

Effectiveness of Element Free Galerkin Method over FEM

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Abstract - Nowadays computer based numerical analysis are widely accepted to solve engineering problems. This reduces time as well as difficulties in solving Partial differential equations. Two commonly used mesh based methods are FDM and FEM. But these methods has several disadvantages like the process of grid generation was tedious and time consuming, difficult to analyse problems with complex geometries and surface etc. to solve these, a new method called meshfree methods were used over decades. Element free Galerkin method (EFGM) is one of the meshfree method. Here I am going to prove its effectiveness over FEM by validating the results with analytical solutions. A simple cantilever beam (Timoshenko beam) was selected to prove EFGM. ANSYS V.14 was used for Finite Element Analysis and EFGM was done in MATLAB platform.

Key Words: EFGM, FEM, Meshfree Methods, MATLAB, Numerical Method

1. INTRODUCTION

A mesh is an open space between the strands of a net that is formed by connecting nodes in a predefined manner [1]. The process of meshing is one of the important steps in FEM because which determines the accuracy of the response obtained from the analysis. The reduction in mesh size increases the accuracy but the time required for analysis was also increases. This is one of the major drawbacks of FEM. In order to solve this problem, meshless method was used. Here in this paper I am going to analyse an isotropic, homogenous cantilever beam under static loading condition using three methods namely FEM, EFGM and Analytical Method.

1.1 Element Free Galerkin Method

EFGM completely replaces the formulation of an element based shape function by a nodal based shape function. For that a Moving Least Square Method was used to develop the shape function from scattered nodal points.

1.2 Scope

The scope of this work is to study the response (displacements and stresses) of cantilever beam under static loading conditions using element free Galerkin method and

Finite element method. Further the results were compared with the analytical solution.

1.3 Objectives

The main objectives are;

1. To study the response of cantilever beam using element free Galerkin method.
2. To study the response using finite element method.
3. To compare the results by FEM and EFGM.
4. To validate the results with its analytical solution.
5. To find the percentage of error between these methods.

2. 2. LITERATURE REVIEW

A detailed study on mesh free methods is done by Liu (1) and it provides a systematic discussion on basic theories, fundamentals of mesh free methods especially mesh free weak form method and a wide range of applications of mesh free method in solid mechanics. In his work different mesh less method such as element free galerkin method, local petrov- Galerkin method, and point interpolation methods and their application in beams, plates and shells are briefly explained.

Pandey *et al* (2) explained the step by step procedure of implementation of Element Free Galerkin Method to beam problem. They analyzed a 2D beam and results are compared with its analytical solution by using Timoshenko beam theory. The algorithm of Element Free Galerkin Method is also developed in MATLAB platform and the results shows that EFG and analytical solutions are exactly same but there is slight variation in results obtained from FEM. According to them element free Galerkin method was the best choice, because it can be deals with complex and difficult problems (problem with deformable boundary, free surface, large deformation etc.) in a better way than finite element method.

Petterson (3) did a study on various types of element free Galerkin method with suitable examples. And found that when number of nodes reaches infinity results converges to

the analytical solution. He concluded that updated Lagrange with constant support domain and the shape functions will improve the analysis and suggest that element free Galerkin method with finite element method is a better choice in future.

Hamed and Idir (4) explains the theory of moving least square method in a detailed way. They prove that MLS method does not satisfy kronecker delta property and did the analysis on a cantilever beam and an infinite plate with central hole. The numerical results demonstrate that the EFG method is easy to implement, convergence and the computational accuracy are good for the displacements and stresses.

Hegen (5), studied about various weak forms of Element Free Galerkin Method. Here a detailed study on the combination of Element Free Galerkin Method and Finite Element Method was done because the EFGM is computationally expensive but require less time in case of FEM, which require more time and less cost. So a combination of these two methods was a better one and found that the rate of convergence is high in this case.

3. METHODOLOGY

The overview of the entire work is shown in Fig 1.

