Fractal Dimension in Medical Imaging: A Review

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Abstract: The latest research works have revealed that fractal geometry, a measure of irregular contours, inconvenient for describing the various pathological architectures like tumors (for yielding insights into the tumor growth), different MR (magnetic-response) image analysis, neuro imaging analysis, bone fracture measurement and many more. Researchers major attraction towards fractal geometry lie in the accuracy and effectiveness of the medical images despite their randomness and roughness. Magnetic-resonance or MR images usually pose a degree of noise and randomness is associated with the natural nature of structure. Most of the image intensity based existing method is either profound to strong noise or not appropriate to diverse imaging modalities. Hence fractal analysis is suitable for MR image analysis and therefore significant research has been done on fractal geometry in various aspects of medical image rather than focusing on modern molecular methods. This article outlines the basic approaches of fractal geometry and debates the significance and application of fractal dimension (FD) in different medical image analysis.

Keywords: fractal geometry, fractal dimension, MR images

1. INTRODUCTION

Most of the human body organs, including tissues and brain, show complex geometric structural properties. Consequently, it becomes difficult to characterize using only the classical Euclidean geometry measures[29]. This complex geometry can be characterized by its shape properties with different scale values. Fractal geometry plays the foremost role when irregular surfaces come in computation. The term fractal means irregular and rough shapes, which was introduced by Mandelbrot in 1982. In fractal geometry, fractal dimension (FD) provides the quantifying way for the shape complexity of an object into a single numerical figure and it can be compared amongst groups of patients and even more among healthy controls and patients. Fractal deals with objects that represent self-similarity, which means magnifying an object into deeper details with varying scale and each individual portion is similar as the whole. FD can be defined as

$$D = -\frac{\log N_r}{\log r}$$

N_r represents number of boxes in the image and r is used for different scale levels. Several fractal dimension estimation methods are proposed by many researchers. Box counting method is one of the interesting and convenient approach towards FD calculation. Different approaches are discussed later.

The aim of this article is to offer a survey of these methods application in medical ground and to discuss the prime results. Again the research may offer assistance to researchers aiming to use fractal geometry in medical imaging applications.

We organized our paper as follow: in the section 2, introduction to fractals is elaborated; Section 3 discusses the FD methodologies and the importance of fractals in various image analyses. In section 4 we presented different application areas where fractal geometry is applied. Finally at the end we conclude our work with importance of FD in medical imaging and its future scope.

2. FRACTAL ANALYSIS

The term 'fractal' is defined as uneven geometric form that can be sub divided into smaller fragments, where each segment is a copy of the whole image[2]. This signifies that self-similarity is an important aspect of the fractal. There are various best known examples of fractals present like Cantor set, Sierpinski gasket, Koch curve etc. The essential characteristic for fractal objects is their measuring metric properties, like length or area, which are treated as function of scale of measurement. When the whole length of the Koch curve is measures, the result is subjected to the scale length. In Similar fashion the length of a coastline is measured by Mandelbrot in 1967. He measured with a spatial scale d and the whole length of coastline L(d) is projected as a set of N number of straight line slices with length d. This leads towards the characterization of fractal dimension, correlated to the number of self-similar sections. This can be seen when scaling down a larger object. More formally, Mandelbrot in 1983defined that fractal set is a set in which the Hausdorff dimensions greater than its topological dimension.

And again according to Mandelbrot, analytically, the relationship among the length (L) and measuring scale (d)can be represented as:

$$L(\sigma) = K * \sigma(1 - D)$$

Here K is treated as constant and Das the fractal dimension which is a fractional number. Numerous techniques exist to calculate this dimension and each method has its own theoretical basis. But all follow the same fundamental concepts briefed by the steps below:

- The object quantities are measured by various step sizes.
- log-log graph is plotted among measured quantities and step sizes and least-squares linear fit of the regression line through the data points is measured.
- The slope of the regression line is measured as fractal dimension (FD).

