²DF system

5. CONCLUSIONS

A new area efficient M²DF architecture for radix 2² parallel FFT has been designed. The inefficient use of adders and multipliers is an important problem in most of feedback architectures. By using this M²DF architecture, the standby time of arithmetic modules can be eliminated. The system

was designed by using Verilog HDL in Xilinx ISE 14.2 Design Suite. The performance of adders can again be enhanced by using high speed adder. The M^2DF architecture system was then modified by using high speed and an area efficient Han Carlson adder. Performance of both system design and proposed system was analyzed. Area and delay are taken as the parameters for performance analysis. Hardware is obtained to be efficient for proposed system by 0.4%.

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