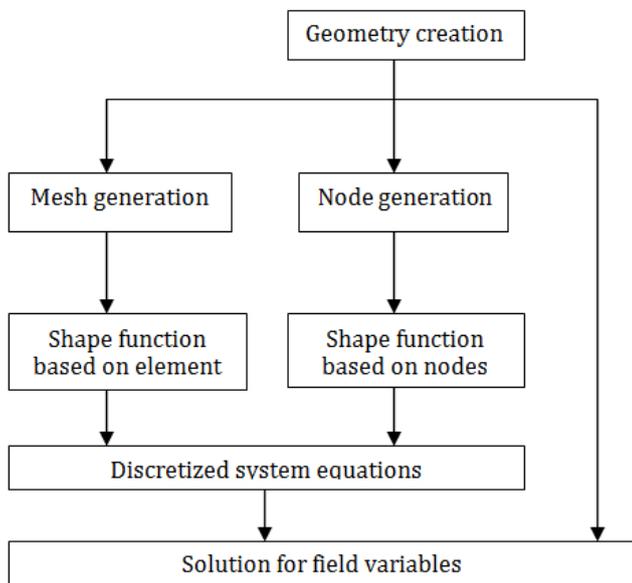


Fig -1: Over view of methodology

A cantilever beam of dimension 48x12x1 m was selected for analysis. Material properties such as young's modulus, $E=30\text{GPa}$ and poissons ratio, $\mu=0.3$ are the input datas. First of all an algorithm is developed in MATLAB to find out the response using EFGM and then the same was analysed in

ANSYS V.14. Finally the results were compared to analytical solution.

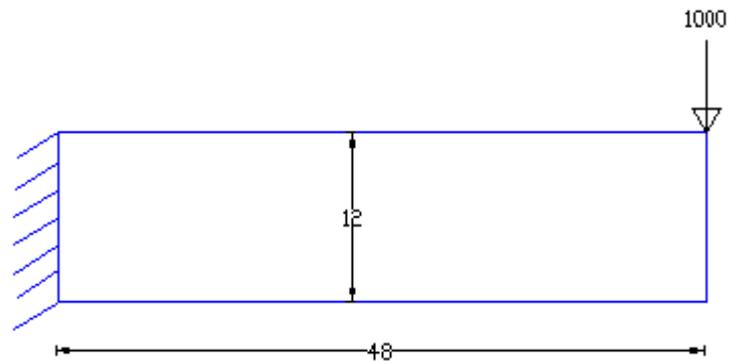


Fig -2: Timoshenko beam used for analysis

The exact solutions are found out by using analytical equations [2].

4. RESULTS AND DISCUSSIONS

4.1 EFGM

MATLAB is a high performance platform for doing engineering calculations especially matrix calculations. The name MATLAB is Derived from MATrix LABoratory. Here in MATLAB an algorithm for EFGM was developed and then analyzed to obtain the value of displacements and stresses at various nodal points. Here Moving Least Square Method (MLS) was used to construct the shape functions from the scattered data (nodal points). This thesis mainly concentrates on parameters such as displacements and stresses developed in the beam when subjected to a static loading condition.

The following table represents the deflection values of cantilever beam under the applied loading condition and the maximum deflection obtained at the free end of the beam is 0.0089 mm.

Table -1: Displacements from EFGM

Distance from fixed end (m)	Displacement in y direction (mm)
0	0
4	0.0001
8	0.0004
12	0.0009
16	0.0014

Distance from fixed end (m)	Displacement in y direction (mm)
20	0.0021
24	0.0029
28	0.0037
32	0.0047
36	0.0057
40	0.0067
44	0.0078
48	0.0089

Distance from fixed end (m)	Displacement in y direction (mm)
0	0
4	0.00020
8	0.00050
12	0.00089
16	0.00150
20	0.00220
24	0.00287
28	0.00384
32	0.00484
36	0.00579
40	0.00687
44	0.00786
48	0.008945

The stress variation diagram from MATLAB is shown below in Fig 2.