3. METHODS USED FOR FD ESTIMATION AND IMPORTANCE OF FRACTALS IN MEDICAL IMAGE ANALUSIS

3.1. Box-counting method

In 1980 Russel et al. has defined this method. It is one of the most commonly used and popular method. In this method the whole image is divided into number of boxes and the division is done with scale value r. according to this method the FD is determined as:

$$FD = -\lim_{r \to 0} \frac{\log(N(r))}{\log(r)} \quad \text{………….. (1)}$$

N(r) represents number of boxes required to cover the image completely. And r indicates the scale

3.2. Differential box-counting method

The extension of box-counting method is the differential box-counting method. It was coined by Chaudhuri and Sarkar in 1995to resolve some of restrictions of box count method. It has a great advantage that it works on grey images. So the binarization step can be avoided. The image is divided into boxes of different scale rand N(r) is calculated as the difference between the maximum and the minimum grey levels on the (i, j)th box. This step is used repeatedly for all boxes. FD is approximate by using in Eq. (1).

3.3. Peleg approach

According to Peleg et al. the tested image is treated as a hilly terrain and its height is proportional to the gray level of the image. Points with distance € from the surface on both sides create a blanket, whose thickness is 2€. The area of the blanket can be estimated as:

$$A(\varepsilon) = F_{\varepsilon}^{2-D} \quad \text{………………..(2)}$$

And therefore the FD can be calculated by using least square linear fit of the log-log plot of A(€) and €.

3.4. Pentland approach

Pentland applied Fourier power spectrum of image intensity surface to estimate fractal dimension. Fourier power spectrum P(f) of fractal Brownian function (f) is proportional to f -2h-1, and h = 2 – D. here D considered as the FD.

3.5. Reticular cell counting approach

Gangepain and Roques-Carmesusedan approximation method to evaluate fractal dimension. This method is well known as reticular cell counting method. According to his methodology, 3-D space is taken where two coordinates x and y represent 2-D position and the third coordinate, z represents image strength.

3.6. Keller's Approach

The modified form of reticular cell counting method is proposed by Keller et al. He used the probabilistic method where P(m, L) be the probability of m intensity points within a L sized box centered on an arbitrary point of image intensity surface [5].

After the concept of fractals being put forwarded by Mandelbrot [2] for describing and analyzing the complex objects, it became one of the prominent method to be applied to numerous scientific areas, including medical imaging [4]. As we have discussed Fractal dimension represents the information about the geometric structures for fractals. As the medical images are complex and irregular in nature and also poses some similarity for different scales, fractal geometry treated as one of the best that can be applied for analyzing various medical images. It also played significant role in several areas of health and medical research like distinguishing pathological tissues from healthy ones [5] and diagnosis verity of diseases [6]. Again it shows a very successful result in quantification of morphological variations in brain images [7]. The major motivation for choosing fractal analysis for medical imaging is because of its relatively insensitiveness towards the image scaling, and its strong correlation nature with human judgment of surface roughness is also a major concern [8].Another important factor that cannot be forgotten is its application towards the noisy medical images. It became powerful tool to characterize this type of medical images where the blurred effect is seen in the edges.

4. APPLICATIONS

Fractal analyses have been significantly applied to study and to describe an extensive range of biological images and medicines. In this section of our work we broadly reviewed different fields where fractal geometry plays a great role.

Fractal theory applied for medical imaging is an active area for research work. Indeed, medical images have a fractal character, particularly the case where repeated sequences, for example: homologies among two separate nucleotide sequences having a self-similar structure in mitochondrial DNA.

4.1. ECG signals

The fractal dimension has shown its capability on characterizing different states or to predict the changes in phenomenon. It is generally effectively used in ECG and EEG samples [9]. Basically box-counting method is applied in this purpose. Fractal analysis is very much useful for distinguishing the classes of EEG pattern, which is correlated to brain activities. Fractal interpolation can also be used for ECG data compression [10].Hsu et al. in 2007proposed a novel procedure known as electro encephalogram (EEG) analyzing system using the differential box counting method [11]. The FD has many advantages in studying EEG data; as it exhibits random fractal pattern. FD has proved its usefulness in various applications like epilepsy, sleeping disorder and Alzheimer disease. Several FD calculation methods are used but and, box-counting methods, provides acceptable results. In recent times, many researches on fractal spectra analysis and fractal dimension predicted the brain activities more accurately than Fourier transform, though fractal spectra analysis has restrictions in experimenting multiple brain activities and stationary brain waves.