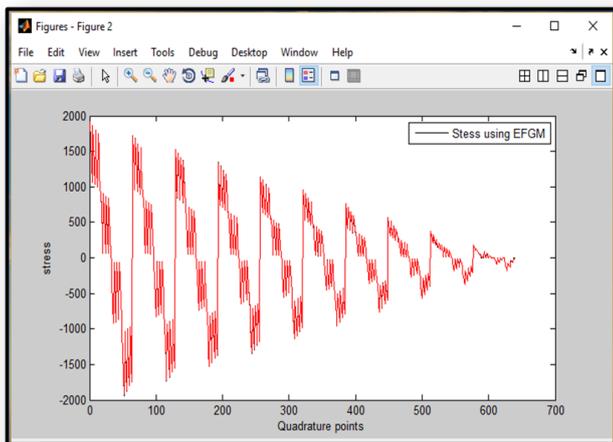


Fig -3: Stress Variation from MATLAB

From this diagram it's clear that the maximum value of stress is 2000N/m² and is at the support.

4.2 FEM

The results obtained from Finite element analysis were shown in Table 2, which is almost similar to the EFG Solution. Fig 2 and Fig 3 represents the deformed model and the displacement contour obtained from ANSYS Software respectively. The maximum displacement obtained from finite element analysis is 0.008945 mm at the free end.

From Table 1 and 2 we can see that the displacement values are almost same and have a difference of about 0.5%.



Fig -4: Deformed model

Table -2: Displacements from FEM

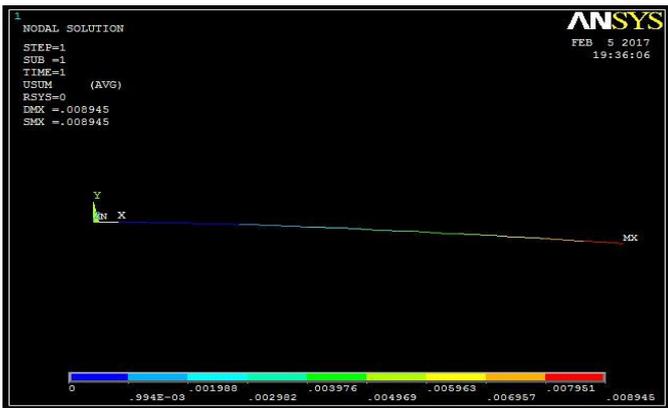


Fig -5: Displacement contour

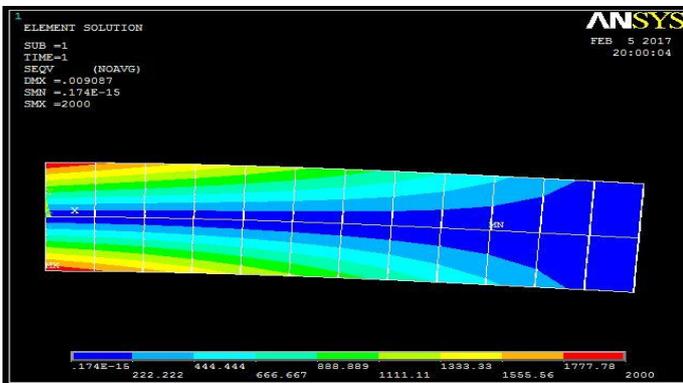


Fig -6: Stress contour

24	0.00285
28	0.00372
Distance from fixed end (m)	Displacement in y direction (mm)
32	0.00466
36	0.00567
40	0.00672
44	0.00780
48	0.00889

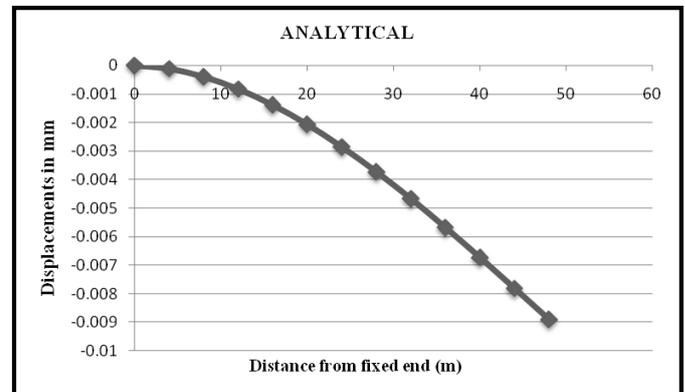


Chart -1: Displacement Variation

4.3 Analytical Method

The displacement corresponding to various points are listed in Table 3 and its variation is plotted in Fig 5. Maximum displacement obtained from analytical equation is 0.00889 mm.