4.2. Brain imaging

Brain image analysis is also one of the interesting areas where fractal dimension can be applied frequently. FDs were successfully used to measure cells morphologies in the brain [12]and the brain shape Magnetic resonance imaging or MRI is basically used for study. It needs special consideration because sometimes FD doesn't work properly to distinguish the patients among the healthy subjects. In the earlier, the box-counting method was used where in pre-processing, binarization was done. So some important information was removed automatically. Then there are limited fractal studies on non-linear structures observed by brain MRI, but this study is limited for only extraction of fractal structures like white matter surface, cortical or sub cortical edges. Nevertheless the FD proved to be a discriminating tool for some structures. FD values are used to identify and locate tumors in the brain [13]. Zook and Iftekharuddin in 2005 introduced an algorithm for detecting tumor in brain using 2DMR images. In this case FD was computed with the help off Bf variance model technique and the tumor progression information extracted with respect to the normal brain data. One example of tumor affected brain MRI with positive FD difference for several methods is given below.

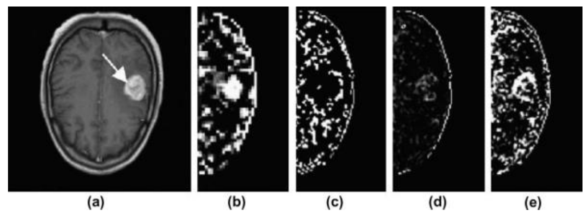

Fig. 1. (a) tumor affected MR image positive and FD difference results for (b) PMBC-32, (c) PMBC-64, (d) PTPSA and (e) the blanket method [13]

4.3. Bone trabeculation

Throughout the earlier decades, continuous researches have been done on the application of fractal analysis in bone trabeculation examination. Numerous applications were studied as the fractal character of the trabecular bone. The most important factors of bone strength are its complex structure. Bone has two elements one is outer part consisting of a tube of very dense material and second one is inside material that looks like a sea sponge that is filled with finely interlaced struts, called trabeculae. Many studies were done to authenticate the use of fractal analysis in this application because it has proved that bone study (trabecular bone micro architecture on calcaneus radiographs) poses self-similarity which is one of the characteristics of fractal geometry. Majumdar et al. in 1999 in his experiment [14], he proposed some important characteristics using fractal analysis. He used three approaches to calculate the FD: surface-area technique, semi-variance technique and Fourier transform technique. In the recent years, research has proved that bone trabeculation through fractal analysis is very dynamic nature [15]. Some experiments have proved that the bone organization is anisotropic in nature for osteoporotic patients than for healthy subjects. Jennane et al. in 2006 calculated the anisotropy degree of fractal for bone X-ray images and showed its complex nature in terms of their textured character, non-stationarity and anisotropy [16].

4.4. Mammography

Mammography is a tool which detects and diagnosis breast cancer. If the breast irregularities are identified at early stages then breast cancer can be fully cured. Presence of small mineral in the breast tissue is one of the most significant signs of breast cancer. This mineral is known as micro-calcifications. Other procedures like X-ray radiography, ultrasound and MR are also for this diagnosis, but micro-calcifications of small sizes, low contrast are still challenging for detection. Application of fractal geometry in detecting breast cancer has given interesting results. Many research works are done using the mammography images [17,18,19].Li et al. in 1996 showed another application of fractal study for micro-calcifications enhancement [20]. In his work, he explained that ductal patterns and general mammographic parenchymal can be modelled properly using fractal geometry. He compared the mammogram and the modelled one and concluded that the fractal modelling technique is an effective approach to enhance micro-calcifications and thereby facilitates the radiologist's diagnosis.

5. CONCLUSION

Like other research area, fractal geometry has provides its efficiency for segmentation and characterization in various application of medical imaging. The main reasons for the usability of fractal geometry in the analysis of medical imaging are its complex nature, discontinuity and fragmented nature. It is prominently used for characterizing healthy and affected states. But choosing the method for FD calculation is an important issue. In the most of the applications box count method is used whereas other method like Differential box Counting, Extended counting Method, Triangular prisms Method, Power spectrum can also be used and compared. One more constraint we can notice with fractal analysis is its particular fractal value but sometimes they show multi-fractal activities.

ACKNOWLEDGEMENT

The authors would like to extend their gratitude towards the department of CSE &IT, VSSUT Burla for extending their cooperation in this research.

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