Table -3: Displacements from analytical method

Distance from fixed end (m)	Displacement in y direction (mm)
0	0
4	0.00011
8	0.00039
12	0.00082
16	0.00138
20	0.00206

Stress varies in linear manner at extreme fibers and at the centre portion of the beam stress is zero because it is a transition zone between the tension phase and compression phase. Stress variation at the support in y direction (cross section at support) is shown in Table 4.

Table -4: Stress variation (σ_x) in y direction at support

Distance from centre (m)	σ_x in y direction (N/m ²)
6	-2000.00
4	-1333.33
2	-666.67
0	0
-2	666.67
-4	1333.33
-6	2000.00

4.4 Comparison of EFGM, FEM and Analytical Method

The results obtained from EFGM, FEM and Analytical solutions were investigated. From that it was clear that, in case of displacements the analytical and EFG solutions gives

almost same result whereas the FE solution was slightly varies from the exact solutions. The Comparison chart of the three methods is shown in Chart 1.

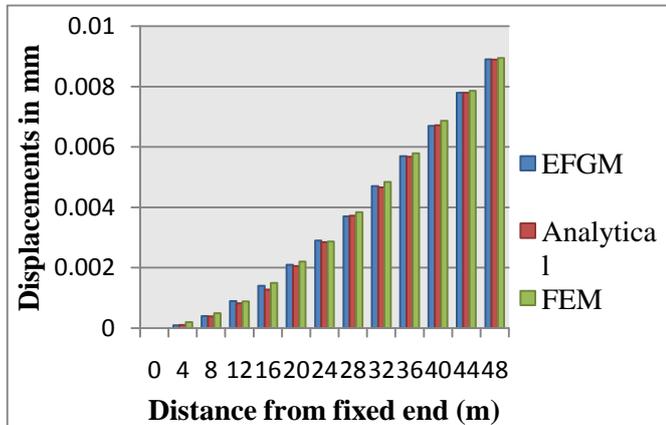


Chart -2: comparison chart between EFGM, FEM and Analytical Method

3. CONCLUSIONS

Important conclusions are drawn from the study are;

- EFGM was capable of analyzing structures and has a high convergence rate (more accurate) than other methods.
- Validation using analytical solution shows that the results incorporate with EFGM, but there was a slight variation in case of displacements.
- ANSYS also gives a better result but compared to other two methods it is less accurate.
- The percentage variation in error is 0.112 between EFGM and analytical solution whereas 0.618% between FEM and analytical solution.
- From all the three analysis, the stress values were same and is 2000 N/m^2
- EFGM is an effective method and is found to be superior to FEM.
- The computational time required for EFGM is less compared to FEM but the algorithm development was tedious and time consuming.

REFERENCES

- [1] G. R. Liu, Meshfree methods: moving beyond the finite element method, CRC Press, 2003.
- [2] S. S. Pandey, P. K. Kasundra and S. D. Daxini, Introduction of Meshfree Methods and Implementation of Element Free Galerkin (EFG) Method to Beam Problem, International Journal on Theoretical and Applied Research in Mechanical Engineering, Vol 2(3), pp.85-89, 2013.

- [3] V.Petersson, An Implementation of Mesh Free Methods for Mechanical Problems at Large Strains, 2007.
- [4] M. M. Hamed and B. Idir, applying element free Galerkin method on beam and plate, International Journal of Chemical, Molecular, Nuclear, Materials and Metallurgical Engineering, Vol 10, pp 627-635,2016.
- [5] D.Hegen, Element-free Galerkin methods in combination with finite element approaches, Computer Methods in Applied Mechanics and Engineering, 135, 143-166, 1995.